OCR
Oxford Cambridge and RSA

# Wednesday 07 October 2020 – Afternoon

## AS Level Mathematics A

### H230/01 Pure Mathematics and Statistics

**Time allowed: 1 hour 30 minutes**

*8232068533*

**You must have:**
- the Printed Answer Booklet
- a scientific or graphical calculator

**INSTRUCTIONS**
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

**ADVICE**
- Read each question carefully before you start your answer.

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# Formulae
# AS Level Mathematics A (H230)

## Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

## Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

## Standard deviation

$$\sqrt{\frac{\Sigma(x-\overline{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2} \quad \text{or} \quad \sqrt{\frac{\Sigma f(x-\overline{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \overline{x}^2}$$

## The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of $X$ is $np$, variance of $X$ is $np(1-p)$

## Kinematics

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

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## Section A: Pure Mathematics

Answer **all** the questions.

**1** **(a)** Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3 - 3x + \frac{5}{x^2}\right)$. **[3]**

**(b)** Find $\int\left(6x^2 - \frac{2}{x^3}\right)\mathrm{d}x$. **[3]**

**2** Points $A$ and $B$ have position vectors $\begin{pmatrix}-3\\4\end{pmatrix}$ and $\begin{pmatrix}1\\2\end{pmatrix}$ respectively.

Point $C$ has position vector $\begin{pmatrix}p\\1\end{pmatrix}$ and $ABC$ is a straight line.

**(a)** Find $p$. **[2]**

Point $D$ has position vector $\begin{pmatrix}q\\1\end{pmatrix}$ and angle $ABD = 90°$.

**(b)** Determine the value of $q$. **[3]**

**3** **In this question you must show detailed reasoning.**

**(a)** Solve the equation $4\sin^2\theta = \tan^2\theta$ for $0° \leqslant \theta \leqslant 180°$. **[5]**

**(b)** Prove that $\frac{\sin^2\theta - 1 + \cos\theta}{1 - \cos\theta} \equiv \cos\theta \quad (\cos\theta \neq 1)$. **[3]**

**4** **(a)** Expand $(1+x)^4$. **[1]**

**(b)** **Use your expansion** to determine the exact value of $1002^4$. **[4]**



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**5** The function f is defined by $f(x) = (x+a)(x+3a)(x-b)$ where $a$ and $b$ are positive integers.

**(a)** On the axes in the Printed Answer Booklet, sketch the curve $y = f(x)$. **[2]**

**(b)** On your sketch show, in terms of $a$ and $b$, the coordinates of the points where the curve meets the axes. **[2]**

It is now given that $a = 1$ and $b = 4$.

**(c)** Find the total area enclosed between the curve $y = f(x)$ and the $x$-axis. **[4]**

**6** **In this question you must show detailed reasoning.**

**(a)** Solve the inequality $x^2 + x - 6 > 0$, giving your answer in set notation. **[4]**

**(b)** Solve the equation $x^3 - 7x^{\frac{3}{2}} - 8 = 0$. **[4]**

**(c)** Find the exact solution of the equation $(3^x)^2 = 3 \times 2^x$. **[5]**

**7** Determine the points of intersection of the curve $3xy + x^2 + 14 = 0$ and the line $x + 2y = 4$. **[5]**

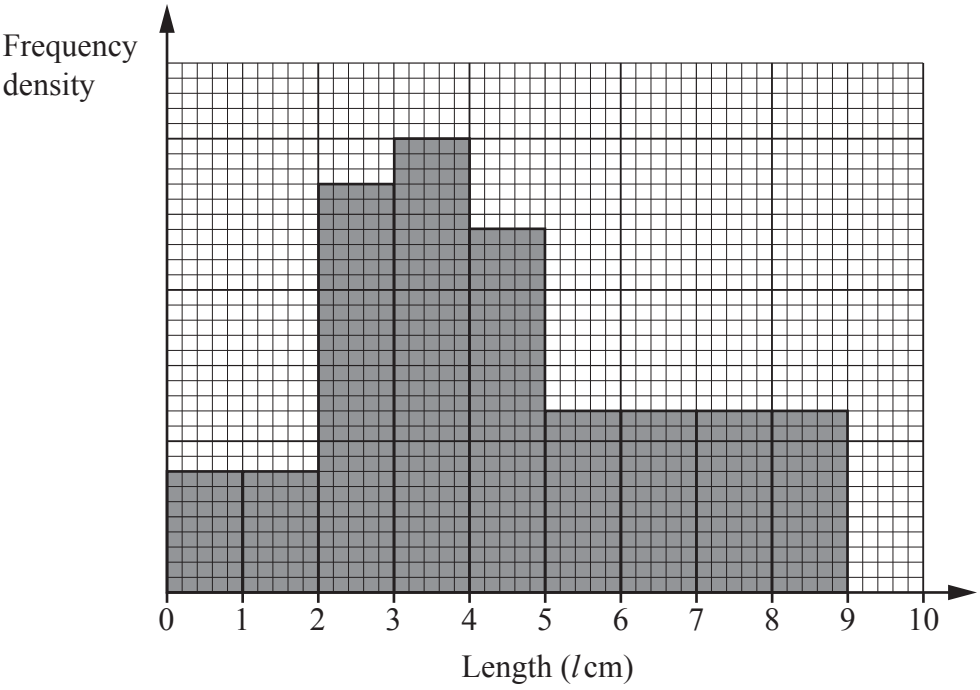


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## Section B: Statistics

Answer **all** the questions.

**8** The histogram shows information about the lengths, $l$ centimetres, of a sample of worms of a certain species.




The number of worms in the sample with lengths in the class $3 \leqslant l < 4$ is 30.

**(a)** Find the number of worms in the sample with lengths in the class $0 \leqslant l < 2$. **[2]**

**(b)** Find an estimate of the number of worms in the sample with lengths in the range $4.5 \leqslant l < 5.5$. **[3]**

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**9** A researcher is studying changes in behaviour in travelling to work by people who live outside London, between 2001 and 2011.

He chooses the 15 Local Authorities (LAs) outside London with the largest decreases in the percentage of people driving to work, and arranges these in descending order.

The table shows the changes in percentages from 2001 to 2011 in various travel categories, for these Local Authorities.

| Local Authority | Work mainly at or from home | Underground, metro, light rail, tram | Train | Bus, minibus or coach | Driving a car or van | Passenger in a car or van | Bicycle | On foot |
|---|---|---|---|---|---|---|---|---|
| Brighton and Hove | 3.2 | 0.1 | 1.5 | 0.8 | −8.2 | −1.5 | 2.1 | 2.3 |
| Cambridge | 2.2 | 0.0 | 1.6 | 1.2 | −7.4 | −1.0 | 3.1 | 0.6 |
| Elmbridge | 2.9 | 0.4 | 4.1 | 0.2 | −6.6 | −0.7 | 0.3 | −0.3 |
| Oxford | 2.0 | 0.0 | 0.6 | −0.4 | −5.2 | −1.1 | 2.2 | 2.1 |
| Epsom and Ewell | 1.6 | 0.4 | 3.9 | 1.1 | −5.2 | −0.9 | 0.0 | −0.6 |
| Watford | 0.7 | 2.0 | 3.1 | 0.4 | −4.5 | −1.2 | 0.0 | −0.1 |
| Tandridge | 3.3 | 0.2 | 4.0 | −0.1 | −4.5 | −1.1 | 0.0 | −1.3 |
| Mole Valley | 3.3 | 0.1 | 1.9 | 0.3 | −4.4 | −0.7 | 0.2 | −0.3 |
| St Albans | 2.3 | 0.3 | 3.4 | −0.3 | −4.3 | −1.2 | 0.3 | −0.2 |
| Chiltern | 2.9 | 1.4 | 1.4 | 0.1 | −4.2 | −0.6 | −0.2 | −0.8 |
| Exeter | 0.7 | 0.0 | 1.0 | −0.6 | −4.2 | −1.5 | 1.7 | 3.4 |
| Woking | 2.1 | 0.1 | 3.7 | 0.0 | −4.2 | −1.3 | −0.1 | 0.0 |
| Reigate and Banstead | 1.8 | 0.1 | 3.2 | 0.6 | −4.1 | −1.0 | 0.1 | −0.2 |
| Waverley | 4.3 | 0.1 | 2.5 | −0.5 | −3.9 | −0.9 | −0.3 | −0.9 |
| Guildford | 2.7 | 0.1 | 2.4 | 0.2 | −3.6 | −1.2 | 0.0 | −0.3 |

**(a)** Explain why these LAs are not necessarily the 15 LAs with the largest decreases in the percentage of people driving to work. **[1]**

**(b)** The researcher wants to talk to those LAs outside London which have been most successful in encouraging people to change to cycling or walking to work.
Suggest four LAs that he should talk to and why. **[2]**

**(c)** The researcher claims that Waverley is the LA outside London which has had the largest increase in the number of people working mainly at or from home.
Does the data support his claim? Explain your answer. **[1]**

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**(d)** Which two categories have replaced driving to work for the highest percentages of workers in these LAs? Support your answer with evidence from the table. **[3]**

**(e)** The researcher suggested that there would be strong correlation between the decrease in the percentage driving to work and the increase in percentage working mainly at or from home. Without calculation, use data from the table to comment briefly on this suggestion. **[1]**

**10** Some packets of a certain kind of biscuit contain a free gift. The manufacturer claims that the proportion of packets containing a free gift is 1 in 4. Marisa suspects that this claim is not true, and that the true proportion is less than 1 in 4. She chooses 20 packets at random and finds that exactly 1 contains the free gift.

**(a)** Use a binomial model to test the manufacturer's claim, at the 2.5% significance level. **[7]**

The packets are packed in boxes, with each box containing 40 packets. Marisa chooses three boxes at random and finds that one box contains 19 packets with the free gift and the other two boxes contain no packets with the free gift.

**(b)** Give a reason why this suggests that the binomial model used in part **(a)** may not be appropriate. **[1]**

**11 In this question you must show detailed reasoning.**

A biased four-sided spinner has edges numbered 1, 2, 3, 4. When the spinner is spun, the probability that it will land on the edge numbered $X$ is given by

$$P(X = x) = \begin{cases} \frac{1}{2} - \frac{1}{10}x & x = 1,2,3,4, \\ 0 & \text{otherwise.} \end{cases}$$

**(a)** Draw a table showing the probability distribution of $X$. **[1]**

The spinner is spun three times and the value of $X$ is noted each time.

**(b)** Find the probability that the third value of $X$ is greater than the sum of the first two values of $X$. **[3]**

**END OF QUESTION PAPER**



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| Surname | StemUp Model Solution |
|---|---|
| Answer booklet | Write the question number in the left-hand margin at the start of each answer. |

Question number

**1 (a)**

$\frac{d}{dx}\left(x^3 - 3x + 5x^{-2}\right) = 3x^2 - 3 - 10x^{-3}$

$= 3x^2 - 3 - \frac{10}{x^3}$

**1 (b)**

$\int \left(6x^2 - 2x^{-3}\right) dx = \frac{6x^3}{3} - \frac{2x^{-2}}{-2} + c$

$= 2x^3 + x^{-2} + c \quad = 2x^3 + \frac{1}{x^2} + c$

**2 (a)**

$AB = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \end{pmatrix} \qquad C = A + t\begin{pmatrix} 4 \\ -2 \end{pmatrix}$

$y$: $4 - 2t = 1 \Rightarrow t = 1.5$

$x$: $p = -3 + 4(1.5) = 3$

$p = 3$

**2 (b)**

$BD = \begin{pmatrix} q \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} q-1 \\ -1 \end{pmatrix}$

Angle $ABD = 90° \Rightarrow AB \cdot BD = 0$

$\begin{pmatrix} 4 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} q-1 \\ -1 \end{pmatrix} = 4(q-1) + 2 = 0$

$4q - 2 = 0 \Rightarrow q = 0.5$

**3 (a)**

$4\sin^2\theta = \tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$

$4\sin^2\theta\cos^2\theta - \sin^2\theta = 0 \Rightarrow \sin^2\theta(4\cos^2\theta - 1) = 0$

$\sin\theta = 0 \quad \text{or} \quad \cos^2\theta = \frac{1}{4} \Rightarrow \cos\theta = \pm\frac{1}{2}$

$\cos\theta = \frac{1}{2}$: $\theta = 60°$ $\qquad$ $\cos\theta = -\frac{1}{2}$: $\theta = 120°$

$\sin\theta = 0$: $\theta = 0° \text{ or } \theta = 180°$

$\theta = 0°, 60°, 120°, 180°$

**3 (b)**

$LHS = \frac{\sin^2\theta - 1 + \cos\theta}{1 - \cos\theta} = \frac{(1 - \cos^2\theta) - 1 + \cos\theta}{1 - \cos\theta} \qquad (\sin^2\theta + \cos^2\theta = 1)$

$\equiv \frac{\cos\theta - \cos^2\theta}{1 - \cos\theta} \equiv \frac{\cos\theta(1 - \cos\theta)}{1 - \cos\theta}$

Question number

$\equiv \cos\theta$ (since $\cos\theta \neq 1$, $1 - \cos\theta \neq 0$) = RHS

4 (a) $(1 + x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$

4 (b) $1002^4 = 1000^4 \times (1.002)^4 = 10^{12} \times (1 + 0.002)^4$

Put $x = 0.002$:

$(1.002)^4 = 1 + 4(0.002) + 6(0.002)^2 + 4(0.002)^3 + (0.002)^4$

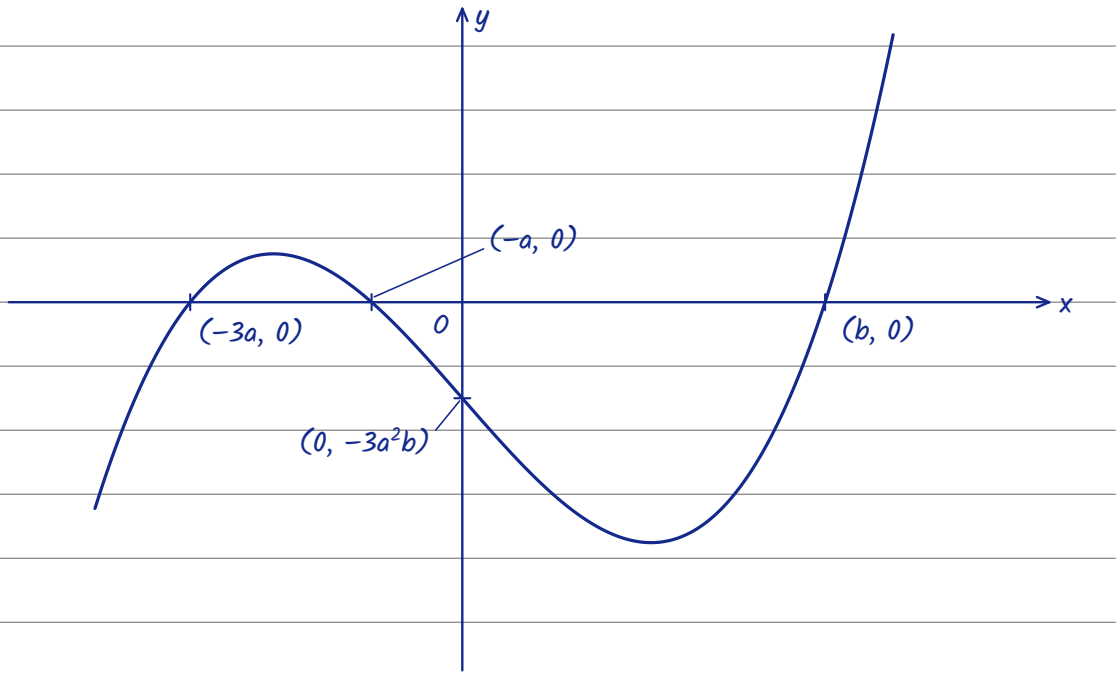
$= 1 + 0.008 + 0.000024 + 0.000000032 + 0.000000000016$

$= 1.008024032016$

$1002^4 = 1.008024032016 \times 10^{12} = 1\ 008\ 024\ 032\ 016$

5 (a)

5 (b)




5 (c) $a = 1$, $b = 4$: $f(x) = (x + 1)(x + 3)(x - 4) = (x^2 + 4x + 3)(x - 4)$

$= x^3 - 13x - 12$

Roots $x = -3, -1, 4$; curve above x-axis on $-3 < x < -1$, below on $-1 < x < 4$

$$\int_{-3}^{-1} (x^3 - 13x - 12)\,dx = \left[\frac{x^4}{4} - \frac{13x^2}{2} - 12x\right]_{-3}^{-1}$$

$$= \left(\frac{1}{4} - \frac{13}{2} + 12\right) - \left(\frac{81}{4} - \frac{117}{2} + 36\right) = 5.75 - (-2.25) = 8$$

$$\int_{-1}^{4} (x^3 - 13x - 12)\,dx = \left[\frac{x^4}{4} - \frac{13x^2}{2} - 12x\right]_{-1}^{4}$$

| Question number | |
|---|---|
| | $= (64 - 104 - 48) - 5.75 = -88 - 5.75 = -93.75$ |
| | Total area $= 8 + 93.75 = 101.75 = \frac{407}{4}$ $(= 102$ to 3 s.f.$)$ |
| | |
| 6 (a) | $x^2 + x - 6 = 0 \Rightarrow (x - 2)(x + 3) = 0 \Rightarrow x = 2$ or $x = -3$ |
| | $(x - 2)(x + 3) > 0$ for $x < -3$ or $x > 2$ (positive quadratic) |
| | $\{x : x < -3\} \cup \{x : x > 2\}$ |
| | |
| 6 (b) | Let $y = x^{3/2}$, so $x^3 = y^2$: $y^2 - 7y - 8 = 0$ |
| | $(y - 8)(y + 1) = 0 \Rightarrow (x^{3/2} - 8)(x^{3/2} + 1) = 0$ |
| | $x^{3/2} = -1$ has no solution, since $x^{3/2} = (\sqrt{x})^3 \geq 0$ |
| | $x^{3/2} = 8 \Rightarrow x^3 = 64 \Rightarrow x = 4$ |
| | |
| 6 (c) | $(3^x)^2 = 3 \times 2^x \Rightarrow 3^{2x} = 3 \times 2^x$ |
| | $\ln(3^{2x}) = \ln(3 \times 2^x)$ |
| | $2x \ln 3 = \ln 3 + x \ln 2$ |
| | $x(2 \ln 3 - \ln 2) = \ln 3 \Rightarrow x \ln \frac{9}{2} = \ln 3$ |
| | $x = \frac{\ln 3}{\ln (9/2)}$ |
| | |
| 7 | $x + 2y = 4 \Rightarrow x = 4 - 2y$ |
| | $3(4 - 2y)y + (4 - 2y)^2 + 14 = 0$ |
| | $12y - 6y^2 + 16 - 16y + 4y^2 + 14 = 0$ |
| | $-2y^2 - 4y + 30 = 0 \Rightarrow y^2 + 2y - 15 = 0$ |
| | $(y + 5)(y - 3) = 0 \Rightarrow y = -5$ or $y = 3$ |
| | $y = -5$: $x = 4 + 10 = 14$ $y = 3$: $x = 4 - 6 = -2$ |
| | Points of intersection are $(14, -5)$ and $(-2, 3)$ |
| | |
| 8 (a) | Using small squares: $3 \leq l < 4$ bar is 30 squares high, width 1 cm |
| | so frequency density 30 per cm (1 small square = 1 worm per cm) |
| | $0 \leq l < 2$: height 8 squares, width 2 cm: frequency $= 8 \times 2 = 16$ |

| Question number | |
|---|---|
| | Number of worms = 16 |
| 8 (b) | $4 \leqslant l < 5$: frequency = $24 \times 1 = 24$; $5 \leqslant l < 9$: fd = 12, so |
| | $5 \leqslant l < 6$: frequency = $12 \times 1 = 12$ |
| | Estimate = $\frac{1}{2} \times 24 + \frac{1}{2} \times 12 = 12 + 6 = 18$ |
| 9 (a) | The table only includes LAs outside London, so LAs inside London |
| | (which are not included) might have had greater decreases. |
| 9 (b) | Brighton and Hove, Cambridge, Oxford and Exeter, because they have |
| | the largest increases in the Bicycle and On foot columns together |
| | (4.4, 3.7, 4.3 and 5.1 percentage points). |
| 9 (c) | No. The data are changes in percentages, not numbers of people. A LA |
| | with a larger population could have a bigger increase in the number |
| | working at home even though its percentage increase is smaller. |
| 9 (d) | Working mainly at or from home and Train. |
| | Both have positive entries for every LA, all above 0.5 (smallest 0.7 |
| | and 0.6), and they have the largest values: home 4.3, train 4.1; |
| | no other category has more than 3 entries above 2.0. |
| 9 (e) | Not the case: the driving decreases fall steadily (they are in order), |
| | but the home figures show no pattern, e.g. Watford −4.5 with 0.7 |
| | and Waverley −3.9 with 4.3. |
| 10 (a) | Let $p$ = proportion of packets containing a free gift. |
| | $H_0$: $p = 0.25$ $H_1$: $p < 0.25$ |
| | Under $H_0$, $X \sim B(20, 0.25)$, observed $X = 1$ |

Question number

$P(X \leqslant 1) = 0.75^{20} + 20 \times 0.25 \times 0.75^{19}$

$= 0.003171... + 0.021141... = 0.024312... = 0.0243$

$0.0243 < 0.025$, so reject $H_0$

There is sufficient evidence at the 2.5% level to suggest that the proportion of packets containing a free gift is less than 0.25.

10 (b) Whether a packet contains a free gift is not independent of whether other packets in the same box contain one (the gifts are not distributed randomly: 19 in one box, 0 in the others).

11 (a)

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $P(X = x)$ | $\frac{2}{5}$ | $\frac{3}{10}$ | $\frac{1}{5}$ | $\frac{1}{10}$ |

11 (b) Third value greater than sum of first two (sum $\geqslant 2$):

third = 3: (1, 1) third = 4: (1, 1), (1, 2), (2, 1)

$P = \left(\frac{2}{5}\right)^2 \times \frac{1}{5} + \left(\frac{2}{5}\right)^2 \times \frac{1}{10} + \frac{2}{5} \times \frac{3}{10} \times \frac{1}{10} + \frac{3}{10} \times \frac{2}{5} \times \frac{1}{10}$

$= 0.032 + 0.016 + 0.012 + 0.012$

$= 0.072 = \frac{9}{125}$