OCR
Oxford Cambridge and RSA

Thursday 18 May 2023 – Afternoon

AS Level Mathematics A

H230/01 Pure Mathematics and Statistics

Time allowed: 1 hour 30 minutes

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You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\overline{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2} \quad \text{or} \quad \sqrt{\frac{\Sigma f(x-\overline{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \overline{x}^2}$$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Kinematics

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

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Section A
Pure Mathematics

1 **(a)** Prove that $\cos x + \sin x \tan x \equiv \dfrac{1}{\cos x}$ (where $x \neq \dfrac{1}{2}n\pi$ for any odd integer n). **[3]**

(b) Solve the equation $2\sin^2 x = \cos^2 x$ for $0° \leqslant x \leqslant 180°$. **[2]**

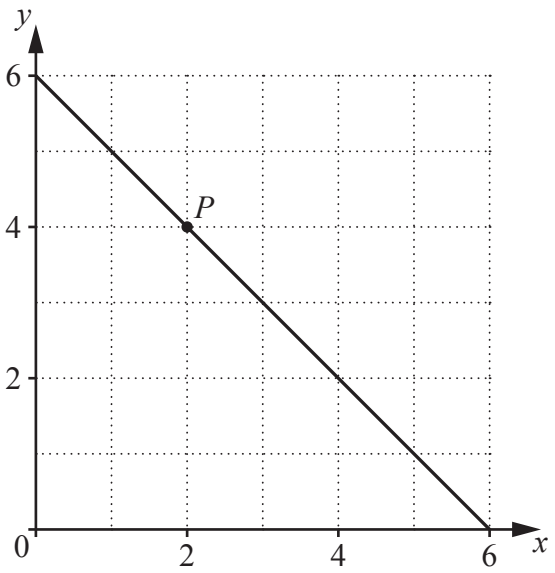
2 **(a)** The points A, B and C have position vectors $\begin{pmatrix}-4\\3\end{pmatrix}$, $\begin{pmatrix}-3\\6\end{pmatrix}$ and $\begin{pmatrix}-1\\12\end{pmatrix}$ respectively.

(i) Show that B lies on AC. **[2]**

(ii) Find the ratio $AB : BC$. **[1]**

(b) The diagram shows the line $x + y = 6$ and the point $P(2, 4)$ that lies on the line.

A copy of the diagram is given in the Printed Answer Booklet.



The distinct point Q also lies on the line $x + y = 6$ and is such that $|\overrightarrow{OQ}| = |\overrightarrow{OP}|$, where O is the origin.

Find the magnitude and direction of the vector $\overrightarrow{PQ}$. **[3]**

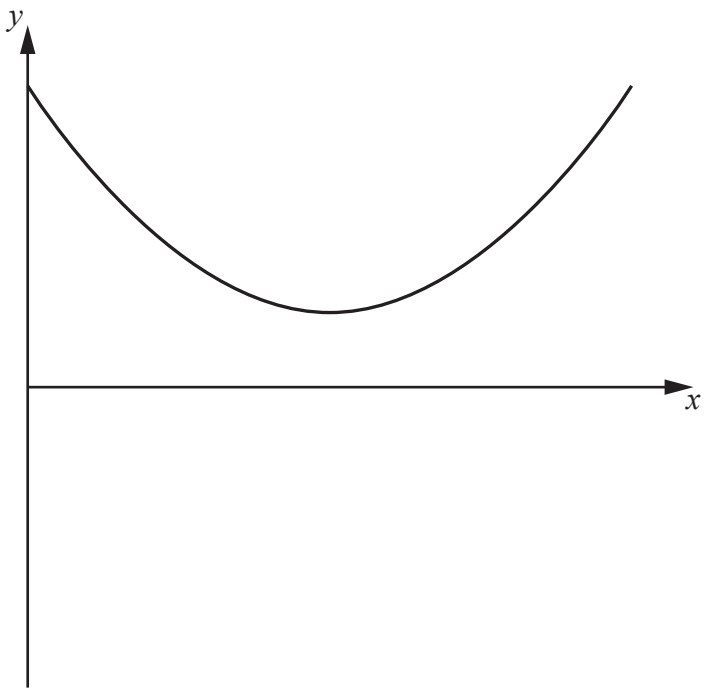
(c) The point R is such that $\overrightarrow{PR}$ is perpendicular to $\overrightarrow{OP}$ and $|\overrightarrow{PR}| = \frac{1}{2}|\overrightarrow{OP}|$.

Write down the position vectors of the **two** possible positions of the point R. **[2]**

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3 The diagram shows the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is a **quadratic** function of x.

A copy of the diagram is given in the Printed Answer Booklet.



(a) On the copy of the diagram in the Printed Answer Booklet, draw a possible graph of the gradient function $y = \mathrm{f}'(x)$. **[3]**

(b) State the gradient of the graph of $y = \mathrm{f}''(x)$. **[1]**

4 A curve has equation $y = \mathrm{e}^{3x}$.

(a) Determine the value of x when $y = 10$. **[2]**

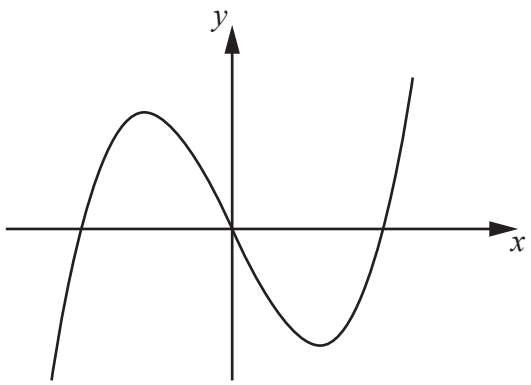
(b) Determine the gradient of the tangent to the curve at the point where $x = 2$. **[2]**

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5 **In this question you must show detailed reasoning.**

The diagram shows part of the graph of $y = x^3 - 4x$.



Determine the total area enclosed by the curve and the x-axis. **[6]**

6 **(a)** Determine the **two** real roots of the equation $8x^6 + 7x^3 - 1 = 0$. **[3]**

(b) Determine the coordinates of the stationary points on the curve $y = 8x^7 + \frac{49}{4}x^4 - 7x$. **[4]**

(c) For each of the stationary points, use the value of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ to determine whether it is a maximum or a minimum. **[4]**

7 **(a)** Two real numbers are denoted by a and b.

(i) Write down expressions for the following.

- The mean of the squares of a and b
- The square of the mean of a and b **[1]**

(ii) Prove that the mean of the squares of a and b is greater than or equal to the square of their mean. **[3]**

(b) You are given that the result in part **(a)(ii)** is true for any two or more real numbers.

Explain what this result shows about the variance of a set of data. **[1]**

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8 In this question you must show detailed reasoning.

A circle has equation $x^2 + y^2 - 6x - 4y + 12 = 0$. Two tangents to this circle pass through the point (0, 1).

You are given that the scales on the x-axis and the y-axis are the same.

Find the angle between these two tangents. **[7]**

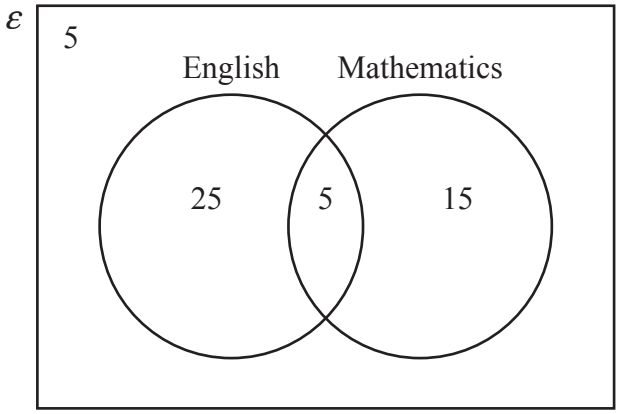
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Section B
Statistics

9 In a survey, 50 people were asked whether they had passed A-level English and whether they had passed A-level Mathematics.

The numbers of people in various categories are shown in the Venn diagram.



(a) A person is chosen at random from the 50 people.

Find the probability that this person has passed A-level Mathematics. **[1]**

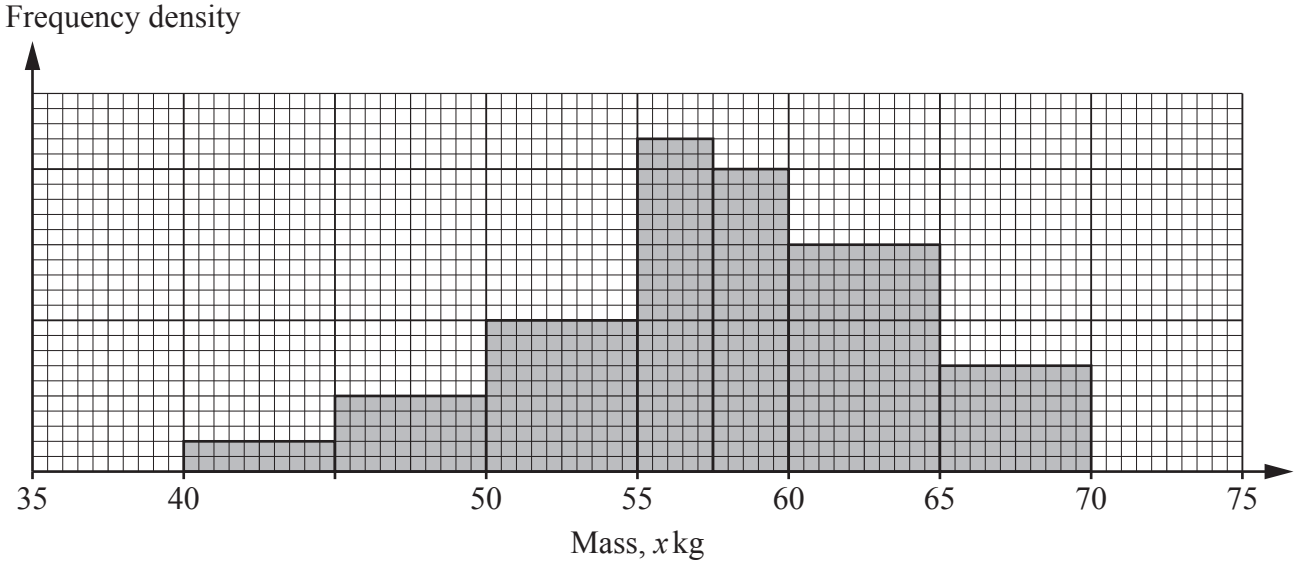
(b) Two people are chosen at random, **without replacement**, from those who have passed A-level in at least one of the two subjects.

Determine the probability that both of these people have passed A-level Mathematics. **[3]**

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10 The masses of a random sample of 120 boulders in a certain area were recorded. The results are summarized in the histogram.



(a) Calculate the number of boulders with masses between 60 and 65 kg. **[2]**

(b) **(i)** Use midpoints to find estimates of the mean and standard deviation of the masses of the boulders in the sample. **[3]**

(ii) Explain why your answers are only estimates. **[1]**

(c) Use your answers to part **(b)(i)** to determine an estimate of the number of outliers, if any, in the distribution. **[2]**

(d) Give **one** advantage of using a histogram rather than a pie chart in this context. **[1]**

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11 Casey and Riley attend a large school. They are discussing the music preferences of the students at their school. Casey believes that the favourite band of 75% of the students is Blue Rocking. Riley believes that the true figure is greater than 75%.

They plan to carry out a hypothesis test at the 5% significance level, using the hypotheses H_0: $p = 0.75$ and H_1: $p > 0.75$.

They choose a random sample of 60 students from the school, and note the number, X, who say that their favourite band is Blue Rocking.

They find that $X = 50$.

(a) Assuming the null hypothesis to be true, Riley correctly calculates that $P(X = 50) = 0.0407$, correct to 3 significant figures.

Riley says that, because this value is less than 0.05, the null hypothesis should be rejected.

Explain why this statement is incorrect. **[1]**

(b) Carry out the test. **[5]**

(c) **(i)** State which mathematical model is used in the calculation in part **(b)**, including the value(s) of any parameter(s). **[1]**

(ii) The random sample was chosen without replacement.

Explain whether this invalidates the model used in part **(b)**. **[1]**

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12 This question deals with information about the populations of Local Authorities (LAs) in the North of England, taken from the 2011 census.

Fig. 1

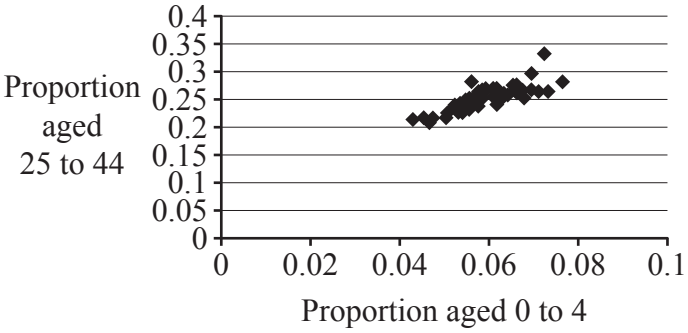


Fig. 2

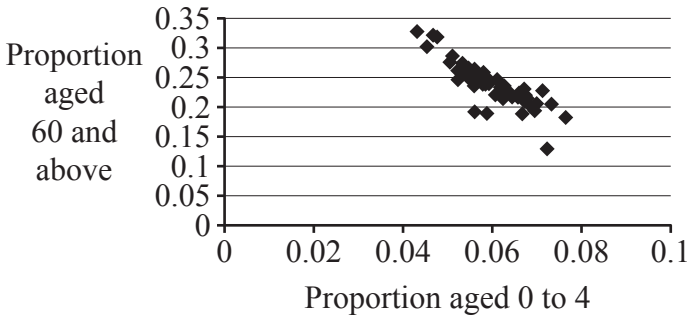


Fig. 1 and **Fig. 2** both show strong correlation, but of two different kinds.

(a) For each diagram, use a single word to describe the kind of correlation shown. **[1]**

(b) For each diagram, suggest a reason, in context, why the correlation is of the particular kind described in part **(a)**. **[2]**

Fig. 3 is the same as **Fig. 2** but with the point *A* marked.
Fig. 4 shows information about the same LAs as **Fig. 2** and **Fig. 3**.

Fig. 3

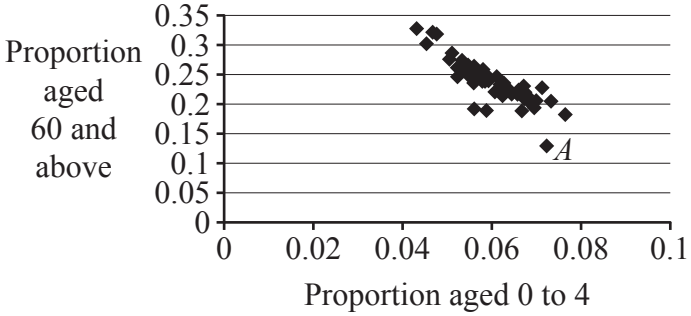
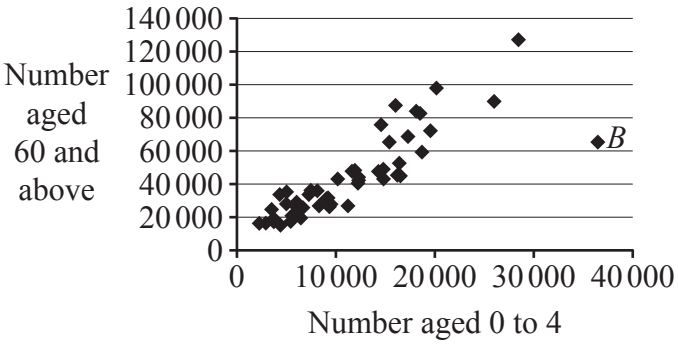


Fig. 4



(c) Point *A* in **Fig. 3** and point *B* in **Fig. 4** represent the same LA.

Explain how you can tell that this LA has a large population. **[1]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a)

$LHS = \cos x + \sin x \times \frac{\sin x}{\cos x} = \cos x + \frac{\sin^2 x}{\cos x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos x}$

$= \frac{1}{\cos x}$ since $\cos^2 x + \sin^2 x \equiv 1$

$= RHS$, so $\cos x + \sin x \tan x \equiv \frac{1}{\cos x}$ as required

1 (b)

$2\sin^2 x = \cos^2 x \Rightarrow \frac{\sin^2 x}{\cos^2 x} = \frac{1}{2} \Rightarrow \tan^2 x = \frac{1}{2}$

$\tan x = \pm \frac{1}{\sqrt{2}} = \pm 0.70710...$

$x = \tan^{-1}(0.70710...) = 35.264...°$

or $x = 180° - 35.264...° = 144.735...°$

$x = 35.3°$ or $x = 145°$

2 (a)(i)

$AB = \begin{pmatrix} -3 \\ 6 \end{pmatrix} - \begin{pmatrix} -4 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ $BC = \begin{pmatrix} -1 \\ 12 \end{pmatrix} - \begin{pmatrix} -3 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$

$BC = 2\begin{pmatrix} 1 \\ 3 \end{pmatrix} = 2AB$, so AB and BC are parallel

and share the point B, hence B lies on AC.

2 (a)(ii)

$AB : BC = 1 : 2$

Question number

2 (b) $|OP| = \sqrt{(2^2 + 4^2)} = \sqrt{20}$. Q on $x + y = 6$ with $|OQ| = \sqrt{20}$:

by symmetry in the line $y = x$, Q is the point (4, 2)

(check: $4 + 2 = 6$ and $4^2 + 2^2 = 20$)

$\overrightarrow{PQ} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

Magnitude $|PQ| = \sqrt{(2^2 + 2^2)} = \sqrt{8} = 2\sqrt{2} = 2.83$

Direction: 45° below the positive x-direction, i.e. −45° (or 315°)

measured from the positive x-axis, along the line $x + y = 6$

2 (c) $\overrightarrow{OP} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$, $|OP| = \sqrt{20}$ so $|PR| = \frac{1}{2}\sqrt{20} = \sqrt{5}$

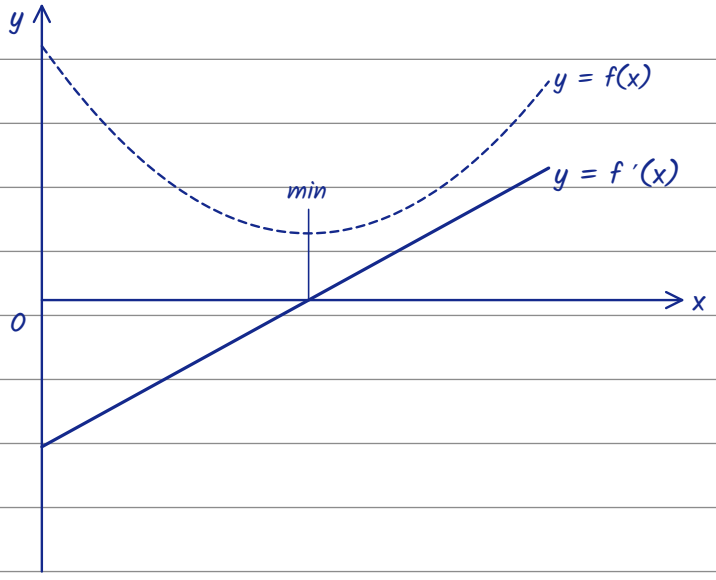
Perpendicular to $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ with magnitude $\sqrt{5}$: $\overrightarrow{PR} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ or $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$\overrightarrow{OR} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ or $\overrightarrow{OR} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$

Position vectors of R: $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$

3 (a) Gradient function of a quadratic is a straight line with positive gradient,

negative where f is decreasing, zero below the minimum point:



Question number

(line crosses the y-axis below 0 and cuts the x-axis directly below the minimum)

3 (b) $f'(x)$ is linear, so $f''(x)$ is a constant, so $y = f''(x)$ is horizontal:

the gradient of the graph of $y = f''(x)$ is 0

4 (a) $10 = e^{3x} \Rightarrow 3x = \ln 10$

$x = \frac{1}{3} \ln 10 = 0.767528... = 0.768$

4 (b) $\frac{dy}{dx} = 3e^{3x}$

At $x = 2$: $\frac{dy}{dx} = 3e^6 = 1210.286...$

Gradient of tangent $= 3e^6 = 1210$ (3 s.f.)

5 Roots: $x^3 - 4x = 0 \Rightarrow x(x^2 - 4) = 0 \Rightarrow x(x - 2)(x + 2) = 0$

$x = 0, -2, 2$. Curve is above the axis on $(-2, 0)$, below on $(0, 2)$.

$\int_0^2 (x^3 - 4x)\,dx = \left[\frac{x^4}{4} - 2x^2\right]_0^2 = (4 - 8) - 0 = -4$

so the area between $x = 0$ and $x = 2$ is $|-4| = 4$

$\int_{-2}^0 (x^3 - 4x)\,dx = \left[\frac{x^4}{4} - 2x^2\right]_{-2}^0 = 0 - (4 - 8) = 4$

so the area between $x = -2$ and $x = 0$ is 4 (also by rotational symmetry)

Total area $= 4 + 4 = 8$

6 (a) Let $u = x^3$: $8u^2 + 7u - 1 = 0 \Rightarrow (8u - 1)(u + 1) = 0$

$u = x^3 = \frac{1}{8}$ or $x^3 = -1$

Question number

$x = \frac{1}{2}$ or $x = -1$

6 (b) $\frac{dy}{dx} = 56x^6 + 49x^3 - 7$

Stationary points: $\frac{dy}{dx} = 0 \Rightarrow 56x^6 + 49x^3 - 7 = 0$

$\div 7$: $8x^6 + 7x^3 - 1 = 0 \Rightarrow x = -1$ or $x = \frac{1}{2}$ (from (a))

$x = -1$: $y = 8(-1) + \frac{49}{4}(1) - 7(-1) = -8 + 12.25 + 7 = 11.25$

$x = \frac{1}{2}$: $y = \frac{8}{128} + \frac{49}{64} - \frac{7}{2} = -\frac{171}{64} = -2.67$ (3 s.f.)

Stationary points: $(-1, 11.25)$ and $(\frac{1}{2}, -\frac{171}{64})$

6 (c) $\frac{d^2y}{dx^2} = 336x^5 + 147x^2$

$x = \frac{1}{2}$: $\frac{d^2y}{dx^2} = \frac{336}{32} + \frac{147}{4} = 47.25 > 0$, so $(\frac{1}{2}, -\frac{171}{64})$ is a minimum

$x = -1$: $\frac{d^2y}{dx^2} = -336 + 147 = -189 < 0$, so $(-1, 11.25)$ is a maximum

7 (a)(i) Mean of the squares $= \frac{a^2 + b^2}{2}$

Square of the mean $= (\frac{a + b}{2})^2$

Question number

7 (a)(ii) $\frac{a^2 + b^2}{2} - \left(\frac{a + b}{2}\right)^2 = \frac{2a^2 + 2b^2}{4} - \frac{a^2 + 2ab + b^2}{4}$

$= \frac{2a^2 + 2b^2 - (a^2 + 2ab + b^2)}{4} = \frac{a^2 - 2ab + b^2}{4}$

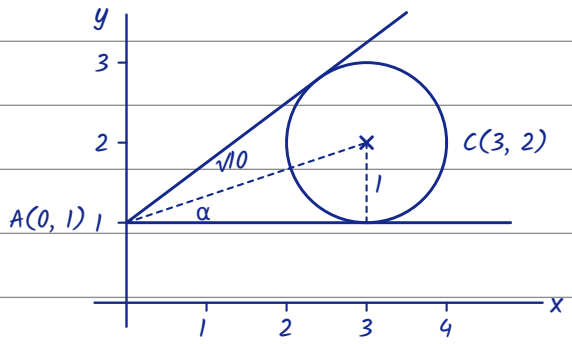
$= \frac{(a - b)^2}{4} \geq 0$ since the square of a real number is never negative

$\therefore \frac{a^2 + b^2}{2} \geq \left(\frac{a + b}{2}\right)^2$: the mean of the squares is greater than or equal to the square of the mean.

7 (b) Variance = mean of the squares − square of the mean, so this shows that the variance of any set of data is always ≥ 0 (variance is never negative).

8 $x^2 - 6x + y^2 - 4y + 12 = 0 \Rightarrow (x - 3)^2 - 9 + (y - 2)^2 - 4 + 12 = 0$

$(x - 3)^2 + (y - 2)^2 = 1$: centre C(3, 2), radius 1



Let A = (0, 1). $AC = \sqrt{((3 - 0)^2 + (2 - 1)^2)} = \sqrt{10}$

Tangent is perpendicular to radius, so each tangent, AC and a radius form a right-angled triangle with hypotenuse AC.

Let α be the angle between AC and each tangent:

Question number

$\sin\alpha = \frac{\text{radius}}{AC} = \frac{1}{\sqrt{10}} \Rightarrow \alpha = \sin^{-1}\frac{1}{\sqrt{10}} = 18.434...°$

Angle between tangents $= 2\alpha = 2\sin^{-1}\frac{1}{\sqrt{10}} = 36.869...°$

Angle between the two tangents = 36.9° (3 s.f.)

9 (a) $P(\text{passed Mathematics}) = \frac{5+15}{50} = \frac{20}{50} = \frac{2}{5}$

9 (b) At least one subject: 25 + 5 + 15 = 45 people, of whom 20 passed Maths

$P(\text{both passed Maths}) = \frac{20}{45} \times \frac{19}{44} = \frac{380}{1980} = \frac{19}{99} = 0.192$ (3 s.f.)

10 (a) Each small square is 0.5 kg wide. Bar widths (squares): 10, 10, 10, 5, 5, 10, 10

Bar heights (squares): 2, 5, 10, 22, 20, 15, 7

Areas = 20, 50, 100, 110, 100, 150, 70 ⇒ total area = 600 squares

600 squares represent 120 boulders

Number between 60 and 65 kg $= 120 \times \frac{150}{600} = 30$

10 (b)(i) Class frequencies (area $\times \frac{120}{600}$):

40–45: 4, 45–50: 10, 50–55: 20, 55–57.5: 22, 57.5–60: 20, 60–65: 30, 65–70: 14

Midpoints x: 42.5, 47.5, 52.5, 56.25, 58.75, 62.5, 67.5

$n = 120$, $\Sigma fx = 6927.5$, $\Sigma fx^2 = 404\,528.125$

$\text{mean} = \frac{6927.5}{120} = 57.729... = 57.7$ kg (3 s.f.)

$\text{s.d.} = \sqrt{\left(\frac{404\,528.125}{120} - 57.729...^2\right)} = \sqrt{38.411...} = 6.1976...$

Question number	
	s.d. = 6.20 kg (3 s.f.)
10 (b)(ii)	The actual masses within each class are not known (the distribution of masses within each class is unknown), so midpoints are assumed.
10 (c)	Outliers: more than 2 s.d. from the mean $57.7 - 2 \times 6.20 = 45.3$ kg, $57.7 + 2 \times 6.20 = 70.1$ kg No masses above 70 kg; class 40–45 (4 boulders) lies below 45.3 So there are about 4 outliers.
10 (d)	From a histogram the actual frequencies can be found (needed to estimate the mean and s.d.); a pie chart only shows relative frequencies.
11 (a)	The test needs the probability of a result at least as extreme as the one observed, i.e. $P(X \geqslant 50)$, not $P(X = 50)$.
11 (b)	H_0: $p = 0.75$, H_1: $p > 0.75$, where p is the proportion of students whose favourite band is Blue Rocking. Under H_0, $X \sim B(60, 0.75)$. Observed $X = 50$. $P(X \geqslant 50) = 1 - P(X \leqslant 49) = 0.085886... = 0.0859$ $0.0859 > 0.05$, so the result is not significant: do not reject H_0. There is insufficient evidence at the 5% level that more than 75% of students have Blue Rocking as their favourite band.
11 (c)(i)	Binomial distribution $B(60, 0.75)$, i.e. $n = 60$, $p = 0.75$
11 (c)(ii)	Yes, strictly: without replacement, whether one student chosen has Blue Rocking as favourite is not independent of the students already chosen (the probability changes as students are chosen), so the

Question number	
	binomial conditions are not exactly met.
12 (a)	Fig. 1: positive Fig. 2: negative
12 (b)	Fig. 1: people aged 25 to 44 are the most likely to be parents of young children, so LAs with a higher proportion of 25 to 44s have a higher proportion of 0 to 4s. Fig. 2: LAs with a high proportion of people aged 60 and above have fewer families with young children, so a lower proportion of 0 to 4s.
12 (c)	In Fig. 4, B has a much larger number of 0 to 4s than any other LA, but in Fig. 3, A shows its proportion of 0 to 4s is similar to several other LAs (and its number of over 60s is similar to many LAs while its proportion is by far the lowest). So its total population must be large.