OXFORD
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INTERNATIONAL A-LEVEL MATHEMATICS

(9660)

Mark scheme

Pure mathematics Unit 2

Specimen

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Principal Examiners have prepared these mark schemes for specimen papers. These mark schemes have not, therefore, been through the normal process of standardising that would take place for live papers.

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

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No method shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

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Q	Answer	Marks	Comments
1(a)	$\left(\frac{dy}{dx}=\right)\ x^3 \times \frac{1}{x} + 3x^2 \ln x$	B2	B1 $px^3 \times \frac{1}{x} + qx^2 \ln x$ where p and q are integers $p = 1$, $q = 3$
1(b)(i)	$\left(\frac{dy}{dx}=\right)\ e^2 + 3e^2 \ln e \quad (= 4e^2)$	M1	Substituting e for x in their $\frac{dy}{dx}$, but must have scored B1 in (a)
	$y = e^3 \ln e\ (= e^3)$	B1	oe but must have evaluated In e (twice) for this mark (must be in exact form, but condone numerical evaluation after correct equation)
	$y - e^3 = 4e^2(x - e)$	A1	
1(b)(ii)	$-e^3 = 4e^2(x - e)$ or $4e^2x = 3e^3$	M1	oe correctly substituting $y = 0$ into a correct tangent equation in (b)(i)
	$x = \frac{3}{4}e$	A1	CSO; ignore subsequent decimal evaluation
	Total	**7**	

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Q	Answer	Marks	Comments
2(a)(i)	**Alternative method 1**		
	$5x - 6 = A(x - 3) + Bx$	M1	Multiply by denominator and use two values of x
	$x = 0 \quad x = 3$ $A = 2 \quad B = 3$	A1	Set up and solve simultaneous equations for values of A and B
	Alternative method 2		
	$-6 = -3A \quad 5 = A + B$	M1	
	$A = 2 \quad B = 3$	A1	
2(a)(ii)	$\left(\int \frac{2}{x} + \frac{3}{x-3}\,dx = \right) 2 \ln x$	B1ft	Their $A \ln x$
	$+ 3 \ln(x - 3) (+ C)$	B1ft	Their $B \ln (x - 3)$ and no other terms; condone $B \ln x - 3$
2(b)(i)	**Alternative method 1**		
	$2x + 1 \overline{)\,4x^3 + 5x - 2}$ $2x^2 - x + 3$ $4x^3 + 2x^2$ $-2x^2 + 5x$ $-2x^2 - x$ $6x - 2$ $6x + 3$ -5	M1	Division as far as $2x^2 + px + q$ with, $p \neq 0$, $q \neq 0$ PI
	$p = -1$	A1	PI by $2x^2 - x + q$ seen
	$q = 3$	A1	PI by $2x^2 - x + 3$ seen and must state $p = -1$, $q = 3$,
	$r = -5$	A1	$r = -5$ explicitly or write out full correct RHS expression

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Q	Answer	Marks	Comments
2(b)(i)	**Alternative method 2**		
	$4x^3 + 5x - 2 =$ $4x^3 + (2 + 2p)\,x^2 + (p + 2q)\,x + q$ $2 + 2p = 0$ $p + 2q = 5$ $q + r = -2$	M1	Clear attempt to equate coefficients, PI by $p = -1$
	$p = -1$	A1	
	$q = 3$	A1	
	$r = -5$	A1	
2(b)(i)	**Alternative method 3**		
	$4x^3 + 5x - 2 = (2x + 1)(2x^2 + px + q) + r$ $x = -\frac{1}{2}$ $\quad 4 \times \left(-\frac{1}{2}\right)^3 + 5\left(-\frac{1}{2}\right) + 2 = r$	M1	$x = -\frac{1}{2}$ used to find a value for r
	$r = -5$	A1	
	$p = -1$	A1	
	$q = 3$	A1	
2(b)(ii)	$\left(\frac{4x^3 + 5x - 2}{2x + 1} =\right) 2x^2 + px + q + \frac{r}{2x + 1}$	M1	ft on p and q
	$\frac{2}{3}x^3 - \frac{1}{2}x^2 + 3x + k\ln(2x + 1)$ (+ C)	A1ft	
	$\frac{2}{3}x^3 - \frac{1}{2}x^2 + 3x + \frac{5}{2}\ln(2x + 1)$ (+ C)	A1	CSO
	Total	**11**	

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Q	Answer	Marks	Comments
3(a)	$R = \sqrt{10}$	B1	Accept 3.2 or better Can be earned in (b)
	tan $\alpha = -3$	M1	oe; M0 if tan $\alpha = -3$ seen
	$\alpha = 71.6°$ or better	A1	$\alpha = 71.56505$
3(b)	$\sin(x \pm \alpha) = \frac{-2}{R}$	M1	or their R and/or their α ; PI
	x (= –39.2 + 71.6) = 32(.333) or x – 71.6 = 219.2	A1 m1	32 or better condone 32.4 must see 219 and 72 or better PI by 291 or better as answer condone extra solutions
	$x = 291°$	A1	condone 290.8 or better CSO withhold final A1 if more than two answers given within interval
	Total	**7**	

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Q	Answer	Marks	Comments
4(a)	f(x) ⩾ 0	M1	f(x) > 0, f ≥ 0, $x \geq 0$, $y > 0$, range ≥ 0
		A1	condone $y \geq 0$
4(b)(i)	$fg(x) = \sqrt{2\left(\frac{10}{x}\right) - 5}$ $\left(= \sqrt{\frac{20}{x} - 5}\right)$ OE	B1	No ISW
4(b)(ii)	$\sqrt{\frac{20}{x} - 5} = 5$ $\frac{20}{x} = 5^2 + 5$	M1	correctly squaring their $fg(x)$ correctly isolating their x term
	$x = \frac{2}{3}$	A1	No ISW
4(c)(i)	$y = \sqrt{2x - 5}$	M1	Swap x and y — Either order
		M1	Correct squaring — Either order
	$(f^{-1}(x) =) \frac{x^2 + 5}{2}$	A1	
4(c)(ii)	$x^2 = 9$ or if $\sqrt{9}$ or 3 seen	M1	Candidate must have scored full marks in (c)(i)(ie no follow through)
	$x = 3$ and $x = -3$ rejected	A1	must see both
	Total	**10**	

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Q	Answer	Marks	Comments
5(a)(i)	$(1-x)^{\frac{1}{3}} = 1 - \frac{1}{3}x$	M1	condone $1^{\frac{1}{3}} + -\frac{1}{3}x$ for M1
	$= 1 - \frac{1}{3}x - \frac{1}{9}x^2$	A1	must simplify coefficients including signs
5(a)(ii)	**Alternative method 1**		
	$(125 - 27x)^{\frac{1}{3}} = 125^{\frac{1}{3}}\left(1 - \frac{27}{125}x\right)^{\frac{1}{3}}$ $\left(1 - \frac{27}{125}x\right)^{\frac{1}{3}} = \left(1 - \frac{1}{3} \times \frac{27}{125}x - \frac{1}{9}\left(\frac{27}{125}x\right)^2\right)$	M1	May have 5 instead of $125^{\frac{1}{3}}$ Attempt to replace x by $\pm\frac{27}{125}x$ condone missing brackets, or start binomial again.
	$= 5 - \frac{9}{25} - \frac{81}{3125}x^2$	A2	Condone $5 + \frac{-9}{25}x + \frac{-81}{3125}x^2$
	Alternative method 2 using $(a + bx)^n$		
	$(125 - 27x)^{\frac{1}{3}} = 125^{\frac{1}{3}} + \frac{1}{3} \times 125^{-\frac{2}{3}} \times (-27x)$ $+ \frac{1}{3}\left(-\frac{2}{3}\right)\frac{1}{2} \times 125^{-\frac{5}{3}} \times (-27x)$	(M1)	Allow one error; condone missing brackets
	$= 5 - \frac{9}{25} - \frac{81}{3125}x^2$	(A2)	
5(b)	$x = \frac{2}{9}$ **used** in answer to (a)(ii)	M1	
	$\sqrt[3]{119} \approx 5 - \frac{9}{25} \times \frac{2}{9} - \frac{81}{3125}\left(\frac{2}{9}\right)^2$ $= 4.91872$	A1	Condone $\frac{6}{27}$ or $x = 0.222$ or better This answer only and must follow from correct expansion
	Total	**7**	

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Q	Answer	Marks	Comments
6(a)(i)	$(\sin^{-1} \pm 0.25 =) \pm 14.5$	M1	PI by sight of 194.5 etc condone 14.4
	$\theta = 194.5°, 345.5°$	A1	no extras in interval, ignore answers outside interval
6(a)(ii)	$2\cot^2(2x + 30) = 2 - 7\text{cosec}(2x + 30)$ $2(\text{cosec}^2(2x + 30) - 1) = 2 - 7\text{cosec}(2x + 30)$	M1	condone replacing $2x + 30$ by Y correct use of $\text{cosec}^2 Y = 1 + \cot^2 Y$
	$2\text{cosec}^2(2x + 30) + 7\text{cosec}(2x + 30) - 4(= 0)$	A1	must be in this form
	$2\text{cosec}(2x + 30) \pm 1)(\text{cosec}(2x + 30) \pm 4)(= 0)$	M1	
	$\text{cosec}(2x + 30) = \frac{1}{2}$ or -4	A1	attempt at factorisation must be in line using f $(2x + 30)$
	$2x + 30 = 194.5, 345.5$	B1	one correct answer, allow 82.3, ignore extra solutions
	$x = 82.2°, 157.8°$	B1	CAO both answers correct and no extras in interval, ignore answers outside interval

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Q	Answer	Marks	Comments
6(b)	**Alternative method 1**		
	stretch (I)	M1	I and either II or III
	scale factor $\frac{1}{2}$ (II)	A1	
	parallel to x-axis (III)	E1	I + II + III
	translate $\begin{bmatrix} -15 \\ 0 \end{bmatrix}$	B1	Condone '15 to left' or '–15 in x (direction)'
	Alternative method 2		
	translate $\begin{bmatrix} -30 \\ 0 \end{bmatrix}$	(E1)	
	stretch (I)	(B1)	
	scale factor $\frac{1}{2}$ (II)	(M1)	as above
	parallel to x-axis (III)	(A1)	as above
	Total	**12**	

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Q	Answer	Marks	Comments
7(a)	$\int xe^{6x}dx$	M1	All 4 terms in this form, $k = \frac{1}{6}$, 1 or 6
	$u = x \quad \frac{dv}{(dx)} = e^{6x}$ $\frac{du}{(dx)} = 1 \quad v = ke^{6x}$	A1	
	$\frac{1}{6}xe^{6x} - \int \frac{1}{6}e^{6x}(dx)$	A1ft	Correct substitution of their terms into parts
	$= \frac{1}{6}xe^{6x} - \frac{1}{36}e^{6x}(+c)$	A1	oe No ISW for incorrect simplification
7(b)	$(V =) \pi\int_0^1 xe^{6x}\,dx$	B1	Must include π, limits and dx
	$= (\pi)\left[\left(\frac{1}{6}e^6 - \frac{1}{36}e^6\right) - \left(-\frac{1}{36}\right)\right]$	M1	Correct substitution of 0 and 1 into their answer in (a), must be of the form $axe^{6x} - be^{6x}$, where $a > 0$, $b > 0$ and F(I) - F(0) seen
	$= (\pi)\left[\frac{5}{36}e^6 + \frac{1}{36}\right]$	A1	CAO; ISW
	Total	**7**	

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Q	Answer	Marks	Comments
8(a)(i)	$\left(\frac{dx}{d\theta}=\right) -6\sin 2\theta$, $\left(\frac{dx}{d\theta}=\right) -2\sin\theta$	M1	$\left(\frac{dx}{d\theta}=\right) p\sin 2\theta$ or $r\sin\theta\cos\theta$ $\left(\frac{dx}{d\theta}=\right) q\sin\theta$
		A1	Both correct
	$\frac{dy}{dx} = \frac{-2\sin\theta}{-6\sin 2\theta}$	M1	Use chain rule $\frac{dy}{d\theta}$; $\frac{dx}{d\theta}$ condone one slip
	$= \frac{2\sin\theta}{6\times 2\sin\theta\cos\theta} = \frac{1}{6\cos\theta}$	A1	k = 6 must come from correct working seen **AG**
8(a)(ii)	$\theta = \frac{\pi}{3}$ $\quad m_T = \frac{1}{3}$	B1ft	ft on k $\left(\frac{1}{k\times\frac{1}{2}}\right)$ k need not be numerical
	$m_N = -3$	B1ft	ft on m_T
	$(x, y) = \left(-\frac{3}{2}, 1\right)$	B1	
	Normal $y - 1 = -3\left(x + \frac{3}{2}\right)$	B1	CAO; any correct form, ISW $2y + 6x + 7 = 0$

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Q	Answer	Marks	Comments
8(b)	**Alternative method 1**		
	$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$	M1 A1	$p + q\cos 2x$; allow different letters for x or mixture eg θ even for A1 and the following A1ft
	$\int p\,dx = px \qquad \int q\cos 2x = \frac{1}{2}q\sin 2x$ $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 \mathrm{d}x = \left[\frac{x}{2} - \frac{1}{4}\right]\sin 2x$	A1ft	Both integrals correct ft on p and q
	$= \left(\frac{\pi}{8} - \frac{1}{4}\right) - \left(-\frac{\pi}{8} - \left(-\frac{1}{4}\right)\right)$	m1	Correct use of limits $F\left(\frac{\pi}{4}\right) - F\left(-\frac{\pi}{4}\right)$ or $2F\left(\frac{\pi}{4}\right)$ $F(x) = px + r\sin 2x$ and $\sin\frac{\pi}{2}$ $\sin\left(-\frac{\pi}{2}\right)$ must be evaluated correctly for m1
	$= \frac{\pi}{4} - \frac{1}{2}$	A1	CSO oe ISW
	Alternative method 2		
	$\int \sin^2 x\,\mathrm{d}x = -\sin x\cos x - \int -\cos x\cos x\,\mathrm{d}x$	M1	Use parts; condone sign slips
	$= -\sin x\cos x + \int 1 - \sin^2 x\,\mathrm{d}x$	m1	Use $\cos^2 x = 1\ \sin^2 x$
	$2\int \sin^2 \mathrm{d}x = -\sin x\cos x + x$	A1	
	$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 \mathrm{d}x = G\left(\frac{\pi}{4}\right) - G\left(-\frac{\pi}{4}\right)$	m1	Correct use of limits
	$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 \mathrm{d}x = \frac{\pi}{4} - \frac{1}{2}$	A1	
	Total	**13**	

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Q	Answer	Marks	Comments
9	$9x^2 - 6xy + 4y^2 = 3$ $18x = 0$	B1	= 0 PI
	$-6y - 6x\frac{dy}{dx}$	B1	or $\frac{d(6xy)}{dx} = 6y + 6x\frac{dy}{dx}$ seen separately
	$+ 8y\frac{dy}{dx}$	B1	$\frac{dy}{dx}(-6x + 8y) = 6y - 18x$
	Use $\frac{dy}{dx} = 0$	M1	
	$\Rightarrow y = 3x$ or $x = \frac{y}{3}$	A1	CSO
	$y = 3x \Rightarrow 9x^2 - 6x \times 3x + 4(3x)^2 = 3$ $27x^2 = 3 \Rightarrow x = \pm\frac{1}{3}$ OE	m1 A1ft	Substitute $y = ax$ into equation and solve for a value of x or y. Condone missing brackets. oe Both values of x or y required. ft on their $y = 3x$
	$\left(\frac{1}{3}, 1\right)$ $\left(-\frac{1}{3}, -1\right)$	A1	CSO Correct corresponding simplified values of x and y Withhold A1 for any additional answers given
	Total	**8**	

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Q	Answer	Marks	Comments
10(a)(i)	$1000 \times 1.03^5 \approx$ (£)1160	B1	condone missing £ sign; 1160 only
10(a)(ii)	$2000 < 1000\left(1+\frac{3}{100}\right)^n$	B1	condone '=' or '<' used throughout take logs, any base, of their initial expression **correctly**
	$\ln 2 < n \ln 1.03$	M1	
	$(n > 23.499\ldots)$ $(N =)$ 24	A1	condone 23
10(b)	$1000 \times \left(1+\frac{3}{100}\right)^n > 1500 \times \left(1+\frac{1.5}{100}\right)^n$	B1	condone use of T for n condone '=' or '<' used throughout
	$\ln 1000 + n \ln 1.03 > \ln 1500 + n \ln 1.015$	M1	take logs, any base, of their initial expression **correctly**
	$n > \frac{\ln(1.5)}{\ln\left(\frac{1.03}{1.015}\right)}$	A1	correct expression for n or T
	$(n > 27.63\ldots)$ $(T =)$ 28	A1	condone 27
	Total	**8**	

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Q	Answer	Marks	Comments
11(a)	$\overrightarrow{AB} = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix} - \begin{bmatrix} 5 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 5 \end{bmatrix}$	B1	$\pm\left(\overrightarrow{OA} - \overrightarrow{OB}\right)$ Coordinate form only is B0 Condone one component incorrect
	Line through A and B	M1	$\overrightarrow{OA} + \lambda\mathbf{d}$ or $\overrightarrow{OB} + \lambda\mathbf{d}$ where $\mathbf{d} = \overrightarrow{AB}$ or $\overrightarrow{BA}$ all in components and identified
	$\mathbf{r} = \begin{bmatrix} 5 \\ 1 \\ -2 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ -2 \\ 5 \end{bmatrix}$ or $\mathbf{r} = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ -2 \\ 5 \end{bmatrix}$	A1	oe **r** or $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ required
11(b)(i)	$5 - \lambda = -8 + 5\mu$ $1 - 2\lambda = 5$ $-2 + 5\lambda = -6 - 2\mu$	M1	Clear attempt to set up and solve at least two simultaneous equations in μ and a different parameter Allow in column vector form
	$\lambda = -2$ $\mu = 3$	A1	One of λ **or** μ correct oe
	$-2 + 5 \times -2 = -12$ $-6 - 2 \times 3 = -12$ Both equal -12 so intersect	E1	Verify intersect, λ **and** μ correct or verify (7, 5, -12) is on both lines **statement required**
	P is (7, 5, -12)	B1	CAO

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Q	Answer	Marks	Comments
11(b)(ii)	$\overrightarrow{BC} = \begin{bmatrix} -8+5\mu \\ 5 \\ -6-2\mu \end{bmatrix} - \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$	M1	$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$ or $\overrightarrow{CB} = \overrightarrow{OB} - OC$
	$\begin{bmatrix} 3 \\ 6 \\ -15 \end{bmatrix} \bullet \overrightarrow{BC} = 0$ $-36 + 15\mu + 36 + 135 + 30\mu = 0$	M1 m1	Clear attempt at $\pm \overrightarrow{BP}$ or $\pm \overrightarrow{AB}$ or $\pm \overrightarrow{AP}$ in components sp with $\overrightarrow{BC} = 0$ Linear equation in μ using their $\overrightarrow{BC}$ and solved for μ Condone one arithmetical or sign slip
	$\mu = -3$	A1	
	C is (–23, 5, 0)	A1	CSO
	Total	**12**	

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Q	Answer	Marks	Comments
12(a)	$\frac{\mathrm{d}h}{\mathrm{d}t}$	B1	Use of $2 - h$
	derivative = $*$ × $(2 - h)$	M1	Is a constant or expression in h and / or t All correct; must be $(2 - h)$
	$\frac{\mathrm{d}h}{\mathrm{d}t} = k(2 - h)$	A1	
12(b)(i)	**Alternative method 1**		
	$\int x\sqrt{2x-1}\,\mathrm{d}x = \int \frac{1}{15}\,\mathrm{d}t$	B1	Correct separation and notation; condone missing integral signs
	$= \frac{1}{15}t$	B1	
	Substitute $u = 2x - 1$	M1	Suitable substitution and attempt to write integral in terms of u including $\mathrm{d}x$ replaced by $\frac{1}{2}$ or $2\,\mathrm{d}u$
	$\int x\sqrt{2x-1}\,\mathrm{d}x = \int \frac{1}{2}(u+1)\sqrt{u}\,\frac{1}{2}\,\mathrm{d}u$	A1	$\frac{1}{4}$ need not be seen
	$= \left(\frac{1}{4}\right)\int u^{\frac{3}{2}} + u^{\frac{1}{2}}$	A1	
	$= \frac{1}{4}\left(\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}\right)$ $(+C)$ $x = 1$, $t = 0$	M1	Integration correct including $\frac{1}{4}$
	$u = 1$, $t = 0$ $\frac{1}{4}\left(\frac{2}{5} + \frac{2}{3}\right) + C = 0$	A1	Use $x = 1$, $t = 0$ to find a value for constant C from equation in x and t
	$C = -\frac{4}{15}$ $t = \frac{1}{2}\left(3(2x-1)^{\frac{5}{2}} + 5(2x-1)^{\frac{3}{2}}\right) - 4$	A1	$C = -0.2666\ldots$ $C = -0.267$ or better ISW $t = (2x-1)^{\frac{3}{2}}(3x+1) - 4$

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Q	Answer	Marks	Comments
12(b)(i)	**Alternative method 2**		
	$\int x\sqrt{2x-1}\,\mathrm{d}x = \int \frac{1}{15}\,\mathrm{d}t$	B1	
	$= \frac{1}{5}t$	B1	
	$u = x,\ \frac{\mathrm{d}y}{\mathrm{d}x} (2x-1)^{\frac{1}{2}}$ $\mathrm{d}u = 1 \quad v = k(2x-1)^{\frac{3}{2}}$	M1	
	$\int x\sqrt{2x-1}\,\mathrm{d}x = x\frac{1}{3}(2x-1)^{\frac{3}{2}} - \int \frac{1}{3}(2x-1)^{\frac{3}{2}}\,\mathrm{d}x$	A1	Attempts to use parts
	$= x\frac{1}{3}(2x-1)^{\frac{3}{2}} - \frac{1}{15}(2x-1)^{\frac{5}{2}} \quad (+C)$	A1	Condone missing dx
	$x = 1,\ t = 0 \qquad \frac{1}{3} - \frac{1}{15} + C = 0$	M1	Use $x = 1$, $t = 0$ to find a value for constant C from an equation in x and t
	$C = -\frac{4}{15}$	A1	
	$t = 5x(2x-1)^{\frac{3}{2}} - (2x-1)^{\frac{5}{2}} - 4$	A1	$C = -0.2666\ldots$ $C = -0.267$ or better ISW $t = (2x-1)^{\frac{3}{2}}(3x+1) - 4$
12(b)(ii)	$x = 2 \quad t = 32.4$ (minutes)	B1	32.4 or better (32.373…)
	Total	**12**	

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Q	Answer	Marks	Comments
13	$u = x^4 + 2$ $\frac{du}{dx} = 4x^3$	B1	or $du = 4x^3\,dx$
	$\int \frac{x^7}{(x^4+2)^2}\,dx$	M1	Either expression all in terms of u including replacing dx, but condone omission of du
	$\int \frac{k(u-2)}{u^2}\,du$ or $\int \frac{k(u-2)^{\frac{7}{4}}}{u^2}\,\frac{du}{(u-2)^{\frac{3}{4}}}$ $= \left(\frac{1}{4}\right)\int \frac{1}{u} - \frac{2}{u^2}\,du$	m1	$k\int au^{-1} + bu^{-2}\,du$ where k, a and b are constants
	$= \left(\frac{1}{4}\right)\left[\ln u + \frac{2}{u}\right]$ $\left(\int = \left(\frac{1}{4}\right)\left[\ln u + \frac{2}{u}\right]_2^3\right)$	A1	Must have seen du on an earlier line where every term is a term in u $\left(\left(\frac{1}{4}\right)\left[\ln(x^4+2) + \frac{2}{(x^4+2)}\right]_0^1\right)$
	$= \left(\frac{1}{4}\right)\left[\left(\ln 3 + \frac{2}{3}\right) - (\ln 2 + 1)\right]$	m1	Dependent on previous A1 Correct change of limits, correct substitution and F(3) – F(2) or Correct replacement of u, correct substitution and F(1) – F(0)
	$= \frac{1}{4}\ln\left(\frac{3}{2}\right) - \frac{1}{12}$	A1	oe in exact form
	Total	**6**	

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