Day 12 **A Tier 10-a-day: Model Solutions**

1 Exchange rates

£1 = €1.2.
(a) Change £80 to euros.
(b) Change €288 to pounds.

(a) 80 × 1.2 = 96
(b) 288 ÷ 1.2 = 240
Answer: (a) €96 (b) £240

2 Loci and constructions

XY is a straight line of length 9 cm.
A point P moves so that it is always less than 3 cm from the point X.
Describe the region where P can be.

Less than 3 cm from X means inside the circle
Centre X, radius 3 cm
Answer: Inside a circle with centre X and radius 3 cm

3 Profit and loss

Femi buys a watch for £200 and sells it for £270.
Work out Femi's percentage profit.

Profit = £270 − £200 = £70

Percentage profit = $\frac{70}{200} \times 100 = 35\%$

Answer: 35%

4 Direct and inverse proportion

y is directly proportional to the square of x.
When x = 4, y = 80.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 6.

(a) $y = kx^2$. Substitute x = 4 and y = 80: 80 = k × 16, so k = 5
$y = 5x^2$
(b) $y = 5 \times 6^2 = 5 \times 36 = 180$
Answer: (a) $y = 5x^2$ (b) 180

5 Terminating and recurring decimals

Here are four fractions: $\frac{5}{16}$ $\frac{5}{12}$ $\frac{11}{25}$ $\frac{6}{20}$

Which one of these fractions is equal to a recurring decimal?

Simplify each fraction: $\frac{5}{16}, \frac{5}{12}, \frac{11}{25}, \frac{6}{20} = \frac{3}{10}$

A fraction in simplest form is a terminating decimal only if its denominator has no prime factors other than 2 and 5.

$\frac{5}{12}$ has denominator 12 = 2 × 2 × 3, which includes a factor other than 2 and 5.

Answer: $\frac{5}{12}$

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6 Iteration

$x_{n+1} = \sqrt[3]{11 - x_n}$

$x_0 = 2$

Work out the values of x_1, x_2 and x_3.

Give your answers correct to 3 decimal places. (Calculator)

Substitute $x_0 = 2$ into the formula, then use each answer in the next step (keep the full calculator display)

$x_1 = 2.08008$

$x_2 = 2.0739$

$x_3 = 2.07438$

Answer: $x_1 = 2.080$, $x_2 = 2.074$, $x_3 = 2.074$

7 Standard form in context

The population of Region P is 3.75×10^6.

The population of Region Q is 1.5×10^5.

How many times bigger is the population of Region P than the population of Region Q?

Divide: $(3.75 \times 10^6) \div (1.5 \times 10^5)$

$= 3.75 \div 1.5 \times 10^1 = 25$

Answer: 25 times

8 Combined means

The mean mark of 8 boys in a test is 66.

The mean mark of 8 girls in the same test is 67.

Work out the mean mark of all 16 people.

Total for boys = $8 \times 66 = 528$

Total for girls = $8 \times 67 = 536$

Total = 1064, and number of students = 16

Mean = $1064 \div 16 = 66.5$

Answer: 66.5

9 Rearranging formulae (subject twice)

Make x the subject of $y = \frac{4x + 6}{x + 5}$

Multiply both sides by $(x + 5)$: $y(x + 5) = 4x + 6$

Expand the bracket: $xy + 5y = 4x + 6$

Collect the x terms on one side: $xy - 4x = -5y + 6$

Factorise: $x(y - 4) = -5y + 6$

Divide by $(y - 4)$: $x = \frac{-5y + 6}{y - 4}$

Answer: $x = \frac{-5y + 6}{y - 4}$

10 The cosine rule: finding a side

Work out the value of x. (Calculator)

Give your answer to 3 significant figures.

The diagram is not drawn accurately.

$x^2 = 12^2 + 13^2 - 2 \times 12 \times 13 \times \cos 32°$

$x^2 = 144 + 169 - 312 \cos 32°$

$x^2 = 48.409$

$x = 6.96$

Answer: x = 6.96 cm