Day 117 **A Tier 10-a-day: Model Solutions**

1 Forming and solving equations

A rectangle has length (2x + 4) cm and width (x + 5) cm.
The perimeter of the rectangle is 84 cm.
(a) Work out the value of x.
(b) Work out the length of the rectangle.

(a) Perimeter = 2 × (length + width), so 2(2x + 4 + x + 5) = 84
2x + 4 + x + 5 = 42 (half of the perimeter)
3x + 9 = 42
3x = 33
x = 11
(b) Length = 2 × 11 + 4 = 26 cm
Answer: (a) x = 11 (b) 26 cm

2 Best buys

A shop sells flour in 2 different sizes.
Small bag: 500 g for £2.05
Large bag: 1000 g for £3.40
Which bag is the best value for money? You must show your working.

Small bag: 205p ÷ 5 = 41p per 100 g
Large bag: 340p ÷ 10 = 34p per 100 g
The large bag has the lowest price per 100 g, so it is the best value
Answer: The large bag

3 Expanding double brackets

Expand and simplify
(3x − 3)(x − 7)

Multiply every term in the first bracket by every term in the second
$= 3x^2 - 21x - 3x + 21$
$= 3x^2 - 24x + 21$
Answer: $3x^2 - 24x + 21$

4 Tree diagrams

The tree diagram shows probabilities for two events.
(a) Work out the values of p and q.
(b) Work out the probability of rain and late.

(a) Probabilities on pairs of branches add up to 1.
p = 1 − 0.2 = 0.8 and q = 1 − 0.6 = 0.4
(b) 0.2 × 0.6 = 0.12
Answer: (a) p = 0.8, q = 0.4 (b) 0.12

5 Standard form in context

One bacterium has a mass of 3×10^{-6} g.
A sample contains 8×10^8 bacteria.
Work out the total mass of the sample.
Give your answer in standard form.

Total mass = $(3 \times 10^{-6}) \times (8 \times 10^8)$
$3 \times 8 = 24$ and $10^{-6} \times 10^8 = 10^2$
$24 \times 10^2 = 2.4 \times 10^3$
Answer: 2.4×10^3 g

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6 Combining ratios

p : q = 8 : 1
q : r = 3 : 4
(a) Work out the ratio p : q : r
(b) p, q and r add up to 186. Work out the value of r.

(a) Make the q parts equal: lowest common multiple of 1 and 3 is 3
p : q = 24 : 3 and q : r = 3 : 4
p : q : r = 24 : 3 : 4
(b) 24 + 3 + 4 = 31 parts, so 1 part = 186 ÷ 31 = 6
r = 4 × 6 = 24
Answer: (a) 24 : 3 : 4 (b) 24

7 Laws of indices

Simplify fully
(a) $\frac{10x^7y^3}{4x^2y^6}$
(b) $(2x^3y^4)^4$

(a) Divide the numbers: $10 \div 4 = \frac{5}{2}$
Subtract the powers: $x^7 \div x^2 = x^5$ and $y^3 \div y^6 = y^{-3}$
$= \frac{5x^5}{2y^3}$
(b) Raise every part to the power 4: $2^4 = 16$, $(x^3)^4 = x^{12}$, $(y^4)^4 = y^{16}$
$= 16x^{12}y^{16}$
Answer: (a) $\frac{5x^5}{2y^3}$ (b) $16x^{12}y^{16}$

8 Circle theorems: cyclic quadrilaterals

A, B, C and D are points on a circle.
Angle B = (4x − 10)° and angle D = (x + 25)°.
(a) Work out the value of x.
(b) Work out the size of angle B.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
4x − 10 + x + 25 = 180
5x + 15 = 180
5x = 165
x = 33
(b) 4 × 33 − 10 = 122
Answer: (a) x = 33 (b) 122°

9 Primes

(a) Write down all the prime numbers between 20 and 34.
(b) Write 91 as the product of two prime numbers.

(a) A prime number has exactly two factors. Checking each number between 20 and 34 gives: 23, 29, 31
(b) 91 ÷ 7 = 13, and both 7 and 13 are prime.
Answer: (a) 23, 29, 31 (b) 7 × 13

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10 The cosine rule: finding an angle

Work out the size of angle x. (Calculator)
Give your answer to 1 decimal place.
The diagram is not drawn accurately.

$$\cos x = \frac{7^2 + 6^2 - 12^2}{2 \times 7 \times 6}$$

$$\cos x = \frac{-59}{84}$$

$x = \cos^{-1}(-0.7024)$

$x = 134.6°$

Answer: x = 134.6°