Day 119 **A Tier 10-a-day: Model Solutions**

1 Angles in parallel lines

The diagram shows two parallel lines crossed by a straight line.
(a) Work out the value of x.
(b) Work out the size of the angle marked (5x + 10)°.
The diagram is not drawn accurately.

(a) Alternate angles are equal
5x + 10 = 4x + 29
x = 19
x = 19
(b) 5 × 19 + 10 = 105
Answer: (a) x = 19 (b) 105°

2 LCM in context

On a stage, a lighthouse flashes every 15 seconds.
A blue beacon flashes every 12 seconds.
They flash together at the start.
After how many seconds will they next flash together?

Find the LCM of 15 and 12.
15 = 3 × 5 and 12 = 2^2 × 3
LCM = 60
They flash together again after 60 seconds.
Answer: 60 seconds

3 Equations with fractions

Solve $\frac{x-4}{4} + \frac{x+8}{3} = 4$

Multiply every term by 12 (the lowest common multiple of 4 and 3)
3(x − 4) + 4(x + 8) = 12 × 4
7x + 20 = 48
7x = 28
x = 4
Answer: x = 4

4 Fractions of an amount

Aisha spends $\frac{1}{4}$ of the money on a phone case and $\frac{2}{5}$ of the money that is left on headphones.
Aisha has £81 left.
How much money did Aisha have to begin with?

After the second purchase $\frac{3}{5}$ of what was left after the first purchase remains.

£81 = $\frac{3}{5}$ of the amount left, so $\frac{1}{5}$ of it is £81 ÷ 3 = £27
So the amount left after the first purchase is £27 × 5 = £135
This is $\frac{3}{4}$ of the original amount, so the original amount is £135 ÷ 3 × 4 = £180
Answer: £180

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5 Capture-recapture

A scientist wants to estimate the number of frogs in a pond.
She catches 80 frogs, marks each one and returns them.
Later she catches a second sample of 50 frogs. 10 of these are marked.
Work out an estimate for the total number of frogs in a pond.

Proportion marked in the second sample = $\frac{10}{50}$

This is the same as the proportion in the whole population: $\frac{80}{N}$

$\frac{80}{N} = \frac{10}{50}$, so N = 80 × 50 ÷ 10 = 400

Answer: 400

6 Transformations of graphs

The graph of y = f(x) has a maximum point at (−1, −6).
Write down the coordinates of the turning point of the graph of
(a) y = f(x − 3)
(b) y = f(x) − 2

(a) Replacing x by (x − 3) moves the graph 3 units right
The maximum point is (2, −6)
(b) Subtracting 2 from f(x) moves the graph 2 units down
The maximum point is (−1, −8)
Answer: (a) (2, −6) (b) (−1, −8)

7 Enlargements and coordinates

Triangle ABC has vertices A(1, 4), B(4, −8), C(−2, 4).
The triangle is enlarged with scale factor $\frac{1}{3}$ and centre (1, −2).
Write down the coordinates of the vertices of the image.

Each vertex moves to: centre + scale factor × (vertex − centre)
A: (1, 4) → (1, 0)
B: (4, −8) → (2, −4)
C: (−2, 4) → (0, 0)
Answer: A'(1, 0), B'(2, −4), C'(0, 0)

8 Inverse proportion formulae

y is inversely proportional to $\sqrt{x}$.
When x = 4, y = 5.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 25.

(a) $y = \frac{k}{\sqrt{x}}$. Substitute: $5 = \frac{k}{2}$, so k = 5 × 2 = 10

$y = \frac{10}{\sqrt{x}}$

(b) $y = \frac{10}{5} = 2$

Answer: (a) $\mathbf{y = \frac{10}{\sqrt{x}}}$ **(b) 2**

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9 Recurring decimals, show that

(a) Show that $0.\dot{6}\dot{0} = \frac{20}{33}$

(b) Hence write $5.\dot{6}\dot{0}$ as a mixed number in its simplest form.

(a) Let x = 0.606060..., so 100x = 60.6060...

100x − x = 60, so 99x = 60 and $x = \frac{60}{99} = \frac{20}{33}$

(b) $5.\dot{6}\dot{0} = 5 + \frac{20}{33} = 5\frac{20}{33}$

Answer: (b) $\mathbf{5\frac{20}{33}}$

10 Frequency density

The table gives information about 47 measurements.
A histogram is drawn using this information.
(a) Work out the missing frequency.
(b) Work out the height of the bar for the class $5 < l \leq 35$.

(a) Missing frequency = 47 − (12 + 14 + 11) = 10
(b) Class width = 35 − 5 = 30
Frequency density = 12 ÷ 30 = 0.4
Answer: (a) 10 (b) 0.4