Day 112 **A Tier 10-a-day: Model Solutions**

1 Area of a sector

The diagram shows a sector of a circle of radius 10 cm.
Work out the area of the sector.
Give your answer in terms of π.
The diagram is not drawn accurately.

Area = $\frac{135}{360} \times \pi \times 10^2$

$= \frac{3}{8} \times 100\,\pi$

$= \frac{75}{2}\pi$

Answer: $\mathbf{\frac{75}{2}\pi\ cm^2}$

2 Straight line graphs

A straight line has equation $-5x + 2y = -15$
(a) Find the gradient of the line.
(b) Find the coordinates of the point where the line crosses the x-axis.

(a) Rearrange into y = mx + c: $2y = 5x - 15$

$y = \frac{5}{2}x - \frac{15}{2}$

The gradient is $\frac{5}{2}$

(b) On the x-axis, y = 0: $-5x = -15$, so x = 3

Answer: (a) $\mathbf{\frac{5}{2}}$ **(b) (3, 0)**

3 Isosceles triangles and exterior angles

PQR is an isosceles triangle with PQ = PR.
QRS is a straight line.
Angle QPR = 50°.
Work out the size of angle x.
The diagram is not drawn accurately.

Base angles of an isosceles triangle are equal
Angle PQR = (180 − 50) ÷ 2 = 65°
Exterior angle of a triangle = sum of the two opposite interior angles
x = 50 + 65 = 115
Answer: x = 115°

4 Laws of indices

Simplify fully
(a) $\frac{18x^6y^2}{5x^5y^6}$
(b) $\frac{(3x^3)^2}{3x^4}$

(a) Divide the numbers: $18 \div 5 = \frac{18}{5}$

Subtract the powers: $x^6 \div x^5 = x$ and $y^2 \div y^6 = y^{-4}$

$= \frac{18x}{5y^4}$

(b) Square the top: $(3x^3)^2 = 9x^6$
Divide the numbers: 9 ÷ 3 = 3. Subtract the powers of x: 6 − 4 = 2
$= 3x^2$

Answer: (a) $\mathbf{\frac{18x}{5y^4}}$ **(b)** $\mathbf{3x^2}$

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5 Standard form calculations

Work out $(3 \times 10^8) \div (2.5 \times 10^{-4})$
Give your answer in standard form.

Divide the numbers and subtract the powers of 10: $3 \div 2.5 = 1.2$ and $10^8 \div 10^{-4} = 10^{12}$
1.2×10^{12}
Answer: 1.2×10^{12}

6 Quadratic sequences

Here are the first five terms of a quadratic sequence.
9, 19, 31, 45, 61
(a) Find an expression for the nth term.
(b) Work out the 10th term.

First differences: 10, 12, 14
Second differences: 2, 2, so $a = 2 \div 2 = 1$
Subtract n^2 from each term (1, 4, 9, 16): 8, 15, 22, 29
This is linear: $7n + 1$
nth term = $n^2 + 7n + 1$
(b) Substitute $n = 10$: $1 \times 10^2 + 7 \times 10 + 1 = 171$
Answer: (a) $n^2 + 7n + 1$ (b) 171

7 Recurring decimals, show that

(a) Show that $0.\dot{8}\dot{4} = \frac{28}{33}$

(b) Hence write $2.\dot{8}\dot{4}$ as a mixed number in its simplest form.

(a) Let $x = 0.848484...$, so $100x = 84.8484...$
$100x - x = 84$, so $99x = 84$ and $x = \frac{84}{99} = \frac{28}{33}$

(b) $2.\dot{8}\dot{4} = 2 + \frac{28}{33} = 2\frac{28}{33}$

Answer: (b) $2\frac{28}{33}$

8 Mixing in a ratio

Jug A contains 11 litres of a mixture of red paint and white paint in the ratio 4 : 7.
Jug B contains 21 litres of a mixture of red paint and white paint in the ratio 6 : 1.
The contents of both jugs are mixed together.
Work out the ratio of red paint to white paint in the mixture. Give your answer in its simplest form.

Jug A: red paint = $4 \times 11 \div 11 = 4$ litres, white paint = 7 litres
Jug B: red paint = 18 litres, white paint = 3 litres
Total: red paint = 22 litres, white paint = 10 litres
Ratio = 22 : 10 = 11 : 5
Answer: 11 : 5

9 Rationalising with conjugates

Rationalise the denominator of $\frac{100}{6 + \sqrt{11}}$

Give your answer in the form $a + b\sqrt{c}$ where a and b are integers.

Multiply top and bottom by the conjugate $6 - \sqrt{11}$: $\frac{100(6 - \sqrt{11})}{(6 + \sqrt{11})(6 - \sqrt{11})}$

Denominator: $36 - 11 = 25$
$\frac{100(6 - \sqrt{11})}{25} = 4(6 - \sqrt{11})$
$= 24 - 4\sqrt{11}$
Answer: $24 - 4\sqrt{11}$

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10 Frequency density

The table gives information about 135 measurements.
A histogram is drawn using this information.
(a) Work out the missing frequency.
(b) Work out the height of the bar for the class $45 < h \leq 75$.

(a) Missing frequency = 135 − (36 + 54 + 12) = 33
(b) Class width = 75 − 45 = 30
Frequency density = 54 ÷ 30 = 1.8
Answer: (a) 33 (b) 1.8