OCR
Oxford Cambridge and RSA

Wednesday 14 October 2020 – Afternoon

AS Level Mathematics B (MEI)

H630/02 Pure Mathematics and Statistics

Time allowed: 1 hour 30 minutes

8170287764

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

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Answer **all** the questions.

1 Solve the inequality $2x + 5 < 6x - 3$. **[2]**

2 A student measures the upper arm lengths of a sample of 97 women. The results are summarised in the frequency table in Fig. 2.1.

Arm length in cm	30 –	31 –	32 –	33 –	34 –	35 –	36 –	37 –	38 –	39 –	40 – 41
Frequency	1	4	5	9	13	19	17	17	4	3	5

Fig. 2.1

The student constructs two cumulative frequency diagrams to represent the data using different class intervals. These are shown in Fig. 2.2 opposite.

One of these diagrams is correct and the other is incorrect.

(a) State which diagram is incorrect, justifying your answer. **[2]**

(b) Use the correct diagram in Fig. 2.2 to find an estimate of the median. **[1]**

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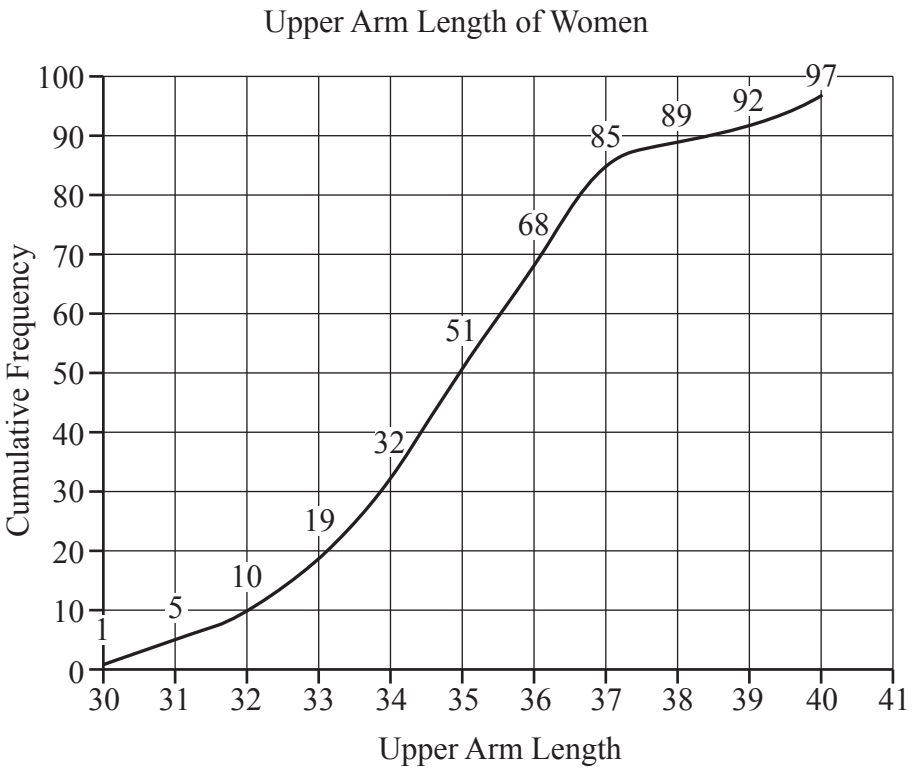


Diagram A

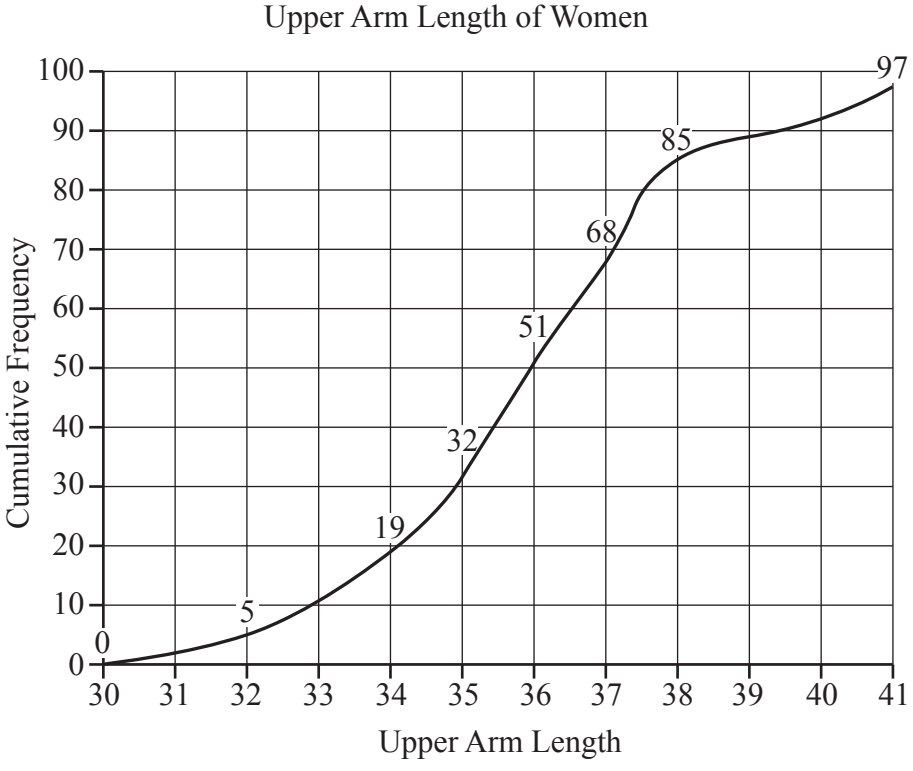


Diagram B

Fig. 2.2

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3 A researcher is conducting an investigation into the number of portions of fruit adults consume each day. The researcher decides to ask 50 men and 50 women to complete a simple questionnaire.

(a) State the type of sampling procedure the researcher is using. **[1]**

(b) Write down one disadvantage of this sampling procedure. **[1]**

The researcher represents the data in Fig. 3.1.

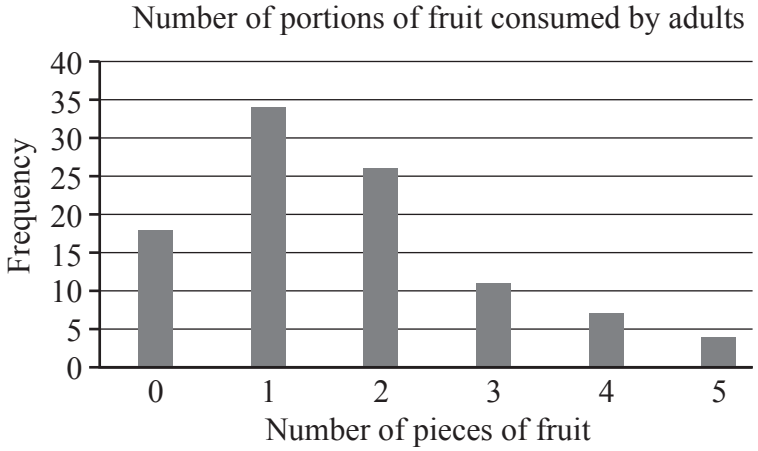


Fig. 3.1

(c) Describe the shape of the distribution. **[1]**

The data are summarised in the frequency table in Fig. 3.2.

Number of portions of fruit	0	1	2	3	4	5
Number of adults	18	34	26	11	7	4

Fig. 3.2

(d) For the data in Fig. 3.2, use your calculator to find

- the mean,
- the standard deviation.

Give your answers correct to 2 decimal places. **[2]**

A second researcher chooses a proportional stratified sample of 100 children from years 5 and 6 in a certain primary school. There are 220 children to choose from. In year 5 there are 125 children, of whom 81 are boys.

(e) How many year 5 girls should be included in the sample? **[1]**

The second researcher found that the mean number of portions of fruit consumed per day by the children in this sample was 1.61 and the standard deviation was 0.53.

(f) Comment on the amount of fruit consumed per day by the children compared to the amount of fruit consumed per day by the adults. **[2]**

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4 In a certain country it is known that 11% of people are left-handed.

(a) Calculate the probability that, in a random sample of 98 people from this country, 5 or fewer are found to be left-handed, giving your answer correct to 3 significant figures. **[1]**

An anthropologist believes that the proportion of left-handed people is lower in a particular ethnic group.

The anthropologist collects a random sample of 98 people from this particular ethnic group in order to test the hypothesis that the proportion of left-handed people is less than 11%.

The anthropologist carries out the test at the 1% level.

(b) Determine the critical region for this test. **[3]**

5 A company needs to appoint 3 new assistants. 8 candidates are invited for interview; each candidate has a different surname. The candidates are to be interviewed one after another. The personnel officer randomly selects the order in which the candidates are to be interviewed by drawing their names out of a hat. One of the candidates is called Mr Browne and another is called Mrs Green.

(a) Calculate the probability that Mr Browne is interviewed first and Mrs Green is interviewed last. **[2]**

5 of the 8 candidates invited for interview are women and the other 3 are men. The chief executive can't make up his mind who to appoint so he randomly selects 3 candidates by drawing their names out of a hat.

(b) Determine the probability that more women than men are selected. **[4]**

6 Use integration to show that the area bounded by the x-axis and the curve with equation $y = (x-1)^2(x-3)$ is $\frac{4}{3}$ square units. **[6]**

7 **In this question you must show detailed reasoning.**

A circle has centre $(2, -1)$ and radius 5.

A straight line passes through the points (1, 1) and (9, 5).

Find the coordinates of the points of intersection of the line and the circle. **[8]**

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8 **In this question you must show detailed reasoning.**

Solve the equation $3\cos\theta + 8\tan\theta = 0$ for $0° < \theta < 360°$, giving your answers correct to the nearest degree. **[6]**

9 The equation of a curve is $y = 24\sqrt{x} - 8x^{\frac{3}{2}} + 16$.

(a) Find $\frac{dy}{dx}$. **[3]**

(b) Find the coordinates of the turning point. **[3]**

(c) Determine the nature of the turning point. **[2]**

10 Fig. 10.1 shows a sample collected from the large data set.

BMI is defined as $\frac{\text{mass of person in kilograms}}{\text{square of person's height in metres}}$.

Sex	Age in years	Mass in kg	Height in cm	BMI
Male	38	77.6	164.8	28.57
Male	17	63.5	170.3	21.89
Male	18	68.0	172.3	22.91
Male	18	57.2	172.2	19.29
Male	19	77.6	191.2	21.23
Male	24	72.7	177.0	23.21
Male	25	92.5	177.9	29.23
Male	26	70.4	159.4	27.71
Male	31	77.5	174.0	25.60
Male	34	132.4	182.2	39.88
Male	38	115.0	186.4	33.10
Male	40	112.1	171.7	38.02

Fig. 10.1

(a) Calculate the mass in kg of a person with a BMI of 23.56 and a height of 181.6 cm, giving your answer correct to 1 decimal place. **[2]**

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Fig. 10.2 shows a scatter diagram of BMI against age for the data in the table. A line of best fit has also been drawn.

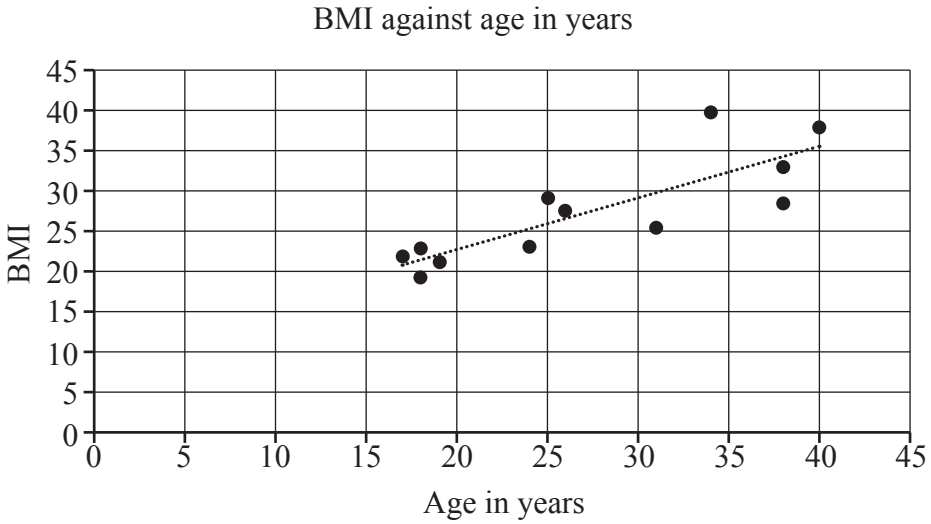


Fig. 10.2

(b) Describe the correlation between age and BMI. **[1]**

(c) Use the line of best fit to estimate the BMI of a 30-year-old man. **[1]**

(d) Explain why it would not be sensible to use the line of best fit to estimate the BMI of a 60-year-old man. **[1]**

(e) Use your knowledge of the large data set to suggest **two** reasons why the sample data in the table may not be representative of the population. **[2]**

(f) Once the data in the large data set had been cleaned there were 196 values available for selection. Describe how a sample of size 12 could be generated using systematic sampling so that each of the 196 values could be selected in the sample. **[2]**

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11 A car is travelling along a stretch of road at a steady speed of $11\,\text{m}\,\text{s}^{-1}$.

The driver accelerates, and t seconds after starting to accelerate the speed of the car, V, is modelled by the formula

$V = A + B(1 - \text{e}^{-0.17t})$.

When $t = 3$, $V = 13.8$.

(a) Find the values of A and B, giving your answers correct to 2 significant figures. **[3]**

When $t = 4$, $V = 14.5$ and when $t = 5$, $V = 14.9$.

(b) Determine whether the model is a good fit for these data. **[2]**

(c) Determine the acceleration of the car according to the model when $t = 5$, giving your answer correct to 3 decimal places. **[2]**

The car continues to accelerate until it reaches its maximum speed.

The speed limit on this road is $60\,\text{km}\,\text{h}^{-1}$. All drivers who exceed this speed limit are recorded by a speed camera and automatically fined £100.

(d) Determine whether, according to the model, the driver of this car is fined £100. **[3]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1

$2x + 5 < 6x - 3$

$5 + 3 < 6x - 2x$

$8 < 4x$

$x > 2$

2 (a)

Diagram A is incorrect.

In Diagram A the cumulative frequencies have been plotted at the left-hand end (lower bound) of each class interval instead of at the right-hand end (upper class boundary), e.g. 1 is plotted at 30 not 31.

2 (b)

$n = 97$, so median is the 48.5th value: read across from 48.5 on Diagram B

Median $\approx$ 35.8 cm

3 (a)

Quota sampling

3 (b)

It is non-random, so the sample may be biased (not representative).

3 (c)

Positive skew

3 (d)

$n = 100$, $\Sigma fx = 167$, $\Sigma fx^2 = 449$

Mean $= 167 \div 100 = 1.67$

$S_{xx} = 449 - 167^2 \div 100 = 170.11$

$s = \sqrt{(170.11 \div 99)} = 1.31083...$

Mean = 1.67, standard deviation = 1.31 (2 d.p.)

Question number	
3 (e)	Year 5 girls = 125 − 81 = 44: $\frac{44}{220} \times 100 = 20$ girls
3 (f)	The means are very similar (1.61 compared with 1.67), so on average the children eat a similar (slightly smaller) number of portions of fruit per day than the adults. The standard deviation for children is lower (0.53 compared with 1.31), so there is less variability in the number of portions of fruit eaten per day by the children.
4 (a)	X = number left-handed, $X \sim B(98, 0.11)$ $P(X \leqslant 5) = 0.0348247\ldots = 0.0348$ (3 s.f.)
4 (b)	H_0: $p = 0.11$ H_1: $p < 0.11$, where p = P(person is left-handed) Under H_0, $X \sim B(98, 0.11)$; lower tail test at 1% level $P(X \leqslant 4) = 0.013337\ldots > 0.01$, so 4 is not in the critical region $P(X \leqslant 3) = 0.004090\ldots < 0.01$, so 3 is in the critical region Critical region is $X \leqslant 3$, i.e. $\{0, 1, 2, 3\}$
5 (a)	P(Browne first and Green last) $= \frac{1}{8} \times \frac{1}{7} = \frac{1}{56}$ $(= 0.0179)$
5 (b)	More women than men: 3W 0M or 2W 1M Ways: 3W: ${}^5C_3 = 10$ 2W 1M: ${}^5C_2 \times {}^3C_1 = 10 \times 3 = 30$ Total ways of choosing 3 from 8: ${}^8C_3 = 56$ P(more women than men) $= \frac{10 + 30}{56} = \frac{40}{56} = \frac{5}{7}$

Question number

6

$y = (x-1)^2(x-3) = (x^2 - 2x + 1)(x-3) = x^3 - 5x^2 + 7x - 3$

$y = 0$ when $x = 1$ (repeated root, touches axis) and $x = 3$,

so the region lies between $x = 1$ and $x = 3$ (below the x-axis).

$$\int_1^3 (x^3 - 5x^2 + 7x - 3)\,dx = \left[\frac{x^4}{4} - \frac{5x^3}{3} + \frac{7x^2}{2} - 3x\right]_1^3$$

$$= \left(\frac{81}{4} - 45 + \frac{63}{2} - 9\right) - \left(\frac{1}{4} - \frac{5}{3} + \frac{7}{2} - 3\right)$$

$$= \left(-\frac{9}{4}\right) - \left(-\frac{11}{12}\right) = -\frac{27}{12} + \frac{11}{12} = -\frac{16}{12} = -\frac{4}{3}$$

Negative since the region is below the x-axis, so area $= \left|-\frac{4}{3}\right| = \frac{4}{3}$ square units

7

Circle: $(x-2)^2 + (y+1)^2 = 5^2 = 25$

Line: $m = \frac{5-1}{9-1} = \frac{4}{8} = \frac{1}{2}$, $\quad y - 1 = \frac{1}{2}(x-1) \Rightarrow y = \frac{1}{2}(x+1)$

Substitute: $(x-2)^2 + \left(\frac{x+1}{2} + 1\right)^2 = 25 \Rightarrow (x-2)^2 + \frac{(x+3)^2}{4} = 25$

$4(x^2 - 4x + 4) + (x^2 + 6x + 9) = 100$

$5x^2 - 10x + 25 = 100$

$5x^2 - 10x - 75 = 0 \Rightarrow x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0 \Rightarrow x = 5$ or $x = -3$

$x = 5$: $y = \frac{1}{2}(5+1) = 3$ $\qquad$ $x = -3$: $y = \frac{1}{2}(-3+1) = -1$

Points of intersection are $(5, 3)$ and $(-3, -1)$

8

$$3\cos\theta + 8\frac{\sin\theta}{\cos\theta} = 0$$

Multiply by $\cos\theta$: $\quad 3\cos^2\theta + 8\sin\theta = 0$

$3(1 - \sin^2\theta) + 8\sin\theta = 0$

$3\sin^2\theta - 8\sin\theta - 3 = 0$

Question number

8 cont. $(3 \sin\theta + 1)(\sin\theta - 3) = 0$

$\sin\theta = -\frac{1}{3}$ or $\sin\theta = 3$ (no solutions since $-1 \leq \sin\theta \leq 1$)

$\sin^{-1}(1/3) = 19.47...°$

$\theta = 180° + 19.47° = 199.47°$ or $\theta = 360° - 19.47° = 340.53°$

$\theta = 199°, 341°$ (nearest degree)

9 (a) $y = 24x^{1/2} - 8x^{3/2} + 16$

$\frac{dy}{dx} = 24 \times \frac{1}{2}x^{-1/2} - 8 \times \frac{3}{2}x^{1/2} = 12x^{-1/2} - 12x^{1/2} = \frac{12}{\sqrt{x}} - 12\sqrt{x}$

9 (b) $\frac{dy}{dx} = 0$: $\frac{12}{\sqrt{x}} = 12\sqrt{x} \Rightarrow 12 = 12x \Rightarrow x = 1$

$y = 24\sqrt{1} - 8 \times 1^{3/2} + 16 = 24 - 8 + 16 = 32$

Turning point is (1, 32)

9 (c) $\frac{d^2y}{dx^2} = -6x^{-3/2} - 6x^{-1/2}$

At $x = 1$: $\frac{d^2y}{dx^2} = -6 - 6 = -12 < 0$, so (1, 32) is a (local) maximum

10 (a) Mass = BMI × (height in m)^2

$= 23.56 \times 1.816^2 = 77.697... = 77.7$ kg (1 d.p.)

10 (b) Positive correlation: as age increases, BMI tends to increase.

10 (c) BMI ≈ 29

Question number	
10 (d)	60 is outside the range of ages in the data (17 to 40), so this would be extrapolation and the relationship may not continue.
10 (e)	1. The sample contains only males, but the large data set (population) contains females as well. 2. The ages in the sample only go from 17 to 40, but the population has a much wider age range.
10 (f)	Number the 196 values 1 to 196. Generate a random number n between 1 and 20 inclusive and select the nth value, then select every 16th value after it: n, $n + 16$, $n + 32$, ..., $n + 176$ (12 values, last at most $20 + 176 = 196$).
11 (a)	$t = 0$: $V = A + B(1 - e^{0}) = A$, so $A = 11$ $t = 3$: $13.8 = 11 + B(1 - e^{-0.51})$ $B = 2.8 \div (1 - e^{-0.51}) = 2.8 \div 0.399504... = 7.0086...$ $A = 11$, $B = 7.0$ (2 s.f.)
11 (b)	$t = 4$: $V = 11 + 7(1 - e^{-0.68}) = 14.4536... \approx 14.5$ (data 14.5) $t = 5$: $V = 11 + 7(1 - e^{-0.85}) = 15.0080... \approx 15.0$ (data 14.9) Both predictions are very close to the data, so the model is a good fit.
11 (c)	$\frac{dV}{dt} = 7 \times 0.17e^{-0.17t} = 1.19e^{-0.17t}$ $t = 5$: acceleration $= 1.19e^{-0.85} = 0.508623... = 0.509$ m s^{-2} (3 d.p.)
11 (d)	As $t \to \infty$, $e^{-0.17t} \to 0$, so $V \to 11 + 7 = 18$ m s^{-1} (maximum speed) 18 m s^{-1} $= 18 \times 3600 \div 1000 = 18 \times 3.6 = 64.8$ km h^{-1} $64.8 > 60$, so according to the model the driver exceeds the speed limit and is fined £100.