Day 105 **A Tier 10-a-day: Model Solutions**

1 Isosceles triangles and exterior angles

ABC is an isosceles triangle with AB = AC.
BCD is a straight line.
Angle BAC = 108°.
Work out the size of angle x.
The diagram is not drawn accurately.

Base angles of an isosceles triangle are equal
Angle ABC = (180 − 108) ÷ 2 = 36°
Exterior angle of a triangle = sum of the two opposite interior angles
x = 108 + 36 = 144
Answer: x = 144°

2 Independent events

The probability that Ravi wins a game is 0.9.
The probability that Omar wins a game is 0.9.
The two events are independent.
Work out the probability that at least one of them will win a game.

P(neither) = 0.1 × 0.1 = 0.01
P(at least one) = 1 − 0.01 = 0.99
Answer: 0.99

3 HCF and LCM in index form

A = $2^4 \times 5$
B = $2^4 \times 3 \times 5^4$
Find the highest common factor (HCF) and the lowest common multiple (LCM) of A and B.
Give your answers as products of prime factors.

HCF: for each prime take the lower power (a missing prime has power 0): $2^4 \times 5$
LCM: for each prime take the higher power: $2^4 \times 3 \times 5^4$
Answer: HCF = $2^4 \times 5$, LCM = $2^4 \times 3 \times 5^4$

4 Percentages in context

A sofa costs £500 plus VAT at 20%.
In a sale, the price including VAT is reduced by 10%.
Work out the sale price of the sofa.

VAT = 20% of £500 = £100, so the price with VAT is £600
Reduction = 10% of £600 = £60
Sale price = £600 − £60 = £540
Answer: £540

5 Income tax

No tax is paid on the first £12,000 of income. Tax of 20% is paid on the next £35,000. Tax of 40% is paid on all income above £47,000. (Calculator)
Mo earns £69,500 a year.
Work out the income tax paid.

Next £35,000 at 20%: £35,000 × 0.2 = £7,000
Above £47,000: £69,500 − £47,000 = £22,500, taxed at 40%: £22,500 × 0.4 = £9,000
Total tax = £7,000 + £9,000 = £16,000
Answer: £16,000

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6 Expanding and simplifying

Expand and simplify
$4(x - 2)(x + 6) - (x + 6)(x - 6)$

$4(x - 2)(x + 6) = 4(x^2 + 4x - 12) = 4x^2 + 16x - 48$
$(x + 6)(x - 6) = x^2 - 36$
Subtract: $3x^2 + 16x - 12$
Answer: $3x^2 + 16x - 12$

7 Loci: area of a region

A goat is tied by a rope of length 4 m to the corner of a square field of side 8 m.
The goat can graze inside the field only.
Work out the area it can graze. Give your answer in terms of π.
The diagram is not drawn accurately.

The rope can sweep a quarter circle (90°)
Area = $\frac{1}{4} \times \pi \times 4^2$
$= 4\pi$
Answer: 4π m^2

8 Perpendicular bisector

A is the point (0, 1) and B is the point (2, 9).
Find the equation of the perpendicular bisector of AB.

Midpoint of AB = $(\frac{0 + 2}{2}, \frac{1 + 9}{2}) = (1, 5)$
Gradient of AB = $\frac{8}{2} = 4$
Perpendicular gradient = $-1 \div 4 = -\frac{1}{4}$
$y = -\frac{1}{4}x + c$ through (1, 5): $5 = -\frac{1}{4} \times 1 + c$, so $c = \frac{21}{4}$
$y = -\frac{1}{4}x + \frac{21}{4}$
Answer: $y = -\frac{1}{4}x + \frac{21}{4}$

9 Bounds in measurement

The length and width of a rectangular poster are 16.4 cm and 9.4 cm.
Each measurement is correct to 1 decimal place.
Work out the lower bound for the perimeter.

Lower bounds: length = 16.4 cm and width = 9.3 cm
Lower bound of perimeter = 2 × (16.35 + 9.35) = 51.4
Answer: 51.4 cm

10 Growth and decay

A new car is worth £8000. (Calculator)
Its value falls by 15% each year.
Work out its value after 2 years.

Multiplier = 100% − 15% = 0.85
$8000 \times 0.85^2 = 5780.00$
= £5780 to the nearest whole number
Answer: £5780