OCR
Oxford Cambridge and RSA

Friday 23 May 2025 – Afternoon

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes

L18O5083L1

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\overline{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2} \quad \text{or} \quad \sqrt{\frac{\Sigma f(x-\overline{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \overline{x}^2}$$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Kinematics

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

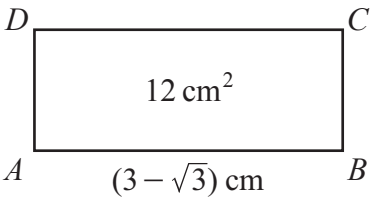
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Section A
Pure Mathematics

1 Determine the equation of the line that passes through the point (1, 3) and is perpendicular to the line with equation $3x + 6y - 5 = 0$. Give your answer in the form $ax + by + c = 0$ where a, b and c are integers to be determined. **[3]**

2 **In this question you must show detailed reasoning.**



The diagram shows the rectangle $ABCD$ where $AB = (3 - \sqrt{3})$ cm.

The area of $ABCD$ is 12 cm^2.

Find the perimeter of $ABCD$. Give your answer in the form $(a + b\sqrt{3})$ cm, where a and b are integers to be determined. **[4]**

3 In a triangle ABC, $AB = 9$ cm, $BC = 7$ cm and $AC = 4$ cm.

(a) Show that $\cos CAB = \frac{2}{3}$. **[2]**

(b) Hence find the exact value of $\sin CAB$. **[2]**

(c) Find the exact area of triangle ABC. **[2]**

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4 The cubic polynomial $2x^3 - kx^2 + 4x + k$, where k is a constant, is denoted by $f(x)$.
It is given that $f'(2) = 16$.

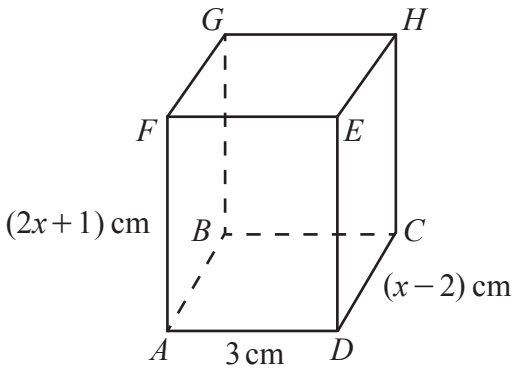
(a) Show that $k = 3$. **[4]**

For the remainder of the question, you should use this value of k.

(b) Use the factor theorem to show that $(2x + 1)$ is a factor of $f(x)$. **[2]**

(c) Hence show that the equation $f(x) = 0$ has only **one** real root. **[4]**

5 **In this question you must show detailed reasoning.**



The diagram shows the cuboid $ABCDEFGH$ where $AD = 3$ cm, $AF = (2x + 1)$ cm and $DC = (x - 2)$ cm.

The volume of the cuboid is at most $9\,\text{cm}^3$.

Find the range of possible values of x. Give your answer in interval notation. **[5]**

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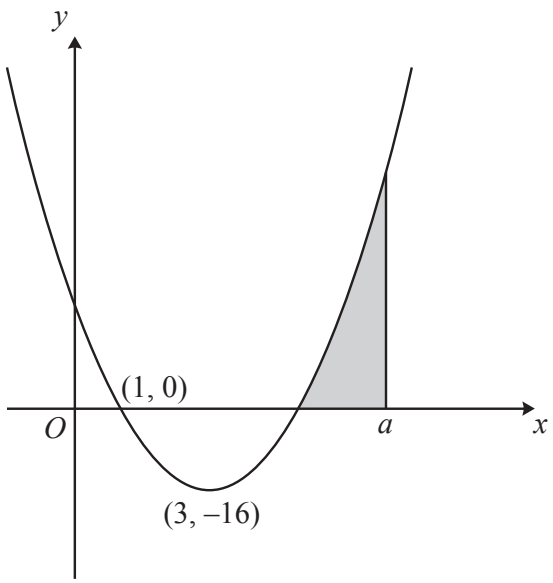
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6 **(a)** The curve $y = \ln x$ is stretched by a scale factor 3 in the x-direction.

State the equation of the curve after it has been stretched. **[2]**

(b) Describe fully the **single** transformation that transforms the curve $y = x^2 + 1$ to the curve $y = x^2 - 6x + 8$. **[4]**

7



The diagram shows the curve with equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a quadratic function. The curve intersects the x-axis at the point $(1, 0)$ and has its minimum at the point $(3, -16)$.

(a) Show that $\mathrm{f}(x) = 4x^2 - 24x + 20$. **[3]**

The shaded region is enclosed by the curve, the x-axis, and the line $x = a$. The area of the shaded region is $\frac{100}{3}$ units^2.

(b) Determine the exact value of the constant a. **[6]**

8 The curves $\mathrm{e}^x - 2\mathrm{e}^y = 1$ and $2\mathrm{e}^x + 3\mathrm{e}^{2y} = 41$ intersect at the point P.

(a) Show that the y-coordinate of P satisfies the equation $3\mathrm{e}^{2y} + 4\mathrm{e}^y - 39 = 0$. **[1]**

(b) **In this question you must show detailed reasoning.**

Hence find the exact coordinates of P. **[5]**

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Section B
Mechanics

9 An athlete runs with velocity $v\,\text{m s}^{-1}$ along a horizontal straight running track AB. At time t seconds after passing A, $v = 0.05t^2 - 0.4t + k$, where k is a constant.

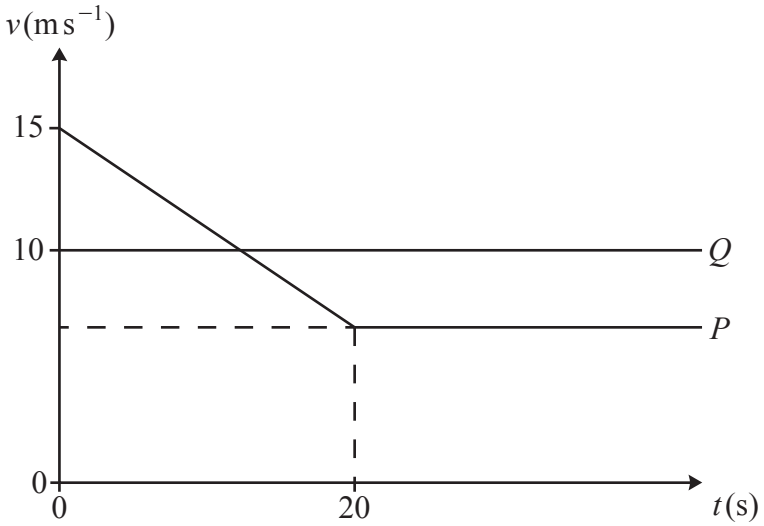
(a) Find the acceleration of the athlete when $t = 6$. **[2]**

(b) Given that the minimum velocity of the athlete is $6.6\,\text{m s}^{-1}$, show that $k = 7.4$. **[3]**

The athlete runs in a straight line from A to B in 13.1 seconds.

(c) Calculate the distance AB. **[2]**

10



The velocity-time graph represents the motion of two cyclists P and Q who are travelling along a straight, horizontal road. At time $t = 0$, the cyclists pass the same point on the road with P cycling at $15\,\text{m s}^{-1}$ and Q cycling at $10\,\text{m s}^{-1}$.

From time $t = 0$, P cycles with constant deceleration $0.4\,\text{m s}^{-2}$ for 20 seconds and Q cycles at a constant velocity of $10\,\text{m s}^{-1}$. From $t = 20$ onwards, P's velocity remains constant.

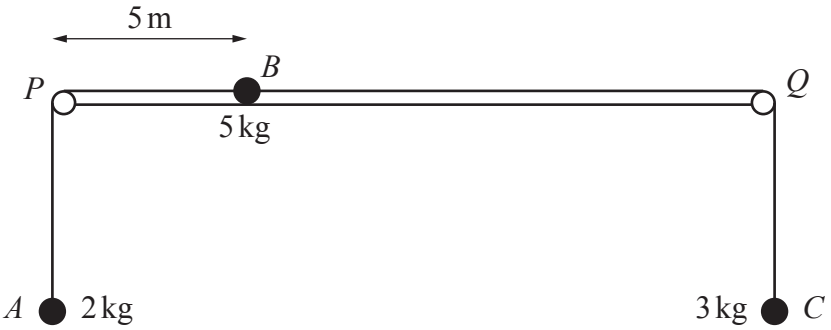
(a) Find the value of t when P and Q have the same velocity. **[2]**

(b) Determine the value of t when Q overtakes P. **[4]**

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11



One end of a light inextensible string is attached to a particle A of mass 2 kg. The other end of the string is attached to a second particle B of mass 5 kg. Particle B is in contact with a smooth horizontal surface. The string passes over a small smooth pulley P which is fixed at one edge of the surface. Particle A hangs vertically below P.

A second light inextensible string is attached to B. The other end of this second string is attached to a third particle C of mass 3 kg. This string passes over a small smooth pulley Q which is fixed at the other edge of the surface. Particle C hangs vertically below Q (see diagram).

The system is released from rest with the strings taut and B a distance 5 m from P. In the subsequent motion A does not reach P.

(a) Determine, in terms of g, the tension in the string connecting B and C. **[4]**

When the particles have been in motion for 3 seconds the string connecting A and B breaks. Particle B reaches Q after a further 2 seconds.

(b) Determine the distance PQ. **[6]**

(c) State what it would mean about the tension in the string connecting B and C if pulley Q could **not** be assumed to be smooth. **[1]**

It is given that in fact the horizontal surface is **not** smooth. When the system, as shown in the diagram, is released, it remains at rest. (You may assume that pulley Q **is** smooth.)

(d) Complete the diagram in the Printed Answer Booklet showing all the forces acting on B. **[1]**

(e) State the magnitude of the frictional force acting on B. Give a brief justification for your answer. **[1]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 $3x + 6y - 5 = 0 \Rightarrow 6y = 5 - 3x \Rightarrow y = \frac{5}{6} - \frac{1}{2}x$

Gradient of given line $= -\frac{1}{2}$, so perpendicular gradient $= 2$

$y - 3 = 2(x - 1)$

$y - 3 = 2x - 2$

$2x - y + 1 = 0$

2 $BC = \frac{12}{3 - \sqrt{3}} = \frac{12}{3 - \sqrt{3}} \times \frac{3 + \sqrt{3}}{3 + \sqrt{3}} = \frac{36 + 12\sqrt{3}}{9 - 3} = \frac{36 + 12\sqrt{3}}{6}$

$BC = 6 + 2\sqrt{3}$

Perimeter $= 2(3 - \sqrt{3}) + 2(6 + 2\sqrt{3})$

$= 6 - 2\sqrt{3} + 12 + 4\sqrt{3}$

$= (18 + 2\sqrt{3})$ cm

3 (a) $\cos CAB = \frac{AC^2 + AB^2 - BC^2}{2 \times AC \times AB} = \frac{4^2 + 9^2 - 7^2}{2(4)(9)}$

$\cos CAB = \frac{16 + 81 - 49}{72} = \frac{48}{72} = \frac{2}{3}$ as required

3 (b) $\sin^2 CAB + \cos^2 CAB = 1 \Rightarrow \sin^2 CAB = 1 - \left(\frac{2}{3}\right)^2 = \frac{5}{9}$

CAB is acute ($\cos > 0$, angle in a triangle), so $\sin CAB = \frac{\sqrt{5}}{3}$

Question number

3 (c)

$\text{Area} = \frac{1}{2} \times AC \times AB \times \sin CAB = \frac{1}{2} \times 4 \times 9 \times \frac{\sqrt{5}}{3}$

$\text{Area} = 6\sqrt{5}\ \text{cm}^2$

4 (a)

$f(x) = 2x^3 - kx^2 + 4x + k$

$f'(x) = 6x^2 - 2kx + 4$

$f(2) = 6(2)^2 - 2k(2) + 4 = 16$

$24 - 4k + 4 = 16$

$28 - 4k = 16 \quad \Rightarrow \quad -4k = -12 \quad \Rightarrow \quad k = 3$ as required

4 (b)

$f(x) = 2x^3 - 3x^2 + 4x + 3$

$f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) + 3$

$= -\frac{1}{4} - \frac{3}{4} - 2 + 3 = 0$

$f\left(-\frac{1}{2}\right) = 0$, so by the factor theorem $(2x + 1)$ is a factor of $f(x)$

4 (c)

$f(x) = (2x + 1)(x^2 + bx + c)$

$2x^3 - 3x^2 + 4x + 3 = 2x^3 + (2b + 1)x^2 + (2c + b)x + c$

$c = 3, \quad 2b + 1 = -3 \quad \Rightarrow \quad b = -2 \quad$ (check: $2c + b = 6 - 2 = 4$ ✓)

$f(x) = (2x + 1)(x^2 - 2x + 3)$

For $x^2 - 2x + 3$: $b^2 - 4ac = (-2)^2 - 4(1)(3) = 4 - 12 = -8 < 0$

so $x^2 - 2x + 3 = 0$ has no real roots.

Hence the only real root of $f(x) = 0$ is from $2x + 1 = 0$, i.e. $x = -\frac{1}{2}$

so $f(x) = 0$ has only one real root.

Question number	
5	Volume $= 3(x-2)(2x+1) \leqslant 9$
	$(x-2)(2x+1) \leqslant 3$
	$2x^2 + x - 4x - 2 \leqslant 3$
	$2x^2 - 3x - 5 \leqslant 0$
	$(2x-5)(x+1) \leqslant 0$
	Critical values: $x = \frac{5}{2}$, $x = -1 \Rightarrow -1 \leqslant x \leqslant \frac{5}{2}$
	Lengths must be positive: $x - 2 > 0 \Rightarrow x > 2$ (and $2x + 1 > 0 \Rightarrow x > -\frac{1}{2}$)
	So $2 < x \leqslant \frac{5}{2}$
	$x \in (2, \frac{5}{2}]$
6 (a)	$y = \ln(\frac{1}{3}x)$
6 (b)	$x^2 - 6x + 8 = (x-3)^2 - 9 + 8 = (x-3)^2 - 1$
	If $f(x) = x^2 + 1$, then $(x-3)^2 - 1 = [(x-3)^2 + 1] - 2 = f(x-3) - 2$
	Translation by vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$
	(3 units in the positive x-direction and 2 units in the negative y-direction)
7 (a)	Minimum at $(3, -16)$: $y = k(x-3)^2 - 16$
	Through $(1, 0)$: $0 = k(1-3)^2 - 16 \Rightarrow 4k = 16 \Rightarrow k = 4$
	$y = 4(x-3)^2 - 16 = 4(x^2 - 6x + 9) - 16$
	$= 4x^2 - 24x + 36 - 16$
	$f(x) = 4x^2 - 24x + 20$ as required

Question number

7 (b) By symmetry about $x = 3$, the curve also crosses the x-axis at $x = 5$

(check: $4x^2 - 24x + 20 = 4(x - 1)(x - 5)$).

$\int_5^a (4x^2 - 24x + 20)\,dx = \left[\frac{4}{3}x^3 - 12x^2 + 20x\right]_5^a = \frac{100}{3}$

$\left(\frac{4}{3}a^3 - 12a^2 + 20a\right) - \left(\frac{4}{3}(5)^3 - 12(5)^2 + 20(5)\right) = \frac{100}{3}$

$\frac{4}{3}a^3 - 12a^2 + 20a - \left(-\frac{100}{3}\right) = \frac{100}{3}$

$\frac{4}{3}a^3 - 12a^2 + 20a = 0 \quad \Rightarrow \quad a^3 - 9a^2 + 15a = 0$

$a(a^2 - 9a + 15) = 0$

$a = 0 \quad \text{or} \quad a = \frac{9 \pm \sqrt{(81 - 60)}}{2} = \frac{9 \pm \sqrt{21}}{2} \quad (= 6.791\ldots \text{ or } 2.208\ldots)$

From the diagram $a > 5$, so $a = \frac{9 + \sqrt{21}}{2}$

8 (a) $e^x - 2e^y = 1 \quad \Rightarrow \quad e^x = 1 + 2e^y$

Substitute: $2(1 + 2e^y) + 3e^{2y} = 41$

$2 + 4e^y + 3e^{2y} = 41$

$3e^{2y} + 4e^y - 39 = 0$ as required

8 (b) Let $u = e^y$: $3u^2 + 4u - 39 = 0$

$(3u + 13)(u - 3) = 0$

$u = 3 \quad \text{or} \quad u = -\frac{13}{3}$

$e^y > 0$ so $e^y \neq -\frac{13}{3}$, $\therefore \; e^y = 3 \quad \Rightarrow \quad y = \ln 3$

$e^x = 1 + 2e^y = 1 + 2(3) = 7 \quad \Rightarrow \quad x = \ln 7$

P is the point $(\ln 7, \ln 3)$

Question number	
9 (a)	$a = \frac{dv}{dt} = 0.1t - 0.4$
	$t = 6$: $a = 0.1(6) - 0.4 = 0.2 \text{ m s}^{-2}$
9 (b)	Minimum velocity when $\frac{dv}{dt} = 0$: $0.1t - 0.4 = 0 \Rightarrow t = 4$
	$v(4) = 0.05(4)^2 - 0.4(4) + k = 6.6$
	$0.8 - 1.6 + k = 6.6$
	$k = 6.6 + 0.8 = 7.4$ as required
9 (c)	$AB = \int_0^{13.1} (0.05t^2 - 0.4t + 7.4)\, dt$ ($v > 0$ throughout)
	$= \left[\frac{0.05}{3} t^3 - 0.2t^2 + 7.4t \right]_0^{13.1}$
	$= \frac{0.05}{3}(13.1)^3 - 0.2(13.1)^2 + 7.4(13.1)$
	$= 37.468... - 34.322 + 96.94 = 100.086...$
	$AB = 100$ m (3 s.f.)
10 (a)	For $0 \leqslant t \leqslant 20$: $v_P = 15 - 0.4t$
	$15 - 0.4t = 10 \Rightarrow 0.4t = 5 \Rightarrow t = 12.5$
10 (b)	At $t = 20$: $v_P = 15 - 0.4(20) = 7 \text{ m s}^{-1}$
	Distance of P in first 20 s $= \frac{1}{2}(15 + 7)(20) = 220$ m
	Q has travelled $10 \times 20 = 200$ m at $t = 20$, so Q has not yet overtaken.
	For $t \geqslant 20$: distance of Q $= 10t$, distance of P $= 220 + 7(t - 20)$
	Q overtakes P when $10t = 220 + 7(t - 20)$
	$10t = 7t + 80 \Rightarrow 3t = 80 \Rightarrow t = \frac{80}{3} = 26.666... \Rightarrow t = 26.7$ s

Question number

11 (a) C (3 kg) is heavier than A (2 kg), so B accelerates towards Q.

N2L for C: $3g - T_{BC} = 3a$

N2L for B: $T_{BC} - T_{AB} = 5a$

N2L for A: $T_{AB} - 2g = 2a$

Adding: $3g - 2g = 10a \Rightarrow g = 10a$

$a = \frac{g}{10}$ $(= 0.98\ m\ s^{-2})$

$T_{BC} = 3g - 3a = 3g - 3(\frac{g}{10}) = \frac{27}{10}g\ N$

11 (b) First 3 seconds ($u = 0$, $a = 0.98$):

$s_1 = \frac{1}{2} \times \frac{g}{10} \times 3^2 = 4.41\ m$, $v = 0 + 0.98 \times 3 = 2.94\ m\ s^{-1}$

After the string breaks (only B and C connected), with acceleration a_1:

N2L for C: $3g - T = 3a_1$ N2L for B: $T = 5a_1$

Adding: $3g = 8a_1 \Rightarrow a_1 = \frac{3}{8}g = 3.675\ m\ s^{-2}$

$s_2 = ut + \frac{1}{2}at^2 = 2.94(2) + \frac{1}{2}(3.675)(2)^2 = 5.88 + 7.35 = 13.23\ m$

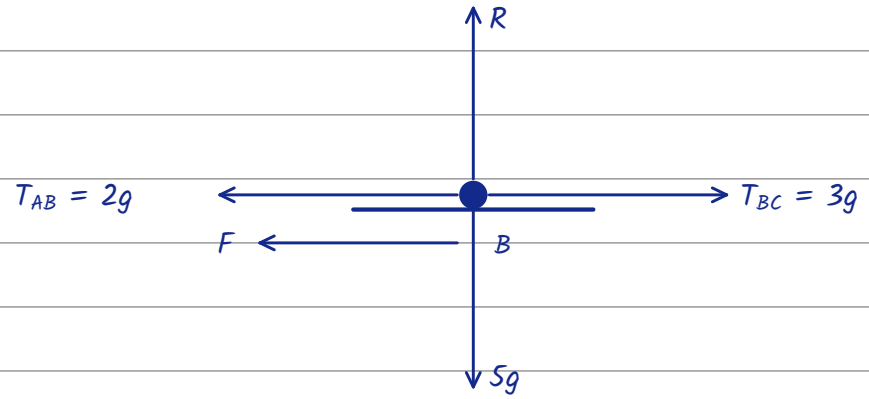
$PQ = 5 + 4.41 + 13.23 = 22.64$

$PQ = 22.6\ m$ (3 s.f.)

11 (c) The tension would not be the same throughout the string BC (the tension on either side of pulley Q would be different).

Question number

11 (d)



11 (e)

$F = g = 9.8$ N. B is in equilibrium, so resolving horizontally:

$F = T_{BC} - T_{AB} = 3g - 2g = g$ (each string tension equals the weight of the particle hanging at rest on it).