Day 107 **A Tier 10-a-day: Model Solutions**

1 Inverse proportion

3 painters take 30 days to paint a hall.
All the painters work at the same rate.
How long would it take 6 painters to paint a hall?

Total work = 3 × 30 = 90 painter-day
Time for 6 painters = 90 ÷ 6 = 15 days
Answer: 15 days

2 Isosceles triangles and exterior angles

ABC is an isosceles triangle with AB = AC.
BCD is a straight line.
Angle ACD = 126°.
Work out the size of angle x.
The diagram is not drawn accurately.

Angles on a straight line add up to 180°
Angle ACB = 180 − 126 = 54°
Base angles are equal, so angle ABC = 54°
x = 180 − 54 − 54 = 72
Answer: x = 72°

3 Functions

$f(x) = 4x - 3$
$g(x) = x^2 - 2$
(a) Find f(5)
(b) Find fg(x). Give your answer in its simplest form.

(a) $f(5) = 4 \times 5 - 3 = 17$
(b) fg(x) means f(g(x)), so replace x in f with $x^2 - 2$
$fg(x) = 4(x^2 - 2) - 3$
$= 4x^2 - 11$
Answer: (a) 17 (b) $4x^2 - 11$

4 Two-way tables

The table shows information about 52 people.
One person is chosen at random.
(a) Find the probability that the person chose chicken.
(b) The person is a woman. Find the probability that they chose pasta.

(a) Chicken total = 18 out of 52 = $\frac{9}{26}$

(b) Women total = 25, of whom 18 are in the Pasta column: $\frac{18}{25}$

Answer: (a) $\frac{9}{26}$ (b) $\frac{18}{25}$

5 Factorising

Factorise fully $5x^2 - 45$

Take out the common factor 5: $5x^2 - 45 = 5(x^2 - 9)$
$x^2 - 9$ is a difference of two squares: $(x - 3)(x + 3)$
$5(x - 3)(x + 3)$
Answer: $5(x - 3)(x + 3)$

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6 Circle theorems: cyclic quadrilaterals

A, B, C and D are points on a circle.
Angle B = (5x + 25)° and angle D = (3x + 51)°.
(a) Work out the value of x.
(b) Work out the size of angle B.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
5x + 25 + 3x + 51 = 180
8x + 76 = 180
8x = 104
x = 13
(b) 5 × 13 + 25 = 90
Answer: (a) x = 13 (b) 90°

7 Recurring decimals to fractions

Write the recurring decimal $0.6\dot{5}$ as a fraction in its simplest form.

Let x = 0.656565...
Multiply by 100: 100x = 65.6565...
Subtract: 99x = 65
$x = \frac{65}{99}$

Answer: $\mathbf{\frac{65}{99}}$

8 Composite functions

f(x) = 3x − 7
g(x) = x^2 − 1
Solve gf(x) = 8

gf(x) = $(3x - 7)^2 - 1$
$(3x - 7)^2 - 1 = 8$, so $(3x - 7)^2 = 9$
3x − 7 = ±3
3x − 7 = 3 or 3x − 7 = −3
$x = \frac{10}{3}$ or $x = \frac{4}{3}$

Answer: $\mathbf{x = \frac{10}{3}}$ **or** $\mathbf{x = \frac{4}{3}}$

9 Bounds

A rectangle has length 4.8 m and width 10.1 m. (Calculator)
Each measurement is correct to the nearest 0.1 m.
Work out the lower bound for the area of the rectangle.

Lower bound of length = 4.8 − 0.05 = 4.75 m
Lower bound of width = 10.1 − 0.05 = 10.05 m
Lower bound of area = 4.75 × 10.05 = 47.7375 m^2
Answer: 47.7375 m^2

10 Pythagoras in three dimensions

The diagram shows a cuboid.
Work out the length of the diagonal AG.
The diagram is not drawn accurately.

$AG^2 = 4^2 + 7^2 + 4^2$
= 16 + 49 + 16 = 81
AG = $\sqrt{81}$ = 9
Answer: 9 cm