Day 10 **A Tier 10-a-day: Model Solutions**

1 Composite functions

f(x) = 4x − 4

$g(x) = x^2 + 4$

Solve gf(x) = 53

$gf(x) = (4x - 4)^2 + 4$

$(4x - 4)^2 + 4 = 53$, so $(4x - 4)^2 = 49$

$4x - 4 = \pm 7$

$4x - 4 = 7$ or $4x - 4 = -7$

$x = \frac{11}{4}$ or $x = -\frac{3}{4}$

Answer: $x = \frac{11}{4}$ or $x = -\frac{3}{4}$

2 Best buys

A shop sells flour in 2 different sizes.

Small bag: 400 g for £3.35

Large bag: 1000 g for £9.20

Which bag is the best value for money? You must show your working.

Small bag: 335p ÷ 4 = 83.75p per 100 g

Large bag: 920p ÷ 10 = 92p per 100 g

The small bag has the lowest price per 100 g, so it is the best value

Answer: The small bag

3 Ordering numbers

Write these numbers in order of size, starting with the smallest.

0.58 $\frac{8}{10}$ 28% $\frac{9}{10}$

Change each to a decimal: 0.58 = 0.58, $\frac{8}{10}$ = 0.8, 28% = 0.28, $\frac{9}{10}$ = 0.9

Order the decimals: 0.28 < 0.58 < 0.8 < 0.9

Answer: 28%, 0.58, $\frac{8}{10}$, $\frac{9}{10}$

4 Straight line graphs

A straight line has equation 3x − 3y = 9

(a) Find the gradient of the line.

(b) Find the coordinates of the point where the line crosses the x-axis.

(a) Rearrange into y = mx + c: −3y = −3x + 9

y = x − 3

The gradient is 1

(b) On the x-axis, y = 0: 3x = 9, so x = 3

Answer: (a) 1 (b) (3, 0)

5 Similar shapes: finding a length

The two triangles are similar.

Work out the value of x.

The diagrams are not drawn accurately.

Scale factor from the large triangle to the small triangle = 15 ÷ 20 = $\frac{3}{4}$

$x = 28 \times \frac{3}{4} = 21$

Answer: x = 21 cm

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6 Recurring decimals to fractions

Write the recurring decimal $0.\dot{1}\dot{9}$ as a fraction in its simplest form.

Let x = 0.191919...
Multiply by 100: 100x = 19.1919...
Subtract: 99x = 19
$x = \frac{19}{99}$

Answer: $\frac{19}{99}$

7 Algebraic rearrangement and change of sign

(a) Show that the equation $x^3 + 3x - 18 = 0$ can be rearranged to give $x = \frac{18 - x^3}{3}$

(b) Show that the equation has a solution between x = 2 and x = 3

(a) Add 18 and subtract 3x from both sides: $3x = 18 - x^3$

Divide both sides by 3: $x = \frac{18 - x^3}{3}$

(b) Let $f(x) = x^3 + 3x - 18$
f(2) = −4
f(3) = 18
There is a change of sign (negative to positive) and f is continuous, so there is a solution between x = 2 and x = 3

Answer: Shown: f(2) = −4, f(3) = 18 (change of sign)

8 Circle area and circumference

The area of a circle is 100π cm^2.
(a) Work out the radius of the circle.
(b) Work out the circumference. Give your answer in terms of π.

(a) $\pi r^2 = 100\pi$, so $r^2 = 100$
r = 10
(b) Circumference = $2\pi r = 2 \times \pi \times 10 = 20\pi$

Answer: (a) 10 cm (b) 20π cm

9 Combined means

The mean of 6 numbers is 70.
Another number is added. The mean of the 7 numbers is now 66.
Work out the number that was added.

Total of the first 6 numbers = 6 × 70 = 420
Total of the 7 numbers = 7 × 66 = 462
Number added = 462 − 420 = 42

Answer: 42

10 Standard form in context

One bacterium has a mass of 4×10^{-3} g.
A sample contains 3×10^8 bacteria.
Work out the total mass of the sample.
Give your answer in standard form.

Total mass = $(4 \times 10^{-3}) \times (3 \times 10^8)$
$4 \times 3 = 12$ and $10^{-3} \times 10^8 = 10^5$
$12 \times 10^5 = 1.2 \times 10^6$

Answer: 1.2×10^6 g