Day 109 **A Tier 10-a-day: Model Solutions**

1 Independent events

The probability that Femi hits the target is 0.4.
The probability that Amy hits the target is 0.2.
The two events are independent.
Work out the probability that at least one of them will hit the target.

P(neither) = 0.6 × 0.8 = 0.48
P(at least one) = 1 − 0.48 = 0.52
Answer: 0.52

2 Surface area of a cuboid

A cuboid has length 12 cm and width 2 cm.
Its total surface area is 272 cm^2.
Work out the height of the cuboid.

2 × (12 × 2 + 12 h + 2 h) = 272
24 + 14h = 136
14h = 112
h = 112 ÷ 14 = 8
Answer: 8 cm

3 Expanding and simplifying

Expand and simplify
$(x - 4)^2 - (x + 1)(x - 5)$

$(x - 4)^2 = x^2 - 8x + 16$
$(x + 1)(x - 5) = x^2 - 4x - 5$
Subtract (take care with the signs): $(x^2 - 8x + 16) - (x^2 - 4x - 5)$
$= -4x + 21$
Answer: $-4x + 21$

4 LCM in context

Bus A leaves a bus station every 15 minutes.
Bus B leaves the same station every 12 minutes.
Both buses leave together at 6:20 am.
At what time will they next leave together?

We need the LCM of 15 and 12.
Multiples of 15: 15, 30, 45, 60
Multiples of 12: 12, 24, 36, 48, 60
LCM = 60, so they leave together again 60 minutes later, at 7:20 am
Answer: 7:20 am

5 Finding a missing value from an average

The mean of the five numbers x, 19, 15, 6, 12 is 13.
Work out the value of x.

Total = 5 × 13 = 65
Known numbers add to 52
x = 65 − 52 = 13
Answer: x = 13

6 Perimeter and area with algebra

A rectangle has length (2x + 2) cm and width (x + 4) cm.
The perimeter of the rectangle is 66 cm.
(a) Work out the value of x.
(b) Work out the area of the rectangle.

(a) $2 \times [(2x + 2) + (x + 4)] = 66$
$(2x + 2) + (x + 4) = 33$
$3x + 6 = 33$
$3x = 27$
$x = 9$
(b) Length = 20 cm and width = 13 cm
Area = $20 \times 13 = 260$
Answer: (a) x = 9 (b) 260 cm^2

7 Circles and tangents

The point P(3, −4) lies on a circle with centre O(0, 0).
(a) Work out the equation of the circle, in the form $x^2 + y^2 = r^2$.
(b) Find the equation of the tangent to the circle at P.

(a) The circle is $x^2 + y^2 = r^2$, so $r^2 = 3^2 + (-4)^2 = 25$

(b) Gradient of the radius OP = $\frac{-4}{3}$

The tangent is perpendicular to the radius, so its gradient = $-1 \div (-\frac{4}{3}) = \frac{3}{4}$

$y = \frac{3}{4}x + c$ through (3, −4): $-4 = \frac{3}{4} \times 3 + c$, so $c = -\frac{25}{4}$

$y = \frac{3}{4}x - \frac{25}{4}$

Answer: (a) $x^2 + y^2 = 25$ (b) $y = \frac{3}{4}x - \frac{25}{4}$

8 Expanding with surds

Expand and simplify $(4 + 2\sqrt{6})^2$
Give your answer in the form $a + b\sqrt{c}$

$(4 + 2\sqrt{6})(4 + 2\sqrt{6}) = 16 + 8\sqrt{6} + 8\sqrt{6} + 24$
Collect: $40 + 16\sqrt{6}$
Answer: $40 + 16\sqrt{6}$

9 Rationalising with conjugates

Rationalise the denominator of $\frac{11}{4 + \sqrt{5}}$
Give your answer in the form $a + b\sqrt{c}$ where a and b are integers.

Multiply top and bottom by the conjugate $4 - \sqrt{5}$: $\frac{11(4 - \sqrt{5})}{(4 + \sqrt{5})(4 - \sqrt{5})}$
Denominator: $16 - 5 = 11$
$\frac{11(4 - \sqrt{5})}{11} = 1(4 - \sqrt{5})$
$= 4 - \sqrt{5}$
Answer: $4 - \sqrt{5}$

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10 Simultaneous equations (one quadratic)

Solve the simultaneous equations

$x^2 + y^2 = 25$

$y = x + 5$

Substitute $y = x + 5$ into $x^2 + y^2 = 25$

$x^2 + (x + 5)^2 = 25$

$2x^2 + 10x = 0$

Divide by 2: $x^2 + 5x = 0$

$(x + 5)(x + 0) = 0$

$x = -5$ or $x = 0$

Use $y = x + 5$ to find y: when $x = -5$, $y = 0$; when $x = 0$, $y = 5$

Answer: $x = -5$, $y = 0$ and $x = 0$, $y = 5$