Oxford Cambridge and RSA

Thursday 18 May 2023 – Afternoon

AS Level Mathematics A

H230/01 Pure Mathematics and Statistics

Time allowed: 1 hour 30 minutes

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Kinematics

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

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Section A
Pure Mathematics

1 **(a)** Prove that $\cos x + \sin x \tan x \equiv \dfrac{1}{\cos x}$ (where $x \neq \frac{1}{2}n\pi$ for any odd integer n). **[3]**

(b) Solve the equation $2\sin^2 x = \cos^2 x$ for $0° \leqslant x \leqslant 180°$. **[2]**

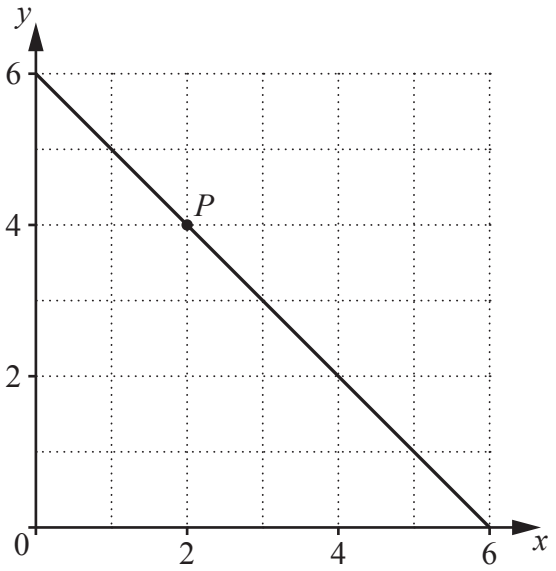
2 **(a)** The points A, B and C have position vectors $\begin{pmatrix}-4\\3\end{pmatrix}$, $\begin{pmatrix}-3\\6\end{pmatrix}$ and $\begin{pmatrix}-1\\12\end{pmatrix}$ respectively.

(i) Show that B lies on AC. **[2]**

(ii) Find the ratio $AB : BC$. **[1]**

(b) The diagram shows the line $x + y = 6$ and the point $P(2, 4)$ that lies on the line.

A copy of the diagram is given in the Printed Answer Booklet.



The distinct point Q also lies on the line $x + y = 6$ and is such that $|\overrightarrow{OQ}| = |\overrightarrow{OP}|$, where O is the origin.

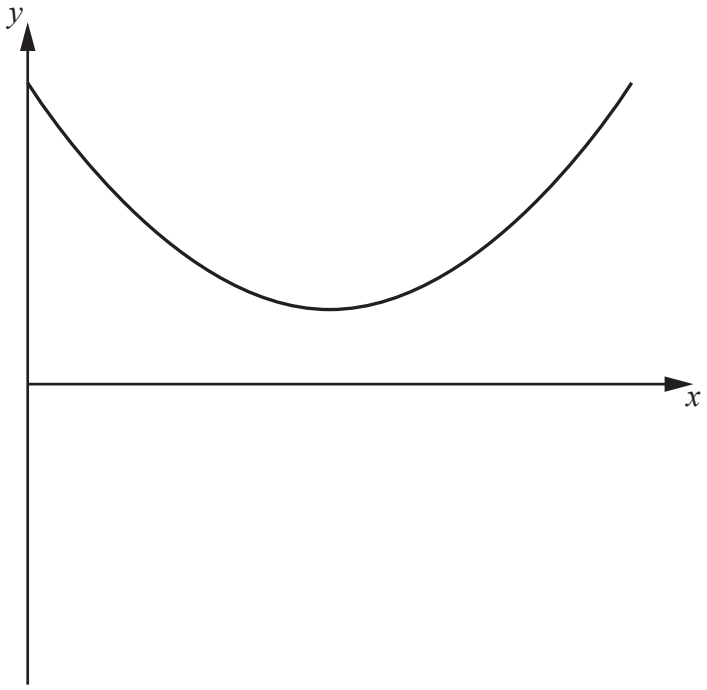
Find the magnitude and direction of the vector $\overrightarrow{PQ}$. **[3]**

(c) The point R is such that $\overrightarrow{PR}$ is perpendicular to $\overrightarrow{OP}$ and $|\overrightarrow{PR}| = \frac{1}{2}|\overrightarrow{OP}|$.

Write down the position vectors of the **two** possible positions of the point R. **[2]**

3 The diagram shows the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is a **quadratic** function of x.

A copy of the diagram is given in the Printed Answer Booklet.



(a) On the copy of the diagram in the Printed Answer Booklet, draw a possible graph of the gradient function $y = \mathrm{f}'(x)$. **[3]**

(b) State the gradient of the graph of $y = \mathrm{f}''(x)$. **[1]**

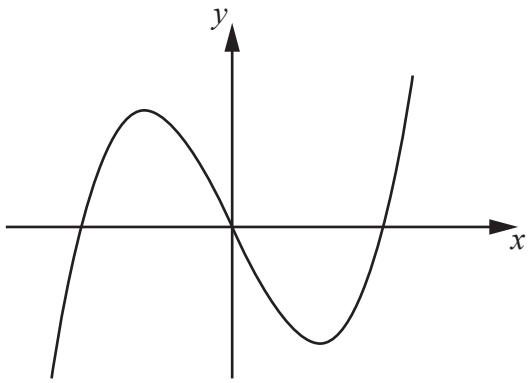
4 A curve has equation $y = \mathrm{e}^{3x}$.

(a) Determine the value of x when $y = 10$. **[2]**

(b) Determine the gradient of the tangent to the curve at the point where $x = 2$. **[2]**

5 **In this question you must show detailed reasoning.**

The diagram shows part of the graph of $y = x^3 - 4x$.



Determine the total area enclosed by the curve and the x-axis. **[6]**

6 **(a)** Determine the **two** real roots of the equation $8x^6 + 7x^3 - 1 = 0$. **[3]**

(b) Determine the coordinates of the stationary points on the curve $y = 8x^7 + \frac{49}{4}x^4 - 7x$. **[4]**

(c) For each of the stationary points, use the value of $\frac{d^2y}{dx^2}$ to determine whether it is a maximum or a minimum. **[4]**

7 **(a)** Two real numbers are denoted by a and b.

(i) Write down expressions for the following.

- The mean of the squares of a and b
- The square of the mean of a and b **[1]**

(ii) Prove that the mean of the squares of a and b is greater than or equal to the square of their mean. **[3]**

(b) You are given that the result in part **(a)(ii)** is true for any two or more real numbers.

Explain what this result shows about the variance of a set of data. **[1]**

8 In this question you must show detailed reasoning.

A circle has equation $x^2 + y^2 - 6x - 4y + 12 = 0$. Two tangents to this circle pass through the point (0, 1).

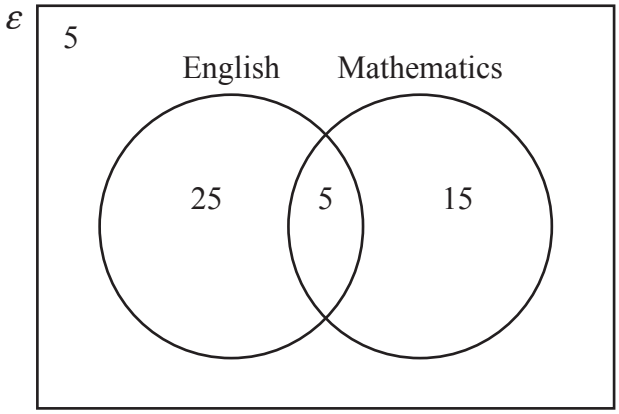
You are given that the scales on the x-axis and the y-axis are the same.

Find the angle between these two tangents. **[7]**

Section B
Statistics

9 In a survey, 50 people were asked whether they had passed A-level English and whether they had passed A-level Mathematics.

The numbers of people in various categories are shown in the Venn diagram.



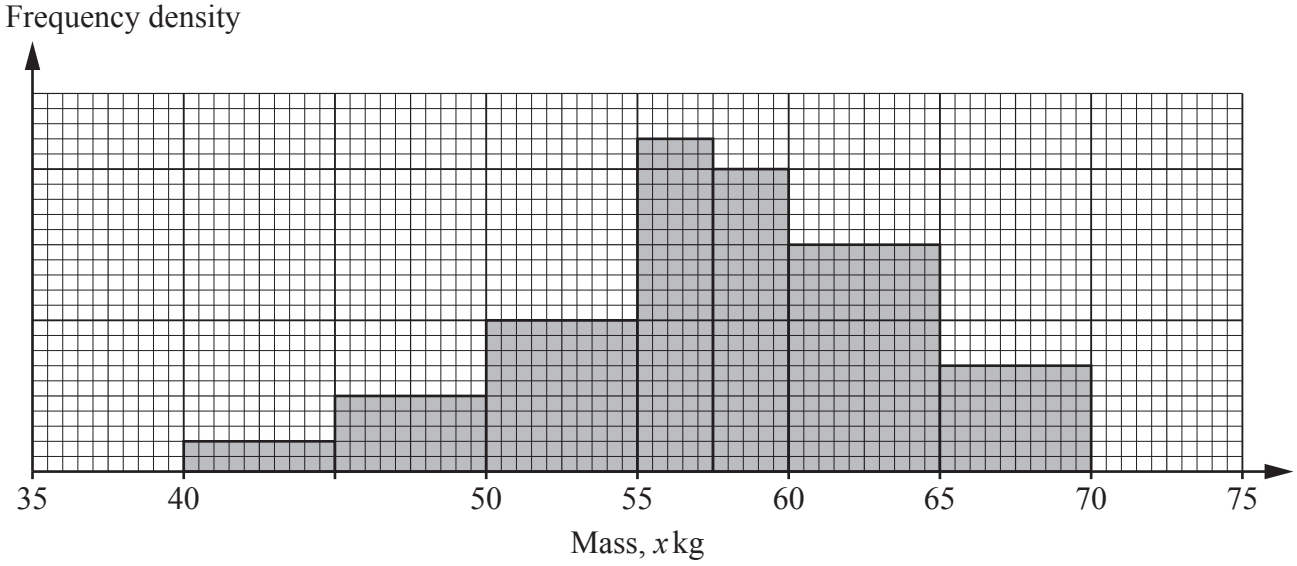
(a) A person is chosen at random from the 50 people.

Find the probability that this person has passed A-level Mathematics. **[1]**

(b) Two people are chosen at random, **without replacement**, from those who have passed A-level in at least one of the two subjects.

Determine the probability that both of these people have passed A-level Mathematics. **[3]**

10 The masses of a random sample of 120 boulders in a certain area were recorded. The results are summarized in the histogram.



(a) Calculate the number of boulders with masses between 60 and 65 kg. **[2]**

(b) **(i)** Use midpoints to find estimates of the mean and standard deviation of the masses of the boulders in the sample. **[3]**

(ii) Explain why your answers are only estimates. **[1]**

(c) Use your answers to part **(b)(i)** to determine an estimate of the number of outliers, if any, in the distribution. **[2]**

(d) Give **one** advantage of using a histogram rather than a pie chart in this context. **[1]**

11 Casey and Riley attend a large school. They are discussing the music preferences of the students at their school. Casey believes that the favourite band of 75% of the students is Blue Rocking. Riley believes that the true figure is greater than 75%.

They plan to carry out a hypothesis test at the 5% significance level, using the hypotheses H_0: $p = 0.75$ and H_1: $p > 0.75$.

They choose a random sample of 60 students from the school, and note the number, X, who say that their favourite band is Blue Rocking.

They find that $X = 50$.

(a) Assuming the null hypothesis to be true, Riley correctly calculates that $P(X = 50) = 0.0407$, correct to 3 significant figures.

Riley says that, because this value is less than 0.05, the null hypothesis should be rejected.

Explain why this statement is incorrect. **[1]**

(b) Carry out the test. **[5]**

(c) **(i)** State which mathematical model is used in the calculation in part **(b)**, including the value(s) of any parameter(s). **[1]**

(ii) The random sample was chosen without replacement.

Explain whether this invalidates the model used in part **(b)**. **[1]**

12 This question deals with information about the populations of Local Authorities (LAs) in the North of England, taken from the 2011 census.

Fig. 1

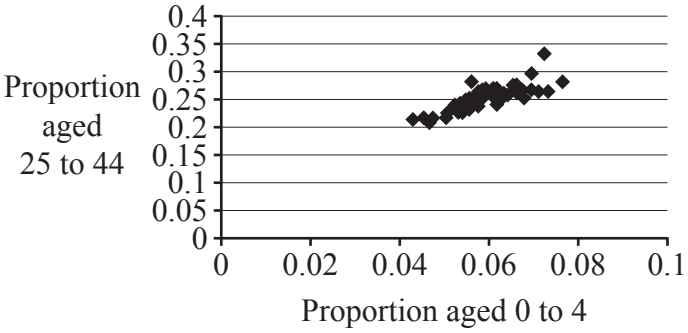


Fig. 2

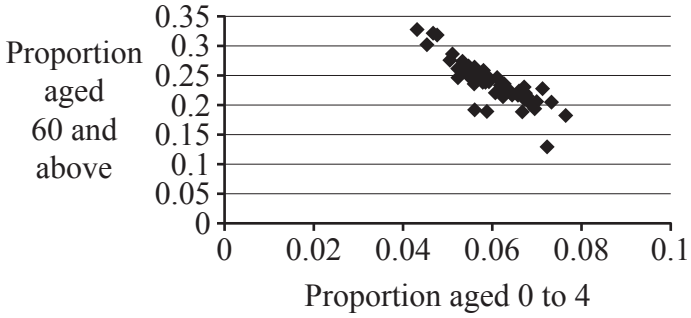


Fig. 1 and **Fig. 2** both show strong correlation, but of two different kinds.

(a) For each diagram, use a single word to describe the kind of correlation shown. **[1]**

(b) For each diagram, suggest a reason, in context, why the correlation is of the particular kind described in part **(a)**. **[2]**

Fig. 3 is the same as **Fig. 2** but with the point *A* marked.
Fig. 4 shows information about the same LAs as **Fig. 2** and **Fig. 3**.

Fig. 3

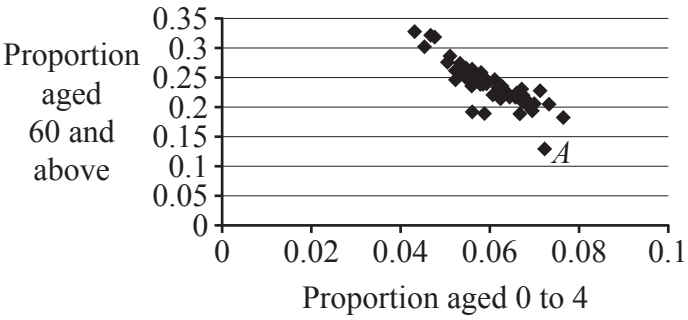
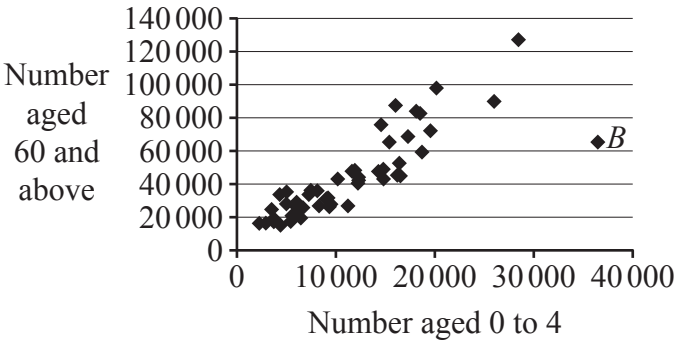


Fig. 4



(c) Point *A* in **Fig. 3** and point *B* in **Fig. 4** represent the same LA.

Explain how you can tell that this LA has a large population. **[1]**

END OF QUESTION PAPER

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