OCR
Oxford Cambridge and RSA

Thursday 16 May 2024 – Afternoon

AS Level Mathematics A

H230/01 Pure Mathematics and Statistics

Time allowed: 1 hour 30 minutes

133481904

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\overline{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2} \quad \text{or} \quad \sqrt{\frac{\Sigma f(x-\overline{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \overline{x}^2}$$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Kinematics

$$v = u + at$$

$$s = ut + \tfrac{1}{2}at^2$$

$$s = \tfrac{1}{2}(u+v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \tfrac{1}{2}at^2$$

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Section A
Pure Mathematics

1 **In this question you must show detailed reasoning.**

Solve the equation $\ln(x^3) - \ln(5x^2) = \ln 2$, where $x > 0$. **[3]**

2 Points A and B have position vectors $\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{OB} = -\mathbf{i} + 5\mathbf{j}$, respectively, where O is the origin.

(a) Find the position vector of the midpoint of AB. **[1]**

(b) Find the exact magnitude of the vector $\overrightarrow{AB}$. **[3]**

(c) Find the angle between the vector $\overrightarrow{OA}$ and the positive x-axis. **[2]**

3 In the triangle ABC, $AB = 3$, $AC = 5$ and angle $BAC = \alpha$.
The area of the triangle is $\frac{5}{2}$.
Determine the **two** possible values of $\cos\alpha$. Give your answers in exact form. **[5]**

4 **(a)** Write down the term in b^5 in the expansion of $(a+b)^{12}$. **[1]**

(b) A variable X has a binomial distribution such that $P(X = 5)$ is given by the term in part **(a)**.

(i) State a condition that must be satisfied by a and b. **[1]**

(ii) State the distribution of X. Give the values of any parameters. **[2]**

5 A curve has equation $y = 4x^2 + \frac{1}{x}$.

(a) Determine the coordinates of the stationary point on the curve. **[5]**

(b) Use $\frac{d^2y}{dx^2}$ to determine whether this turning point is a maximum or a minimum. **[2]**

(c) Find the values of x for which the curve is increasing. **[2]**

6 A curve has the following properties:

- The gradient of the curve is given by $\frac{dy}{dx} = -2x$.
- The curve passes through the point $(4, -13)$.

Determine the coordinates of the points where the curve meets the line $y = 2x$. **[7]**

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7 **In this question you must show detailed reasoning.**

(a) Show that $(x+2)^4 - 6(x+2)^2 - 16 \equiv x^4 + 8x^3 + 18x^2 + 8x - 24$. **[1]**

(b) Hence solve the equation $x^4 + 8x^3 + 18x^2 + 8x - 24 = 0$. **[4]**

8 A circle, C, has equation $x^2 - 6x + y^2 = 16$.

A second circle, D, has the following properties:

- The line through the centres of circle C and circle D has gradient 1.
- Circle D touches circle C at exactly one point.
- The centre of circle D lies in the first quadrant.
- Circle D has the same radius as circle C.

Find the coordinates of the centre of circle D. **[5]**

9 Express $\dfrac{a^{\frac{7}{2}} - a^{\frac{5}{2}}}{a^{\frac{3}{2}} - a}$ in the form $a^m + \sqrt{a^n}$, where m and n are integers and $a \neq 0$ or 1. **[5]**

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Section B
Statistics

10 There are 60 members in a sports club. The table shows the numbers of members in various categories.

	Beginner	Advanced
Junior	35	5
Senior	7	13

A member of the sports club is chosen at random.

(a) Find the probability that this member is a Beginner. **[1]**

A Junior member of the club is chosen at random.

(b) Find the probability that this Junior member is a Beginner. **[1]**

Two different members of the club are chosen at random.

(c) Determine the probability that both these members are Juniors and exactly one is a Beginner. **[2]**

11 A biased six-sided dice is thrown several times. The results are shown in the table, where a is a constant.

Score x	1	2	3	4	5	6
Frequency f	1	3	10	9	0	a

You are given that the mean score is 3.4.

(a) (i) Find the value of a. **[3]**

(ii) Determine the standard deviation of the scores. You should use one of the formulae given on page 2. **[3]**

A fair 6-sided dice with faces numbered 1, 2, 3, 4, 5, 6 is thrown a large number of times, and the standard deviation s of the scores is found.

(b) Without calculation, explain whether s is likely to be larger, smaller, or about the same size as the standard deviation found in part **(a)(ii)**. **[1]**

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12 The table gives the percentage increases in the numbers of usual residents in five Local Authorities (LAs) in the West Midlands between 2001 and 2011.

	Age ranges					
LA	**0 to 9**	**10 to 19**	**20 to 59**	**60 to 64**	**65 to 74**	**75 or above**
Lichfield	−1%	0%	0%	33%	50%	29%
Redditch	2%	−7%	3%	77%	31%	14%
Rugby	11%	11%	12%	33%	29%	17%
Stafford	4%	1%	3%	35%	28%	19%
Stratford-on-Avon	−2%	8%	−1%	40%	34%	29%

(a) Use the data to suggest, with a reason, which of these LAs was likely to have the most urgent need for more provision for elderly people in the years from 2012 to 2020. **[1]**

In 2011, local government officers in the West Midlands attempted to judge which LAs would be most in need of extra provision for secondary schools in the years from 2012 to 2020.

(b) One officer said

"In Rugby, the figures for 0 to 9 and 10 to 19 are the same, so there will be no need for extra provision for secondary schools in Rugby in the years 2012 to 2020."

Explain whether you agree with this statement. **[1]**

(c) Use the data to explain the issues likely to have affected provision for secondary schools in 2012 to 2020 in the following LAs.

(i) Redditch **[1]**

(ii) Stratford-on-Avon **[1]**

(d) State an assumption you needed to make in answering parts **(a)**, **(b)** and **(c)**. **[1]**

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13 Some students at a large school are doing a survey about attitudes to school uniform. They want to give a questionnaire to a random sample of 50 students at their school.

The school has three entrances. Gabi suggests giving the questionnaire to the first 50 students who arrive at a particular entrance on a randomly chosen morning.

(a) Give **two** reasons why this method of sampling will **not** produce a random sample. **[2]**

The students are also doing a survey about attitudes to school lunches. Jane claims that 30% of students think the lunches are satisfactory. Dev claims that the true percentage is less than 30%. The students decide to test Jane's claim by taking a **random** sample of 50 students and noting the number who think the lunches satisfactory.

Out of the 50 students in the sample, 9 said that they thought the lunches are satisfactory.

(b) Using a binomial distribution as a model, test Jane's claim against Dev's claim at the 5% significance level. **[7]**

(c) The sample used by the students was taken without replacement. One student suggests that this means that one of the assumptions underlying the model is **not** valid.

Explain whether you agree with this statement. **[1]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1

$\ln(x^3) - \ln(5x^2) = \ln 2$

$\ln\left(\frac{x^3}{5x^2}\right) = \ln 2$

$\ln\left(\frac{x}{5}\right) = \ln 2$

$\frac{x}{5} = 2$

$x = 10$

2 (a)

$\overrightarrow{OM} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB}) = \frac{1}{2}(2\mathbf{i} + 3\mathbf{j} - \mathbf{i} + 5\mathbf{j}) = \frac{1}{2}(\mathbf{i} + 8\mathbf{j})$

Position vector of midpoint $= 0.5\mathbf{i} + 4\mathbf{j}$

2 (b)

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (-\mathbf{i} + 5\mathbf{j}) - (2\mathbf{i} + 3\mathbf{j}) = -3\mathbf{i} + 2\mathbf{j}$

$|\overrightarrow{AB}| = \sqrt{(-3)^2 + 2^2} = \sqrt{9 + 4}$

$|\overrightarrow{AB}| = \sqrt{13}$

2 (c)

$\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j}$: $\tan\theta = \frac{3}{2}$

$\theta = \tan^{-1}(1.5) = 56.309...° = 56.3°$ (3 s.f.)

3

Area $= \frac{1}{2} \times 3 \times 5 \times \sin\alpha = \frac{5}{2}$

$\frac{15}{2}\sin\alpha = \frac{5}{2} \quad \Rightarrow \quad \sin\alpha = \frac{1}{3}$

$\cos^2\alpha = 1 - \sin^2\alpha = 1 - \left(\frac{1}{3}\right)^2 = 1 - \frac{1}{9} = \frac{8}{9}$

$\cos\alpha = \pm\sqrt{8/9} = \pm\frac{\sqrt{8}}{3}$

$\cos\alpha = \frac{2\sqrt{2}}{3}$ or $\cos\alpha = -\frac{2\sqrt{2}}{3}$

4 (a)

${}^{12}C_5\, a^7 b^5 = 792 a^7 b^5$

4 (b)(i)

$a + b = 1$

Question number	
4 (b)(ii)	$X \sim B(12, b)$ (binomial, $n = 12$, $p = b$)
5 (a)	$y = 4x^2 + x^{-1} \Rightarrow \frac{dy}{dx} = 8x - x^{-2}$
	Stationary point: $8x - \frac{1}{x^2} = 0$
	$8x^3 = 1 \Rightarrow x^3 = \frac{1}{8} \Rightarrow x = \frac{1}{2}$
	$y = 4 \times \left(\frac{1}{2}\right)^2 + \frac{1}{(1/2)} = 1 + 2 = 3$
	Stationary point is $\left(\frac{1}{2}, 3\right)$
5 (b)	$\frac{d^2y}{dx^2} = 8 + 2x^{-3} = 8 + \frac{2}{x^3}$
	At $x = \frac{1}{2}$: $\frac{d^2y}{dx^2} = 8 + \frac{2}{(1/8)} = 8 + 16 = 24 > 0$
	Hence the turning point is a minimum
5 (c)	Increasing when $\frac{dy}{dx} \geq 0$: $8x - \frac{1}{x^2} \geq 0$
	Multiply by $x^2 > 0$: $8x^3 - 1 \geq 0 \Rightarrow x^3 \geq \frac{1}{8}$
	$x \geq \frac{1}{2}$
6	$y = \int -2x \, dx = -x^2 + c$
	Through $(4, -13)$: $-13 = -4^2 + c = -16 + c \Rightarrow c = 3$
	$y = 3 - x^2$
	Meets $y = 2x$: $3 - x^2 = 2x$
	$x^2 + 2x - 3 = 0$
	$(x + 3)(x - 1) = 0 \Rightarrow x = 1$ or $x = -3$
	$x = 1$: $y = 2 \times 1 = 2$ $x = -3$: $y = 2 \times (-3) = -6$
	Points are $(1, 2)$ and $(-3, -6)$
7 (a)	$(x + 2)^4 = x^4 + 4(2)x^3 + 6(4)x^2 + 4(8)x + 16$
	$= x^4 + 8x^3 + 24x^2 + 32x + 16$
	$6(x + 2)^2 = 6(x^2 + 4x + 4) = 6x^2 + 24x + 24$
	$(x + 2)^4 - 6(x + 2)^2 - 16$

Question number	
	$= x^4 + 8x^3 + 24x^2 + 32x + 16 - 6x^2 - 24x - 24 - 16$
	$= x^4 + 8x^3 + 18x^2 + 8x - 24$ as required
7 (b)	$x^4 + 8x^3 + 18x^2 + 8x - 24 = 0 \Leftrightarrow (x + 2)^4 - 6(x + 2)^2 - 16 = 0$
	Let $y = (x + 2)^2$: $y^2 - 6y - 16 = 0$
	$(y - 8)(y + 2) = 0 \Rightarrow (x + 2)^2 = 8$ or $(x + 2)^2 = -2$
	$(x + 2)^2 = -2$ has no real roots, since $(x + 2)^2 \geqslant 0$
	$(x + 2)^2 = 8 \Rightarrow x + 2 = \pm\sqrt{8} = \pm 2\sqrt{2}$
	$x = -2 + 2\sqrt{2}$ or $x = -2 - 2\sqrt{2}$
8	C: $x^2 - 6x + y^2 = 16 \Rightarrow (x - 3)^2 - 9 + y^2 = 16 \Rightarrow (x - 3)^2 + y^2 = 25$
	Centre of C is (3, 0), radius 5
	D has radius 5 and touches C once, so distance between centres = 10
	Line of centres has gradient 1, so centre of D = $(3 + k, k)$,
	with $k > 0$ since the centre is in the first quadrant
	$k^2 + k^2 = 10^2 \Rightarrow 2k^2 = 100 \Rightarrow k = \sqrt{50} = 5\sqrt{2}$ $(= 10\cos 45°)$
	Centre of D is $(3 + 5\sqrt{2},\ 5\sqrt{2})$ $(\approx (10.1, 7.07))$
9	$\frac{a^{7/2} - a^{5/2}}{a^{3/2} - a} = \frac{a^{5/2}(a - 1)}{a(a^{1/2} - 1)} \times \frac{a^{1/2} + 1}{a^{1/2} + 1}$
	$= \frac{a^{5/2}(a - 1)(a^{1/2} + 1)}{a(a - 1)}$ since $(a^{1/2} - 1)(a^{1/2} + 1) = a - 1$
	$= a^{3/2}(a^{1/2} + 1)$
	$= a^2 + a^{3/2}$
	$= a^2 + \sqrt{a^3}$ $(m = 2,\ n = 3)$
10 (a)	$P(\text{Beginner}) = \frac{35 + 7}{60} = \frac{42}{60} = \frac{7}{10}$ $(= 0.7)$
10 (b)	$P(\text{Beginner} \mid \text{Junior}) = \frac{35}{35 + 5} = \frac{35}{40} = \frac{7}{8}$ $(= 0.875)$
10 (c)	$P = P(\text{JB then JA}) + P(\text{JA then JB}) = \frac{35}{60} \times \frac{5}{59} \times 2$

Question number	
	$= \frac{350}{3540} = \frac{35}{354} = 0.098870...$
	$= 0.0989$ (3 s.f.)
11 (a)(i)	$\Sigma f = 1 + 3 + 10 + 9 + 0 + a = 23 + a$
	$\Sigma xf = 1 + 6 + 30 + 36 + 0 + 6a = 73 + 6a$
	$\text{Mean} = \frac{73 + 6a}{23 + a} = 3.4 \Rightarrow 73 + 6a = 78.2 + 3.4a$
	$2.6a = 5.2 \Rightarrow a = 2$
11 (a)(ii)	$\Sigma x^2 f = 1 + 12 + 90 + 144 + 0 + 72 = 319, \quad \Sigma f = 25$
	$sd = \sqrt{319/25 - 3.4^2} = \sqrt{12.76 - 11.56} = \sqrt{1.2}$
	$= 1.0954... = 1.10$ (3 s.f.)
11 (b)	s is likely to be larger. With a fair dice the scores will be spread
	evenly over 1 to 6, whereas the scores in the table are concentrated
	towards the middle (mostly 3 and 4).
12 (a)	Redditch: it had a large (77%) increase in the 60 to 64 age range,
	and these people will be elderly (65 and over) in the years 2012 to 2020.
12 (b)	No, I disagree. There was an 11% increase in the 0 to 9 age range,
	and these children will reach secondary school age in 2012 to 2020,
	so more provision will be needed (the increase in 10 to 19 has
	already happened, regardless of this).
12 (c)(i)	Redditch: the 10 to 19 numbers fell by 7%, so secondary school
	facilities may have been reduced; but the 0 to 9 numbers rose by 2%,
	so as these children reach secondary age, extra provision may still
	be a problem.

Question number	
12 (c)(ii)	Stratford-on-Avon: the 10 to 19 numbers rose by 8%, so extra secondary provision has already been made; the 0 to 9 numbers fell by 2%, so there will be spare capacity.
12 (d)	That there is little movement of people into or out of the LAs (no significant change in the population other than ageing) between 2011 and 2020.
13 (a)	1. Only students who use that one entrance can be chosen; students arriving at the other two entrances are not included. 2. Only the first 50 to arrive, on one morning, are chosen; students who arrive later or on other days are not included, and early arrivers may not be representative.
13 (b)	Let p = proportion of students at the school who think the lunches are satisfactory. H_0: $p = 0.3$ H_1: $p < 0.3$ Let X = number in the sample who think the lunches are satisfactory. Under H_0, $X \sim B(50, 0.3)$. Observed $X = 9$ $P(X \leq 9) = 0.040231\ldots = 0.0402$ (3 s.f.) $0.0402 < 0.05$, so reject H_0 There is sufficient evidence at the 5% level to suggest that the proportion of students who think the lunches are satisfactory is less than 0.3, which supports Dev's claim.
13 (c)	Yes, I agree. Since the sample is taken without replacement, the probability that each chosen student thinks the lunches are satisfactory changes slightly after each selection, so the trials are not independent (p is not constant).