OCR
Oxford Cambridge and RSA

Thursday 25 May 2023 – Afternoon

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes

988746455O

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\overline{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2} \quad \text{or} \quad \sqrt{\frac{\Sigma f(x-\overline{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \overline{x}^2}$$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Kinematics

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

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Section A

Pure Mathematics

1 The quadratic equation $kx^2 + 3x + k = 0$ has no real roots.

Determine the set of possible values of k. **[3]**

2 **In this question you must show detailed reasoning.**

Solve the equation $x\sqrt{5} + 32 = x\sqrt{45} + 2x$. Give your answer in the form $a\sqrt{5} + b$, where a and b are integers to be determined. **[4]**

3 A Ferris wheel at a fairground rotates in a vertical plane. The height above the ground of a seat on the wheel is h metres at time t seconds after the seat is at its lowest point.

The height is given by the equation $h = 15 - 14\cos(kt)°$, where k is a positive constant.

(a) **(i)** Write down the greatest height of a seat above the ground. **[1]**

(ii) Write down the least height of a seat above the ground. **[1]**

(b) Given that a seat first returns to its lowest point after 150 seconds, calculate the value of k. **[2]**

(c) Determine for how long a seat is 20 metres or more above the ground during one complete revolution. Give your answer correct to the nearest tenth of a second. **[4]**

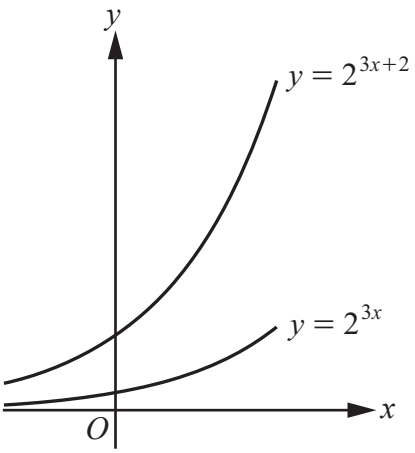
4 **(a)** Find and simplify the first three terms in the expansion, in ascending powers of x, of $\left(2 + \frac{1}{3}kx\right)^6$, where k is a constant. **[3]**

(b) In the expansion of $(3 - 4x)\left(2 + \frac{1}{3}kx\right)^6$, the constant term is equal to the coefficient of x^2.

Determine the exact value of k, given that k is positive. **[3]**

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The diagram shows the graphs of $y = 2^{3x}$ and $y = 2^{3x+2}$. The graph of $y = 2^{3x}$ can be transformed to the graph of $y = 2^{3x+2}$ by means of a stretch.

(a) Give details of the stretch. **[2]**

The point A lies on $y = 2^{3x}$ and the point B lies on $y = 2^{3x+2}$. The line segment AB is parallel to the y-axis and the difference between the y-coordinates of A and B is 36.

(b) Determine the x-coordinate of A. Give your answer in the form $m \log_2 n$ where m and n are constants to be determined. **[3]**

6 The vertices of triangle ABC are $A(-3, 1)$, $B(5, 0)$ and $C(9, 7)$.

(a) Show that $AB = BC$. **[2]**

(b) Show that angle ABC is **not** a right angle. **[2]**

(c) Find the coordinates of the midpoint of AC. **[1]**

(d) Determine the equation of the line of symmetry of the triangle, giving your answer in the form $px + qy = r$, where p, q and r are integers to be determined. **[2]**

(e) Write down an equation of the circle with centre A which passes through B. **[2]**

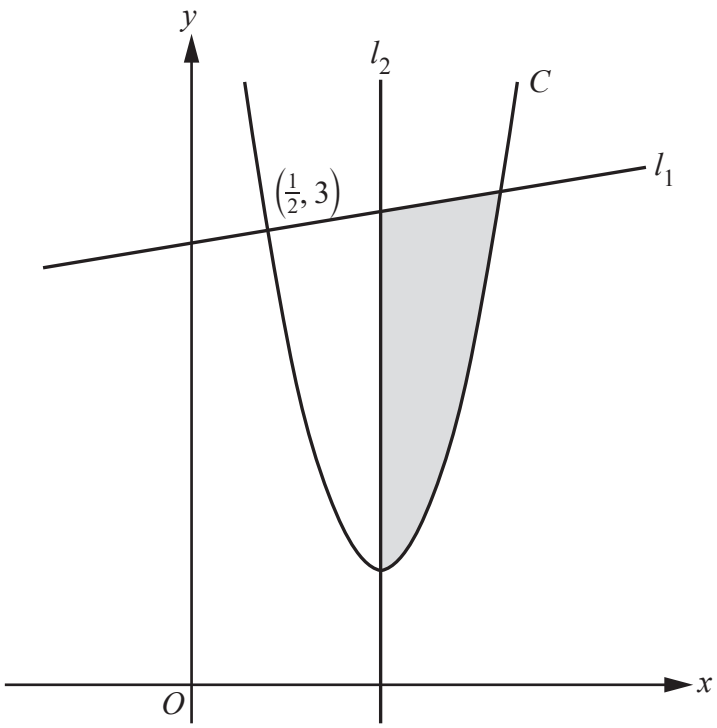
This circle intersects the line of symmetry of the triangle at B and at a second point.

(f) Find the coordinates of this second point. **[1]**

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The diagram shows the curve C with equation $y = 4x^2 - 10x + 7$ and two straight lines, l_1 and l_2. The line l_1 is the normal to C at the point $\left(\frac{1}{2}, 3\right)$. The line l_2 is the normal to C at the minimum point of C.

(a) Determine the equation of l_1, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be determined. **[4]**

The shaded region shown in the diagram is bounded by C, l_1 and l_2.

(b) Determine the inequalities that define the shaded region, including its boundaries. **[3]**

8 **In this question you must show detailed reasoning.**

Given that $\int_4^a \left(\frac{4}{\sqrt{x}} + 3\right) \mathrm{d}x = 7$, find the value of a. **[7]**

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Section B

Mechanics

9 A cyclist travels along a straight horizontal road between house A and house B.

The cyclist starts from rest at A and moves with constant acceleration for 20 seconds, reaching a velocity of $15\,\text{m}\,\text{s}^{-1}$. The cyclist then moves at this constant velocity before decelerating at $0.3\,\text{m}\,\text{s}^{-2}$, coming to rest at B.

(a) Find the time, in seconds, for which the cyclist is decelerating. **[1]**

(b) Sketch a velocity-time graph for the motion of the cyclist between A and B. [Your sketch need not be drawn to scale; numerical values need not be shown.] **[1]**

The total distance between A and B is 1950 m.

(c) Find the time, in seconds, for which the cyclist is moving at constant velocity. **[2]**

10 A particle P is moving in a straight line. At time t seconds, where $t \geqslant 0$, P has velocity $v\,\text{m}\,\text{s}^{-1}$ and acceleration $a\,\text{m}\,\text{s}^{-2}$ where $a = 4t - 9$. It is given that $v = 2$ when $t = 1$.

(a) Find an expression for v in terms of t. **[3]**

The particle P is instantaneously at rest when $t = t_1$ and $t = t_2$, where $t_1 < t_2$.

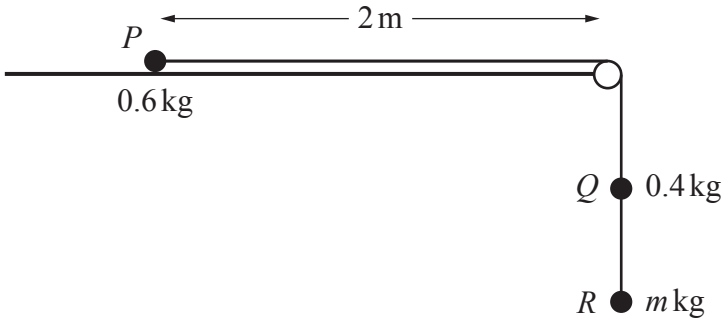
(b) Find the values of t_1 and t_2. **[2]**

(c) Determine the total distance travelled by P between times $t = 0$ and $t = t_2$. **[3]**

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11 Two balls P and Q have masses 0.6 kg and 0.4 kg respectively. The balls are attached to the ends of a string. The string passes over a pulley which is fixed at the edge of a rough horizontal surface. Ball P is held at rest on the surface 2 m from the pulley. Ball Q hangs vertically below the pulley. Ball Q is attached to a third ball R of mass m kg by another string and R hangs vertically below Q (see diagram).



The system is released from rest with the strings taut. Ball P moves towards the pulley with acceleration $3.5\,\text{m}\,\text{s}^{-2}$ and a constant frictional force of magnitude 4.5 N opposes the motion of P.

The balls are modelled as particles, the pulley is modelled as being small and smooth, and the strings are modelled as being light and inextensible.

(a) By considering the motion of P, find the tension in the string connecting P and Q. **[2]**

(b) Hence determine the value of m. Give your answer correct to **3** significant figures. **[4]**

When the balls have been in motion for 0.4 seconds the string connecting Q and R breaks.

(c) Show that, according to the model, P does not reach the pulley. **[6]**

It is given that in fact ball P does reach the pulley.

(d) Identify one factor in the modelling that could account for this difference. **[1]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1

No real roots $\Rightarrow$ $b^2 - 4ac < 0$

$\Delta = 3^2 - 4(k)(k) = 9 - 4k^2$

$9 - 4k^2 < 0 \quad \Rightarrow \quad k^2 > 9/4$

Critical values $k = \pm 3/2$

$k < -3/2$ or $k > 3/2$

2

$x\sqrt{5} + 32 = x\sqrt{45} + 2x$

$32 = x\sqrt{45} + 2x - x\sqrt{5} \quad \Rightarrow \quad x(\sqrt{45} + 2 - \sqrt{5}) = 32$

$\sqrt{45} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$, so $x(3\sqrt{5} + 2 - \sqrt{5}) = 32 \quad \Rightarrow \quad x(2\sqrt{5} + 2) = 32$

$x = 32/(2\sqrt{5} + 2) \times (2\sqrt{5} - 2)/(2\sqrt{5} - 2)$

$x = 32(2\sqrt{5} - 2)/((2\sqrt{5})^2 - 2^2) = 32(2\sqrt{5} - 2)/(20 - 4) = 32(2\sqrt{5} - 2)/16$

$x = 2(2\sqrt{5} - 2)$

$x = 4\sqrt{5} - 4 \qquad (a = 4, \ b = -4)$

3 (a)(i)

Greatest height = 15 + 14 = 29 m

3 (a)(ii)

Least height = 15 − 14 = 1 m

3 (b)

At $t = 150$, $h = 1$: $15 - 14\cos(150k) = 1 \quad \Rightarrow \quad \cos(150k) = 1$

First return to lowest point, so $150k = 360$

$k = 2.4$

Question number	
3 (c)	$15 - 14\cos(2.4t) = 20 \Rightarrow \cos(2.4t) = -5/14$
	$2.4t = 110.9248...°$ or $2.4t = 360 - 110.9248... = 249.0751...°$
	$t = 46.2186...$ or $t = 103.7813...$
	$h \geq 20$ between these times: $103.7813... - 46.2186... = 57.5626...$
	Time at 20 m or more = 57.6 s
4 (a)	$(2 + \frac{1}{3}kx)^6 = 2^6 + {}^6C_1\, 2^5(\frac{1}{3}kx) + {}^6C_2\, 2^4(\frac{1}{3}kx)^2 + ...$
	$= 64 + 6 \times 32 \times \frac{1}{3}kx + 15 \times 16 \times (1/9)k^2x^2 + ...$
	$= 64 + 64kx + (80/3)k^2x^2 + ...$
4 (b)	$(3 - 4x)(64 + 64kx + (80/3)k^2x^2 + ...)$
	Constant term = $3 \times 64 = 192$
	Coefficient of $x^2 = 3 \times (80/3)k^2 - 4 \times 64k = 80k^2 - 256k$
	$80k^2 - 256k = 192 \Rightarrow 80k^2 - 256k - 192 = 0 \Rightarrow 5k^2 - 16k - 12 = 0$
	$k = (16 \pm \sqrt{(16^2 + 4 \times 5 \times 12)})/10 = (16 \pm \sqrt{496})/10 = (16 \pm 4\sqrt{31})/10$
	$k > 0$, so $k = (8 + 2\sqrt{31})/5$
5 (a)	$2^{3x+2} = 2^2 \times 2^{3x} = 4 \times 2^{3x}$
	Stretch parallel to the y-axis, scale factor 4
5 (b)	Same x-coordinate: $2^{3x+2} - 2^{3x} = 36$
	$4 \times 2^{3x} - 2^{3x} = 36 \Rightarrow 3 \times 2^{3x} = 36 \Rightarrow 2^{3x} = 12$
	$\log_2(2^{3x}) = \log_2 12 \Rightarrow 3x = \log_2 12$
	$x = \frac{1}{3}\log_2 12$
6 (a)	$AB = \sqrt{((-3 - 5)^2 + (1 - 0)^2)} = \sqrt{(64 + 1)} = \sqrt{65}$
	$BC = \sqrt{((5 - 9)^2 + (0 - 7)^2)} = \sqrt{(16 + 49)} = \sqrt{65}$
	$\therefore AB = BC = \sqrt{65}$

Question number	
6 (b)	$AC = \sqrt{((-3 - 9)^2 + (1 - 7)^2)} = \sqrt{(144 + 36)} = \sqrt{180}$
	$AB^2 + BC^2 = 65 + 65 = 130, \quad AC^2 = 180$
	$130 \neq 180$, so Pythagoras does not hold: angle ABC is not a right angle.
6 (c)	Midpoint of $AC = ((-3 + 9)/2, (1 + 7)/2) = (3, 4)$
6 (d)	Line of symmetry passes through $B(5, 0)$ and the midpoint of AC, $(3, 4)$
	Gradient $= (4 - 0)/(3 - 5) = -2$
	$y - 0 = -2(x - 5) \quad \Rightarrow \quad y = -2x + 10$
	$2x + y = 10$
6 (e)	$\text{Radius}^2 = AB^2 = 65$
	$(x + 3)^2 + (y - 1)^2 = 65$
6 (f)	$y = 10 - 2x$: $(x + 3)^2 + (9 - 2x)^2 = 65 \quad \Rightarrow \quad 5x^2 - 30x + 25 = 0$
	$x^2 - 6x + 5 = 0 \quad \Rightarrow \quad (x - 1)(x - 5) = 0, \quad x = 5$ is B
	Second point is $(1, 8)$
7 (a)	$y = 4x^2 - 10x + 7 \quad \Rightarrow \quad dy/dx = 8x - 10$
	At $(½, 3)$: $m_T = 8(½) - 10 = -6 \quad \Rightarrow \quad m_N = -1/(-6) = 1/6$
	$y - 3 = (1/6)(x - ½)$
	$6y - 18 = x - ½ \quad \Rightarrow \quad 12y - 36 = 2x - 1$
	$2x - 12y + 35 = 0$
7 (b)	Minimum point: $dy/dx = 8x - 10 = 0 \quad \Rightarrow \quad x = 1.25$, so l_2 is $x = 1.25$
	Shaded region (right of l_2, above C, below l_1):
	$x \geq 1.25, \quad y \geq 4x^2 - 10x + 7, \quad 2x - 12y + 35 \geq 0$

Question number

8

$\int_4^a (4x^{-\frac{1}{2}} + 3)\,dx = [\,8x^{\frac{1}{2}} + 3x\,]_4^a$

$= (8a^{\frac{1}{2}} + 3a) - (8 \times 2 + 12) = (8a^{\frac{1}{2}} + 3a) - (16 + 12)$

$8a^{\frac{1}{2}} + 3a - 28 = 7 \quad \Rightarrow \quad 3a + 8a^{\frac{1}{2}} - 35 = 0$

Quadratic in $a^{\frac{1}{2}}$: $(3a^{\frac{1}{2}} - 7)(a^{\frac{1}{2}} + 5) = 0$

$a^{\frac{1}{2}} = 7/3$ or $a^{\frac{1}{2}} = -5$

$a^{\frac{1}{2}} \neq -5$ since $a^{\frac{1}{2}}$ cannot be negative

$a^{\frac{1}{2}} = 7/3 \quad \Rightarrow \quad a = 49/9$

9 (a)

$v = u + at$: $0 = 15 - 0.3t \quad \Rightarrow \quad t = 15/0.3 = 50$ s

9 (b)



9 (c)

Distance = area under graph, with T = time at constant velocity:

$\frac{1}{2}(15)(20) + 15T + \frac{1}{2}(15)(50) = 1950$

$150 + 15T + 375 = 1950 \quad \Rightarrow \quad 15T = 1425$

$T = 95$ s

10 (a)

$v = \int (4t - 9)\,dt = 2t^2 - 9t + c$

$t = 1, v = 2$: $2 = 2 - 9 + c \quad \Rightarrow \quad c = 9$

$v = 2t^2 - 9t + 9$

Question number	
10 (b)	$v = 0$: $2t^2 - 9t + 9 = 0 \Rightarrow (2t - 3)(t - 3) = 0$
	$t_1 = 1.5, \quad t_2 = 3$
10 (c)	$\int (2t^2 - 9t + 9)\,dt = \frac{2}{3}t^3 - 4.5t^2 + 9t$
	$\int_0^{1.5} (2t^2 - 9t + 9)\,dt = 2.25 - 10.125 + 13.5 - 0 = 45/8$
	$\int_{1.5}^{3} (2t^2 - 9t + 9)\,dt = (18 - 40.5 + 27) - 45/8 = 4.5 - 5.625 = -9/8$
	Total distance $= 45/8 + 9/8 = 54/8$
	$= 6.75$ m
11 (a)	N2L for P: $T_1 - 4.5 = 0.6 \times 3.5$
	$T_1 = 2.1 + 4.5 = 6.6$ N
11 (b)	For Q: $T_2 + 0.4g - T_1 = 0.4 \times 3.5 \Rightarrow T_2 + 3.92 - 6.6 = 1.4$
	$T_2 = 4.08$ N
	For R: $mg - T_2 = m \times 3.5 \Rightarrow 9.8m - 4.08 = 3.5m$
	$6.3m = 4.08 \Rightarrow m = 0.647619...$
	$m = 0.648$ (3 s.f.)
11 (c)	Before the string breaks (0.4 s): $s = \frac{1}{2}(3.5)(0.4)^2 = 0.28$ m
	Speed of P when the string breaks: $v = 3.5 \times 0.4 = 1.4$ m s^{-1}
	After it breaks, acceleration a towards the pulley:
	For P: $T - 4.5 = 0.6a$, For Q: $0.4g - T = 0.4a$
	Adding: $3.92 - 4.5 = a \Rightarrow a = -0.58$ m s^{-2}
	P comes to rest: $v^2 = u^2 + 2as \Rightarrow 0 = 1.4^2 + 2(-0.58)s$
	$s = 1.96/1.16 = 1.689655...$ m
	Total distance $= 0.28 + 1.689655... = 1.9696...$ m
	$1.9696... < 2$, so according to the model P does not reach the pulley.
11 (d)	The string may not be inextensible (it could stretch), so P could travel further.