OCR
Oxford Cambridge and RSA

Wednesday 14 October 2020 – Afternoon

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes

8232206127

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\overline{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2} \quad \text{or} \quad \sqrt{\frac{\Sigma f(x-\overline{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \overline{x}^2}$$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Kinematics

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

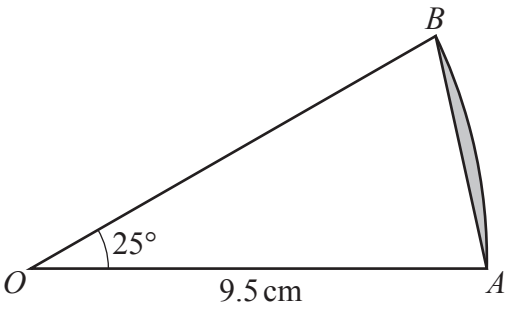
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Section A: Pure Mathematics

Answer **all** the questions.

1



The diagram shows a sector AOB of a circle with centre O and radius 9.5 cm. The angle AOB is 25°.

(a) Calculate the length of the straight line AB. **[2]**

(b) Find the area of the segment shaded in the diagram. **[3]**

2 Two curves have equations $y = \ln x$ and $y = \frac{k}{x}$, where k is a positive constant.

(a) Sketch the curves on a **single** diagram. **[3]**

(b) Explain how your diagram shows that the equation $x \ln x - k = 0$ has exactly one real root. **[2]**

3 In this question you must show detailed reasoning.

Find the equation of the normal to the curve $y = 4\sqrt{x} - 3x + 1$ at the point on the curve where $x = 4$. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[7]**

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4 **In this question you must show detailed reasoning.**

The cubic polynomial $6x^3 + kx^2 + 57x - 20$ is denoted by $f(x)$. It is given that $(2x - 1)$ is a factor of $f(x)$.

(a) Use the factor theorem to show that $k = -37$. **[2]**

(b) Using this value of k, factorise $f(x)$ completely. **[3]**

(c) **(i)** Hence find the three values of t satisfying the equation $6e^{-3t} - 37e^{-2t} + 57e^{-t} - 20 = 0$. **[2]**

(ii) Express the sum of the three values found in part **(c)(i)** as a single logarithm. **[2]**

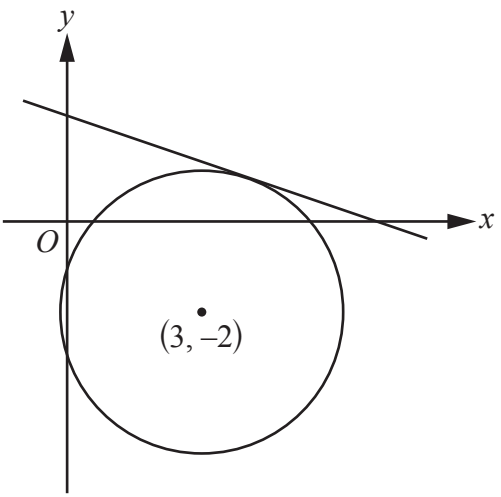
5 A curve has equation $y = a(x + b)^2 + c$, where a, b and c are constants. The curve has a stationary point at $(-3, 2)$.

(a) State the values of b and c. **[2]**

When the curve is translated by $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$ the transformed curve passes through the point $(3, -18)$.

(b) Determine the value of a. **[3]**

6 **In this question you must show detailed reasoning.**



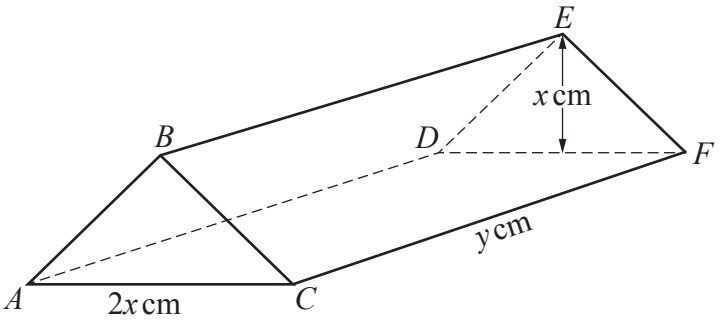
The diagram shows the line $3y + x = 7$ which is a tangent to a circle with centre $(3, -2)$.

Find an equation for the circle. **[6]**

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7



The diagram shows a model for the roof of a toy building. The roof is in the form of a solid triangular prism $ABCDEF$. The base $ACFD$ of the roof is a horizontal rectangle, and the cross-section ABC of the roof is an isosceles triangle with $AB = BC$.

The lengths of AC and CF are $2x$ cm and y cm respectively, and the height of BE above the base of the roof is x cm.

The total surface area of the **five** faces of the roof is 600 cm^2 and the volume of the roof is $V\,\text{cm}^3$.

(a) Show that $V = kx(300 - x^2)$, where $k = \sqrt{a} + b$ and a and b are integers to be determined. **[6]**

(b) Use differentiation to determine the value of x for which the volume of the roof is a maximum. **[4]**

(c) Find the maximum volume of the roof. Give your answer in cm^3, correct to the nearest integer. **[1]**

(d) Explain why, for this roof, x must be less than a certain value, which you should state. **[2]**

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Section B: Mechanics

Answer **all** the questions.

8 A particle is in equilibrium under the action of the following three forces:

$(2p\mathbf{i} - 4\mathbf{j})$ N, $(-3q\mathbf{i} + 5p\mathbf{j})$ N and $(-13\mathbf{i} - 6\mathbf{j})$ N.

Find the values of p and q. **[3]**

9 A crane lifts a car vertically. The car is inside a crate which is raised by the crane by means of a strong cable. The cable can withstand a maximum tension of 9500 N without breaking. The crate has a mass of 55 kg and the car has a mass of 830 kg.

(a) Find the maximum acceleration with which the crate and car can be raised. **[2]**

(b) Show on a clearly labelled diagram the forces acting on the **crate** while it is in motion. **[1]**

(c) Determine the magnitude of the reaction force between the crate and the car when they are ascending with maximum acceleration. **[3]**

10 A particle P is moving in a straight line. At time t seconds P has velocity $v\,\text{m s}^{-1}$ where $v = (2t+1)(3-t)$.

(a) Find the deceleration of P when $t = 4$. **[2]**

(b) State the positive value of t for which P is instantaneously at rest. **[1]**

(c) Find the total distance that P travels between times $t = 0$ and $t = 4$. **[3]**

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11 A car starts from rest at a set of traffic lights and moves along a straight road with constant acceleration $4\,\text{m}\,\text{s}^{-2}$. A motorcycle, travelling parallel to the car with constant speed $16\,\text{m}\,\text{s}^{-1}$, passes the same traffic lights exactly 1.5 seconds after the car starts to move. The time after the car starts to move is denoted by t seconds.

(a) Determine the two values of t at which the car and motorcycle are the same distance from the traffic lights. **[6]**

These two values of t are denoted by t_1 and t_2, where $t_1 < t_2$.

(b) Describe the relative positions of the car and the motorcycle when $t_1 < t < t_2$. **[1]**

(c) Determine the maximum distance between the car and the motorcycle when $t_1 < t < t_2$. **[3]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a) $AB^2 = 9.5^2 + 9.5^2 - 2(9.5)(9.5)\cos 25°$

$AB^2 = 16.9114...$

$AB = 4.1123... = 4.11$ cm

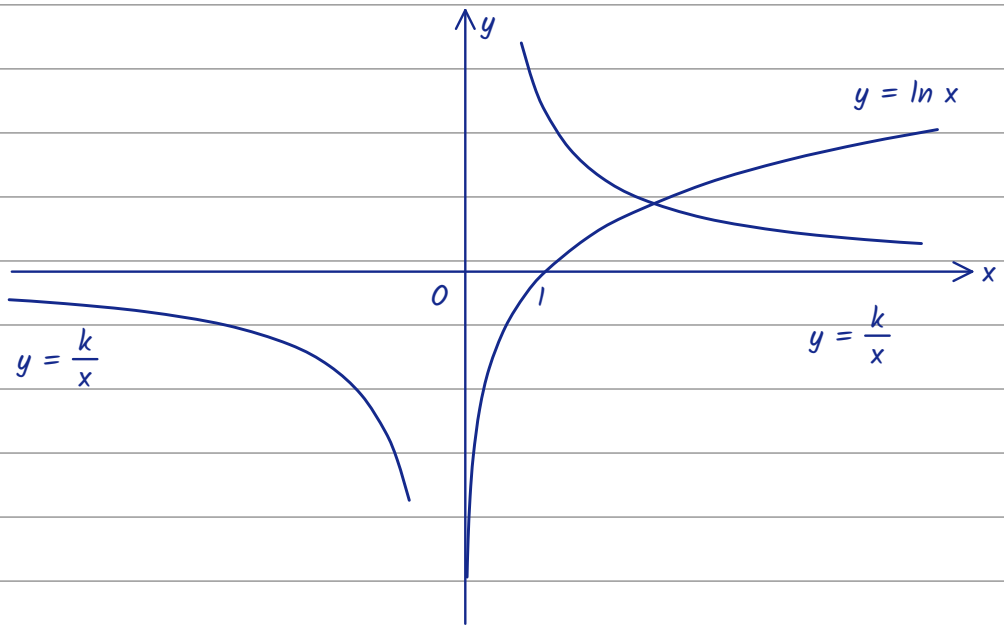
1 (b) Sector area $= \frac{25}{360} \times \pi \times 9.5^2 = 19.6894...$

Triangle area $= \frac{1}{2} \times 9.5^2 \times \sin 25° = 19.0706...$

Segment area $= 19.6894... - 19.0706... = 0.61884...$

Area $= 0.619$ cm^2

2 (a)



2 (b) $x \ln x - k = 0 \Leftrightarrow x \ln x = k \Leftrightarrow \ln x = k/x \quad (x > 0)$

so the roots are the x-coordinates where $y = \ln x$ meets $y = \frac{k}{x}$.

Question number	
2 cont.	The two curves intersect at exactly one point (in the first quadrant), so $x \ln x - k = 0$ has exactly one real root.
3	$y = 4x^{\frac{1}{2}} - 3x + 1$
	$\frac{dy}{dx} = 4 \times \frac{1}{2}x^{-\frac{1}{2}} - 3 = \frac{2}{\sqrt{x}} - 3$
	When $x = 4$: $\frac{dy}{dx} = \frac{2}{\sqrt{4}} - 3 = 1 - 3 = -2$
	Gradient of normal $= -\frac{1}{-2} = \frac{1}{2}$
	When $x = 4$: $y = 4\sqrt{4} - 3(4) + 1 = 8 - 12 + 1 = -3$
	Normal: $y - (-3) = \frac{1}{2}(x - 4)$
	$2y + 6 = x - 4$
	$x - 2y - 10 = 0$
4 (a)	$(2x - 1)$ is a factor $\Rightarrow f(\frac{1}{2}) = 0$
	$f(\frac{1}{2}) = 6(\frac{1}{2})^3 + k(\frac{1}{2})^2 + 57(\frac{1}{2}) - 20 = 0$
	$\frac{3}{4} + \frac{k}{4} + \frac{57}{2} - 20 = 0 \Rightarrow \frac{k}{4} = -\frac{37}{4}$
	$k = -37$ as required
4 (b)	$f(x) = 6x^3 - 37x^2 + 57x - 20$
	$= (2x - 1)(3x^2 - 17x + 20)$
	$= (2x - 1)(3x - 5)(x - 4)$
4 (c)(i)	Let $x = e^{-t}$: $6x^3 - 37x^2 + 57x - 20 = 0$, i.e. $f(x) = 0$

Question number	
4 cont.	$e^{-t} = \frac{1}{2}$, $e^{-t} = \frac{5}{3}$, $e^{-t} = 4$
	$t = -\ln(\frac{1}{2}) = \ln 2$, $t = -\ln(\frac{5}{3}) = \ln(\frac{3}{5})$, $t = -\ln 4$
4 (c)(ii)	Sum $= \ln 2 + \ln(\frac{3}{5}) - \ln 4 = \ln(\frac{2 \times 3}{5 \times 4})$
	Sum $= \ln(\frac{3}{10})$
5 (a)	Stationary point $(-b, c) = (-3, 2)$: $b = 3$, $c = 2$
5 (b)	Curve: $y = a(x + 3)^2 + 2$
	Translated curve: $y = a(x - 4 + 3)^2 + 2 = a(x - 1)^2 + 2$
	Passes through $(3, -18)$: $-18 = a(3 - 1)^2 + 2$
	$4a = -20$
	$a = -5$
6	$3y + x = 7 \Rightarrow y = -\frac{1}{3}x + \frac{7}{3}$, gradient $m = -\frac{1}{3}$
	Radius to point of contact is perpendicular to tangent: gradient 3
	Line through centre: $y + 2 = 3(x - 3) \Rightarrow y = 3x - 11$
	Point of contact: $3(3x - 11) + x = 7 \Rightarrow 10x = 40$
	$x = 4$, $y = 3(4) - 11 = 1$, i.e. $(4, 1)$
	$r^2 = (4 - 3)^2 + (1 - (-2))^2 = 1 + 9 = 10$
	$(x - 3)^2 + (y + 2)^2 = 10$
7 (a)	$V = \frac{1}{2} \times 2x \times x \times y = x^2y$

Question number

7 cont.

Slant edge $AB = \sqrt{(x^2 + x^2)} = x\sqrt{2}$

$S = 2 \times \frac{1}{2}(2x)(x) + 2xy + 2(x\sqrt{2})y = 2x^2 + 2xy + 2\sqrt{2}xy = 600$

$2xy(1 + \sqrt{2}) = 600 - 2x^2 \quad \Rightarrow \quad y = \frac{300 - x^2}{x(1 + \sqrt{2})}$

$V = x^2 \times \frac{300 - x^2}{x(1 + \sqrt{2})} = x(300 - x^2) \times \frac{1}{1 + \sqrt{2}}$

$\frac{1}{1 + \sqrt{2}} \times \frac{1 - \sqrt{2}}{1 - \sqrt{2}} = \frac{1 - \sqrt{2}}{1 - 2} = \sqrt{2} - 1$

$V = (\sqrt{2} - 1)x(300 - x^2)$, so $k = \sqrt{2} - 1$, $a = 2$, $b = -1$

7 (b)

$V = k(300x - x^3)$

$\frac{dV}{dx} = k(300 - 3x^2) = 0 \quad \Rightarrow \quad x^2 = 100 \quad \Rightarrow \quad x = 10 \quad (x > 0)$

$\frac{d^2V}{dx^2} = -6kx = -60(\sqrt{2} - 1) < 0$ at $x = 10$, so maximum

$x = 10$ cm

7 (c)

$V = (\sqrt{2} - 1)(10)(300 - 100) = 2000(\sqrt{2} - 1) = 828.427...$

Maximum volume = 828 cm^3

7 (d)

y (and V) must be positive, so $300 - x^2 > 0$

$x < \sqrt{300}$, so x must be less than $10\sqrt{3} = 17.3$ cm (3 s.f.)

8

Equilibrium: resultant force is zero.

j: $-4 + 5p - 6 = 0 \quad \Rightarrow \quad 5p = 10 \quad \Rightarrow \quad p = 2$

i: $2p - 3q - 13 = 0 \quad \Rightarrow \quad 4 - 3q - 13 = 0 \quad \Rightarrow \quad 3q = -9$

$p = 2, \quad q = -3$

Question number

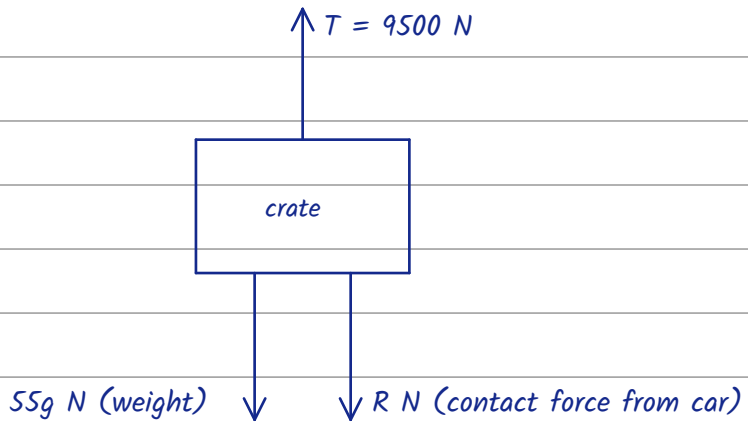
9 (a) N2L for crate and car (mass 55 + 830 = 885 kg):

$9500 - 885g = 885a$

$9500 - 8673 = 885a \quad \Rightarrow \quad a = 827/885 = 0.93446...$

Maximum acceleration = 0.934 m s^{-2}

9 (b)



9 (c) N2L for the crate:

$9500 - 55g - R = 55a$

$R = 9500 - 539 - 55(0.93446...) = 8909.60...$

$R = 8910$ N

10 (a) $v = (2t + 1)(3 - t) = -2t^2 + 5t + 3$

$a = \frac{dv}{dt} = -4t + 5$; at $t = 4$: $a = -16 + 5 = -11$

Deceleration = 11 m s^{-2}

10 (b) $v = 0 \Rightarrow t = 3$ s ($t = -\frac{1}{2}$ rejected)

10 (c) $\int_0^3 (-2t^2 + 5t + 3)\,dt = \left[-\frac{2t^3}{3} + \frac{5t^2}{2} + 3t \right]_0^3 = -18 + 22.5 + 9 = \frac{27}{2}$

$\int_3^4 (-2t^2 + 5t + 3)\,dt = \left(-\frac{128}{3} + 40 + 12 \right) - \frac{27}{2} = -\frac{25}{6}$

Question number

10 cont.

Total distance $= \frac{27}{2} + \frac{25}{6} = \frac{53}{3} = 17.666...$

Total distance $= 17.7$ m

11 (a)

Car $(u = 0, a = 4)$: $s = \frac{1}{2}(4)t^2 = 2t^2$

Motorcycle (constant speed 16): $s = 16(t - 1.5)$

Same distance: $2t^2 = 16(t - 1.5) = 16t - 24$

$2t^2 - 16t + 24 = 0 \Rightarrow t^2 - 8t + 12 = 0$

$(t - 2)(t - 6) = 0$

$t = 2$ and $t = 6$

11 (b)

The motorcycle is ahead of the car.

11 (c)

Distance between them: $d = 16(t - 1.5) - 2t^2 = 16t - 24 - 2t^2$

$\frac{dd}{dt} = 16 - 4t = 0 \Rightarrow t = 4$

$d = 16(4) - 24 - 2(4)^2 = 64 - 24 - 32$

Maximum distance = 8 m