

GCE

Mathematics A

H230/01: Pure Mathematics and Statistics

Advanced Subsidiary GCE

Mark Scheme for November 2020

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Text Instructions

1. Annotations and abbreviations

Annotation in RM assessor	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

- a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
- Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value **is given** in the paper only accept an answer correct to at least as many significant figures as the given value.

- When a value **is not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.

NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads “2 s.f.”.

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold “In this question you must show detailed reasoning”, or the command words “Show” or “Determine”. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance
1	(a)		$3x^2 - 3 - \frac{10}{x^3}$ oe	M1 B1 A1 [3]	1.1 1.1 1.1	Allow M1 for $\pm \frac{k}{x^3}$, B1 for either $3x^2$ or -3 A1 for all correct
1	(b)		$2x^3 + \frac{1}{x^2}$ or $2x^3 + x^{-2}$ oe + c	M1 A1 B1 [3]	1.1 1.1 1.1	Allow M1 for ax^3 or $\pm \frac{b}{x^2}$ ($a, b \neq 0$) A1 for both terms correct. Allow unsimplified form, eg $-\frac{2}{-2x^2}$
2	(a)		$\frac{p-1}{1-2} = \frac{-3-1}{4-2}$ or $\frac{p+3}{1-4} = -2$ oe $p = 3$	M1 A1 [2]	1.1 2.2a	Correct equation involving ratios seen. or $y = -\frac{1}{2}x + \frac{5}{2}$ correct, and substitute $y = 1$ or clear correct diagram drawn, from $x = -3$ to $x = 3$ Allow M1A1 with unclear working or no working
2	(b)		$\overrightarrow{AB} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$, $\overrightarrow{BD} = \begin{pmatrix} q-1 \\ -1 \end{pmatrix}$, $\overrightarrow{DA} = \begin{pmatrix} -3-q \\ 3 \end{pmatrix}$ $(-3-q)^2 + 3^2 = (q-1)^2 + 1 + 16 + 4$ oe $q = 0.5$ Alternative method Gradient of $AB = -\frac{1}{2}$, gradient of $BD = 2$ BD is $y-2 = 2(x-1)$ or $y = 2x+c$ & $c = 0$ M1 BD is $y = 2x$ A1 When $y = 1$, $x = 0.5$ $q = 0.5$ A1	M1 M1 A1 [3]	3.1a 1.1 2.2a	Attempt to find vectors along 2 or 3 sides. Allow errors Their $DA^2 = AB^2 + BD^2$ or $\begin{pmatrix} 4 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} q-1 \\ -1 \end{pmatrix} = 0$ ft their $\overrightarrow{AB}$ & $\overrightarrow{BD}$ Must follow from correct working seen Attempt find gradient and equation of BD . Allow errors

Question		Answer	Marks	AO	Guidance
3	(a)	DR			
		$4\sin^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta}$	B1	1.1	Not incorrect notation, eg $\left(\frac{\sin}{\cos}\right)^2 \theta$
		$\cos^2 \theta = \frac{1}{4}$	M1	1.1	Attempt $\div$ bs by $\sin^2 \theta$ & $\sqrt{\text{bs}}$, rearrange to this form. Allow errors.
		$\cos \theta = \pm \frac{1}{2}$ or $2\cos \theta = \pm 1$			
		Alternative method for M1 $4\sin^2 \theta \cos^2 \theta = \sin^2 \theta$ $4\sin^4 \theta - 3\sin^2 \theta = 0$ $\sin^2 \theta = \frac{3}{4}$ M1 $\sin \theta = \pm \frac{\sqrt{3}}{2}$ Allow $\sin \theta = \frac{\sqrt{3}}{2}$			Similar for finding quartic equation in $\cos \theta$ Attempt use $s^2 + c^2 = 1$, rearrange to quartic in $\sin \theta$ & obtain $\sin^2 \theta = \dots$ or $\sin \theta = \dots$ Allow errors . Other methods, see below.
		$\theta = 60^\circ$ or 120°	A1 A1	1.1 1.1	Allow 240° and/or 300° but no other extras
		or $\sin \theta = 0$, $\theta = 0^\circ$ or 180°	B1	1.1	Allow 360° but no other extras
		Summary Any largely correct method obtaining $\cos^2 \theta = \dots$ or $\sin^2 \theta = \dots$ B1M1 60° and 120° A1A1 0° and 180° A1	[5]		or $\cos \theta = \dots$ or $\sin \theta = \dots$ Allow errors

Question			Answer	Marks	AO	Guidance
3	(b)		DR			
			$\frac{1-\cos^2 \theta-1+\cos \theta}{1-\cos \theta} \quad (\equiv \frac{\cos \theta-\cos^2 \theta}{1-\cos \theta})$	M1	1.1	Use of $\sin^2 \theta + \cos^2 \theta = 1$ to obtain correct fraction in cos only
			$\equiv \frac{\cos \theta(1-\cos \theta)}{1-\cos \theta}$	A1	1.1	Correct factorised numerator
			$\equiv \cos \theta$	A1	2.2a	Must see previous line and result. Allow = instead of $\equiv$ throughout Allow no mention that $\cos \theta \neq 1$.
			[3]			
			Example of an alternative method			
			$\frac{1-\cos^2 \theta-1+\cos \theta}{1-\cos \theta} = \cos \theta$	M1		
			$-\cos^2 \theta + \cos \theta = \cos \theta(1-\cos \theta)$	A1		
			$-\cos^2 \theta + \cos \theta = \cos \theta - \cos^2 \theta$	A1		Any correct manipulation of the original identity that finishes with a statement that is correct
4	(a)		$1+4 x+6 x^2+4 x^3+x^4$	B1 [1]	1.1	
4	(b)		$(1+0.002)^4 =$	M1	3.1a	Attempt subst $x=0.002$ in their expansion
			$1+0.008+0.000024+0.000000032+1.6 \times 10^{-11}$	A1	1.1	Correct values for all terms, not just correct expressions
			$=1.008024032016$	A1	1.1	
			$1002^4=1\,008\,024\,032\,016$	A1	2.1	cao. No working, or inadequate working, no marks
			or $1.008\,024\,032\,016 \times 10^{12}$			$(1000+2)^4$ scores no marks $(1001+1)^4$ unless a complete solution is seen to an exact answer
				[4]		

Question		Answer	Marks	AO	Guidance
5	(a)	Cubic curve, correct orientation cuts x-axis twice to left of O and once to right	B1 B1 [2]	1.2 1.1	Allow cubic of incorrect orientation
5	(b)	$(-3a, 0)$, $(-a, 0)$, $(b, 0)$ all shown correctly $(0, -3a^2b)$ shown correctly	B1 B1 [2]	1.1 1.1	allow $-3a$, $-a$, b marked on x-axis allow $-3a^2b$ marked on y-axis
5	(c)	$\int_{-3}^{-1} (x+3)(x+1)(x-4)dx$ $\left[\frac{x^4}{4} - \frac{13x^2}{2} - 12x \right]_{-3}^{-1}$ $= 8$ $\int_{-1}^4 (x^3 - 13x - 12)dx$ $\left[\frac{x^4}{4} - \frac{13x^2}{2} - 12x \right]_{-1}^4$ $= -\frac{375}{4} \text{ or } -93.75$ $8 - (-93.75) \quad \text{or } 8 + 93.75$ $\frac{407}{4} \text{ or } 101.75 \text{ or } 102 \text{ (3 sf)}$	M1 A1 M1 A1 [4]	1.1 1.1 2.1 2.2a	State or imply integrating $f(x)$ with any limits or none Allow incorrect expansion A1 for either answer or for $\frac{375}{4}$. BC Allow A1 for $\pm \frac{343}{4}$ Attempt subtract their integrals with correct limits, one +ve, one -ve ie $I_1 - I_2$ or $I_1 + (-I_2)$ or $I_1 + I_2 $ dep I_2 being -ve NB $\int_{-3}^{-1} + \int_{-1}^4 = 8 + 93.75$ M1A1M0A0 (Two errors)
6	(a)	DR $(x-2)(x+3) > 0$ $x < -3$, $x > 2$ $\{x: x < -3\} \cup \{x: x > 2\}$	M1 A1 A1 B1ft [4]	3.1a 1.1 1.1 2.5	Attempt factorise Correct factors and > 0 or $y > 0$ Any notation. Allow "and", "or", comma etc But NOT $-3 < x > 2$ $x < -3$, $x > 2$ seen, with no working or muddled working; SC B1 Allow () instead of { } Allow $x \in (-\infty, -3) \cup (2, \infty)$ ft their factors, dep two separate ranges.

Question			Answer	Marks	AO	Guidance
6	(b)		DR $(x^{\frac{3}{2}} + 1)(x^{\frac{3}{2}} - 8) = 0$	M1	1.1	Attempt factors of form $(x^{\frac{3}{2}} \pm k)$ or $(y \pm k)$ or $(y + 1)(y - 8)$ or $y = -1$ or $y = 8$ AND $y = x^{\frac{3}{2}}$ soi
			$x^{\frac{3}{2}} = -1$ gives no solution	B1	3.2b	Allow $(x + 1)(x - 8)$ AND $x = x^{\frac{3}{2}}$ seen Condone inadequate reason
			$x^{\frac{3}{2}} = 8$ or $x^3 = 64$	A1	1.1	$y = 8$ not enough for this mark
			$x = 4$	A1 [4]	1.1	Indep of previous A1
6	(c)		DR $\ln[(3^x)^2] = \ln[3 \times 2^x]$	M1	3.1a	Attempt take logs. Allow errors, eg RHS = $\ln 3 \times \ln(2^x)$
			$2\ln(3^x)$ or $\ln(3^{2x})$ or $x\ln(3^2)$ or $\ln(9^x)$ or $2x\ln 3$	A1	1.1	LHS correct after one further step
			$= \ln 3 + \ln(2^x)$ oe	A1	1.1	RHS correct after one further step
			$2x\ln 3$ or $x\ln 9 = \ln 3 + x\ln 2$	A1	2.1	Both sides correct with x removed from index
			$x = \frac{\ln 3}{\ln \frac{9}{2}}$	A1	2.2a	or any equivalent correct form e.g. $\frac{1}{2 - \log_3 2}$ Must be exact ISW
			Alternative methods			
			$\ln(3^{2x-1}) = \ln(2^x)$ or $\log_3(2^x) = 2x - 1$ M1			Attempt take logs.
			$(2x - 1)\ln 3 = x\ln 2$ or $x\log_3 2 = 2x - 1$ A1			LHS correct after one further step
				A1		RHS correct after one further step
				A1		Both sides correct with x removed from index
			$x = \frac{\ln 3}{2\ln 3 - \ln 2}$ or $x = \frac{1}{2 - \log_3 2}$ A1			ISW
			$9^x = 3 \times 2^x$ M1			
			$\left(\frac{9}{2}\right)^x = 3$ M1			Divide by 2^x and arrange into $a^x = b$ form
			$4.5^x = 3$ A1A1			A1 for each side correct
			$x = \log_{4.5}(3)$ A1			ISW
				[5]		

Question			Answer	Marks	AO	Guidance
7			Example method: $3(4 - 2y)y + (4 - 2y)^2 = -14$ $12y - 6y^2 + 16 - 16y + 4y^2 = -14$ $2y^2 + 4y - 30 = 0$ $y^2 + 2y - 15 = 0$ $(y + 5)(y - 3) = 0$ $y = -5$ or 3 eg $x + 2(-5) = 4$ and $x + 2 \times 3 = 4$ Points of intersection are $(14, -5)$ & $(-2, 3)$	M1 A1 A1 M1 A1 [5]	3.1a 1.1 1.1 2.2a 1.1	Other methods score similarly Attempt substitution from (ii) into (i) or (i) into (ii) and obtain equation in one letter Obtain correct 3-term quadratic equation eg $x^2 - 12x - 28 = 0$ Method may not be seen cao or $x = 14$ or -2 Substitute their y values into either equation or their x values or $x = 14, y = -5; x = -2, y = 3$. Must be clearly paired cao
8	(a)		$30 \times 2 \times 1.6 / 6$ or $30 \times 2 \times 8 / 30$ oe $= 16$	M1 A1 [2]	1.1 1.1	or $15 \times 80 / 150$ oe Correct answer without working or unclear working: allow M1A1
8	(b)		Freq 4-5 = 24 or freq 5-6 = 12 $\frac{1}{2}(\text{freq 4-5} + \text{freq 5-6})$ or $\frac{1}{2} \times 24 + \frac{1}{2} \times 12$ $= 18$	B1 M1 A1 [3]	3.1a 1.1 1.1	or freq 5-9 = 48 OR similar with frequency density or $\frac{1}{2}(\text{freq 4-5}) + \frac{1}{8}(\text{freq 5-9})$ oe Correct answer without working or unclear working: allow B1M1A1

Question			Answer	Marks	AO	Guidance
9	(a)		The table does not include London LAs, so one of them might have had a greater increase. These areas may not have as large decrease as areas such as London	B1 [1]	2.2b	NOT Not include London plus incorrect, eg "small sample" Not include London and other cities There could be other LAs with bigger decreases in other years
9	(b)		Brighton and Hove, Oxford, Cambridge, Exeter. E.g. they have relatively high (positive) values in the Bicycle and Walking columns	B1* B1_{dep} [2]	1.1 2.2a	All four required or, eg, they have the largest increases (or changes) in percentage cycling & walking or they have largest total increase for cycling and walking Must mention both cycling and walking
9	(c)		No, eg Data given is proportions, so there may be LAs with large populations where the absolute change is larger but the proportion of total population is smaller. No, eg data does not show population sizes	A1 [1]	2.2b	

Question		Answer	Marks	AO	Guidance
9	(d)	Work mainly at or from home Train Eg: These two categories have entries above 1.0 except in 3 cases. These two categories' entries all above 0.6 No other method of travel has more than 3 entries greater than 2.0 Both are all positive <u>and</u> have largest home (4.3) and largest train (4.1) Both are all over 0.5. Smallest home 0.7, smallest train 0.6 Average increases: Home 2.4, Train 2.55 (must be correct) Total increases: Home 36, Train 38.3 (correct)	B1 B1 B1 [3]	2.2b 2.2b 2.4	Subtract B1 for each extra category quoted NOT These two categories are positive for all LAs This B1 can only be awarded if B1B1 already scored Good explanation of why these two are significantly different from the others. Must quote at least two figures from table, from any two categories, or two correct totals or averages
9	(e)	Not the case. The "driving" figures increase, but the "home" figures have no pattern.	B1 [1]	2.3	Not enough to quote individual LAs. Not Weak correlation
10	(a)	Allow 2 sf throughout $H_0: p = 0.25$ where $p = P(\text{a packet contains gift})$ $H_1: p < 0.25$ $B(20, 0.25) \text{ \& } X = 1$ $P(X \leq 1) = 0.0243$ comp 0.025 Reject H_0 Sufficient evidence that proportion containing gift is less than 0.25	B1 B1 M1 A1 A1 M1 A1 [7]	1.1 2.5 3.3 3.4 1.1 1.1 2.2b	or $p = \text{proportion of packets containing gift}$ One error, eg undefined p B1B0 soi Condone $P(X = 1) = 0.0243$ but not $P(X = 1) = 0.0211$ or other incorrect dep 0.0243 and 0.025 Allow eg "H_0 is incorrect" Dep 0.0243 or $P(X \leq 1)$ stated or 0.0211 Can be implied by correct conclusion as for A1 below In context, not definite, eg not "Proportion is less"

Question			Answer	Marks	AO	Guidance
10	(b)		EITHER whether a packet contains a free gift is not independent of whether other nearby packets contain the free gift OR eg The probability that a packet contains a gift is not the same for each packet or The proportion of packets with gifts in each box is not constant OR Free gifts not distributed randomly	B1 [1]	3.5b	Allow The probability of packet containing a gift is not independent Explanation, in context of why either the independence condition or the constant probability condition is not met. NOT The number of gifts in each box is not constant
11	(a)		$\frac{1}{5} \quad \frac{2}{10} \quad \frac{3}{5} \quad \frac{4}{10}$ oe	B1 [1]	1.1	B1
11	(b)		DR 1,1,3; 1,1,4; 1,2,4; 2,1,4 $(\frac{2}{5})^2 \times \frac{1}{5} + (\frac{2}{5})^2 \times \frac{1}{10} + \frac{2}{5} \times \frac{3}{10} \times \frac{1}{10} + \frac{3}{10} \times \frac{2}{5} \times \frac{1}{10}$ $= \frac{9}{125}$ or 0.072	B1 M1 A1 [3]	3.1a 2.1 1.1	B1 for any three of these soi, eg on tree diagram Any two correct products added: ft their table

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