Day 115 **A Tier 10-a-day: Model Solutions**

1 Direct proportion

Here are the ingredients for 6 cupcakes.
Sugar: 150 g
Flour: 180 g
Milk: 300 ml
(a) Work out how much sugar is needed for 36 cupcakes.
(b) Hannah has 601 g of flour. What is the greatest number of cupcakes she can make?

(a) sugar for one: 150 ÷ 6 = 25 g
For 36: 25 × 36 = 900 g
(b) flour for one: 180 ÷ 6 = 30 g
601 ÷ 30 = 20 remainder 1, so she can make 20
Answer: (a) 900 g (b) 20

2 nth term of a linear sequence

Here are the first four terms of a sequence.
−2, −9, −16, −23
(a) Find an expression for the nth term.
(b) Is −120 a term of this sequence? You must show your working.

The terms change by −7 each time, so the nth term starts with −7n
−2 − (−7) = 5, so the nth term is −7n + 5
(b) −7n + 5 = −120
−7n = −125, so n = $\frac{125}{7}$
n is not a positive whole number, so −120 is not a term of the sequence
Answer: (a) −7n + 5 (b) No

3 HCF and LCM

(a) Write 25 and 55 as products of prime factors.
(b) Find the highest common factor (HCF) and the lowest common multiple (LCM) of 25 and 55.

(a) $25 = 5^2$ and $55 = 5 \times 11$
(b) HCF: multiply the prime factors in both lists, taking the smaller power: 5
LCM: take the larger power of every prime: $5^2 \times 11 = 275$
Answer: (a) $25 = 5^2$, $55 = 5 \times 11$ (b) HCF = 5, LCM = 275

4 Is it a right-angled triangle?

A triangle has sides of length 25 cm, 7 cm and 24 cm.
Is the triangle right-angled?
You must show your working.

Longest side^2 = 25^2 = 625
Sum of the squares of the other two sides = $7^2 + 24^2 = 49 + 576 = 625$
625 = 625, so the triangle is right-angled
Answer: Yes, it is right-angled

5 Factorising quadratics

Factorise $x^2 + 7x + 12$

Find two numbers that multiply to 12 and add to 7
The numbers are 3 and 4
(x + 3)(x + 4)
Answer: (x + 3)(x + 4)

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6 Counting

A cafe menu has 2 starters, 3 main courses and 2 desserts.
(a) How many different three-course meals are possible?
(b) A customer chooses a main course and either a starter or a dessert (not both). How many different meals are possible?

(a) 2 × 3 × 2 = 12
(b) 3 choices of main course × (2 + 2) choices of starter or dessert = 3 × 4 = 12
Answer: (a) 12 (b) 12

7 Circle theorems: tangents

PA and PB are tangents to a circle with centre O.
Angle APB = 66°.
Work out the size of angle x (angle AOB).
Give a reason for each step of your working.

A tangent is perpendicular to the radius, so angles OAP and OBP are both 90°
The angles in quadrilateral OAPB add up to 360°
x = 360 − 90 − 90 − 66 = 114
Answer: x = 114°

8 Mixing in a ratio

Jug A contains 28 litres of a mixture of concentrate and water in the ratio 6 : 1.
Jug B contains 9 litres of a mixture of concentrate and water in the ratio 2 : 1.
The contents of both jugs are mixed together.
Work out the ratio of concentrate to water in the mixture. Give your answer in its simplest form.

Jug A: concentrate = 6 × 28 ÷ 7 = 24 litres, water = 4 litres
Jug B: concentrate = 6 litres, water = 3 litres
Total: concentrate = 30 litres, water = 7 litres
Ratio = 30 : 7 = 30 : 7
Answer: 30 : 7

9 Trigonometry in context

A boat is at sea. (Calculator)
From the top of a vertical cliff 60 m high, the angle of depression of the boat is 25°.
Work out the horizontal distance of the boat from the foot of the cliff. Give your answer to 3 significant figures.

The angle of depression equals the angle of elevation from the boat

$\tan 25° = \frac{60}{d}$

d = 60 ÷ tan 25°
= 129
Answer: 129 m

10 Percentages of percentages

75% of a number is 225. (Calculator)
Work out 90% of the same number.

75% = 225, so 1% = 225 ÷ 75 = 3
90% = 3 × 90 = 270
Answer: 270