Day 104 **A Tier 10-a-day: Model Solutions**

1 Rounding

(a) Write 93494 correct to 2 significant figures.
(b) Write 5.8035 correct to 1 decimal place.

(a) Count 2 significant figures and look at the next digit: 93494 rounds to 93000
(b) Look at the digit after decimal place 1: 5.8035 rounds to 5.8
Answer: (a) 93000 (b) 5.8

2 Two dice

Two fair dice are thrown.
The two scores are added together.
Work out the probability that the sum is 7.

Outcomes with a sum that is 7: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)
There are 6 of these out of 36 equally likely outcomes
$\frac{6}{36} = \frac{1}{6}$

Answer: $\frac{1}{6}$

3 nth term of a linear sequence

Here are the first four terms of a sequence.
9, 14, 19, 24
(a) Find an expression for the nth term.
(b) Is 104 a term of this sequence? You must show your working.

The terms go up by 5 each time, so the nth term starts with 5n
9 − 5 = 4, so the nth term is 5n + 4
(b) 5n + 4 = 104
5n = 100, so n = 20
n is a whole number, so 104 is a term of the sequence
Answer: (a) 5n + 4 (b) Yes (n = 20)

4 Rates of flow

A tank is in the shape of a cuboid measuring 120 cm by 50 cm by 40 cm.
It already contains 100 litres of water.
Water flows in at 5 litres per minute until the tank is full.
Work out how long this takes. Give your answer in minutes.

Volume = 120 × 50 × 40 = 240000 cm^3 = 240 litres
Water needed = 240 − 100 = 140 litres
Time = 140 ÷ 5 = 28 minutes
Answer: 28 minutes

5 Terminating and recurring decimals

Here are four fractions: $\frac{7}{12}$ $\frac{27}{60}$ $\frac{8}{10}$ $\frac{21}{24}$

Which one of these fractions is equal to a recurring decimal?

Simplify each fraction: $\frac{7}{12}$, $\frac{27}{60} = \frac{9}{20}$, $\frac{8}{10} = \frac{4}{5}$, $\frac{21}{24} = \frac{7}{8}$

A fraction in simplest form is a terminating decimal only if its denominator has no prime factors other than 2 and 5.

$\frac{7}{12}$ has denominator 12 = 2 × 2 × 3, which includes a factor other than 2 and 5.

Answer: $\frac{7}{12}$

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6 Similar triangles with parallel lines

In triangle ABC, DE is parallel to BC.
D is on AB and E is on AC.
Work out the value of x.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar (DE is parallel to BC)

AE ÷ AC = AD ÷ AB, so AC = $6 \times \frac{5}{3} = 10$

x = EC = 10 − 6 = 4

Answer: x = 4

7 Direct and inverse proportion

y is inversely proportional to the square of x.
When x = 1, y = 36.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 6.

(a) $y = \frac{k}{x^2}$. Substitute x = 1 and y = 36: $36 = \frac{k}{1}$, so $k = 36 \times 1 = 36$

$y = \frac{36}{x^2}$

(b) $y = \frac{36}{36} = 1$

Answer: (a) $\mathbf{y = \frac{36}{x^2}}$ **(b) 1**

8 Bearings and distances

A plane leaves an airport and travels 9 km on a bearing of 070°. (Calculator)
It then travels 12 km on a bearing of 160°.
(a) Work out how far it is now from the airport.
(b) Work out the bearing of its final position from the airport. Give your answer to the nearest degree.

The two bearings differ by 90°, so the path makes a right angle

(a) $\text{Distance}^2 = 9^2 + 12^2 = 225$, so distance = 15 km

(b) The angle at the port between the first leg and the line to the end: $\tan^{-1}(\frac{12}{9}) = 53.1°$

Bearing = 70° + 53.1° = 123.1°
Bearing = 123°

Answer: (a) 15 km (b) 123°

9 Rationalising with conjugates

Rationalise the denominator of $\frac{8}{3+\sqrt{7}}$

Give your answer in the form $a + b\sqrt{c}$ where a and b are integers.

Multiply top and bottom by the conjugate $3 - \sqrt{7}$: $\frac{8(3-\sqrt{7})}{(3+\sqrt{7})(3-\sqrt{7})}$

Denominator: 9 − 7 = 2

$\frac{8(3-\sqrt{7})}{2} = 4(3-\sqrt{7})$

$= 12 - 4\sqrt{7}$

Answer: $\mathbf{12 - 4\sqrt{7}}$

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10 Probability with three selections

A bag contains 5 red counters and 2 blue counters.
Three counters are taken at random without replacement.
Work out the probability that exactly one of the three counters is blue.

One order, for example blue, red, red: $\frac{2}{7} \times \frac{5}{6} \times \frac{4}{5} = \frac{4}{21}$

There are 3 possible orders, each with the same probability

$3 \times \frac{4}{21} = \frac{4}{7}$

Answer: $\mathbf{\frac{4}{7}}$