Oxford Cambridge and RSA

Wednesday 07 October 2020 – Afternoon

AS Level Mathematics B (MEI)

H630/01 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

© OCR 2020 [603/0991/X]
DC (ST/CB) 184883/4
OCR is an exempt Charity

Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$,

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

Differentiation from first principles

$\mathrm{f}'(x) = \lim\limits_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

© OCR 2020

Answer **all** the questions.

1 Celia states that $n^2 + 2n + 10$ is always odd when n is a prime number.

Prove that Celia's statement is false. **[2]**

2 Fig. 2 shows a quadrilateral ABCD. The lengths AB and BC are 5 cm and 6 cm respectively. The angles ABC, ACD and DAC are 60°, 60° and 75° respectively.

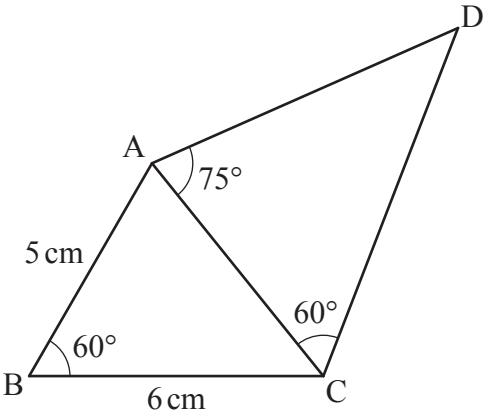

Fig. 2

Calculate the exact value of the length AD. **[4]**

3 Fig. 3 shows a triangle PQR. The vector $\overrightarrow{PQ}$ is $\mathbf{i} + 7\mathbf{j}$ and the vector $\overrightarrow{QR}$ is $4\mathbf{i} - 12\mathbf{j}$.

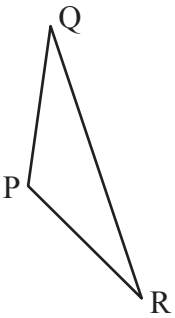

Fig. 3

(a) Show that the triangle PQR is isosceles. **[3]**

The point P has position vector $-3\mathbf{i} - \mathbf{j}$. The point S is added so that PQRS is a parallelogram.

(b) Find the position vector of S. **[2]**

4 In this question, the x and y directions are horizontal and vertically upwards respectively.

A particle of mass 1.5 kg is in equilibrium under the action of its weight and forces $\mathbf{F}_1 = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$N and $\mathbf{F}_2$.

(a) Find the force $\mathbf{F}_2$. **[3]**

The force $\mathbf{F}_2$ is changed to $\begin{pmatrix} 2 \\ 20 \end{pmatrix}$N.

(b) Find the acceleration of the particle. **[2]**

5 Fig. 5.1 shows part of the curve $y = x^{\frac{1}{2}}$. P is the point (1, 1) and Q is the point on the curve with x-coordinate $1 + h$.

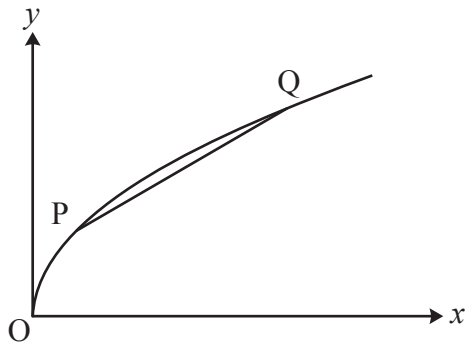

Fig. 5.1

Table 5.2 shows, for different values of h, the coordinates of P, the coordinates of Q, the change in y from P to Q and the gradient of the chord PQ.

x for P	y for P	h	x for Q	y for Q	change in y	gradient PQ
1	1	1				
1	1	0.1	1.1	1.048 809	0.048 809	0.488 088
1	1	0.01	1.01	1.004 988	0.004 988	0.498 756
1	1	0.001	1.001	1.000 500	0.000 500	0.499 875

Table 5.2

(a) Fill in the missing values for the case $h = 1$ in the copy of Table 5.2 in the Printed Answer Booklet. Give your answers correct to 6 decimal places where necessary. **[1]**

(b) Explain how the sequence of values in the last column of Table 5.2 relates to the gradient of the curve $y = x^{\frac{1}{2}}$ at the point P. **[1]**

(c) Use calculus to find the gradient of the curve at the point P. **[2]**

6 **In this question you must show detailed reasoning.**

A particle moves in a straight line. Its velocity $v\,\mathrm{m\,s^{-1}}$ after t s is given by $v = t^3 - 5t^2$.

(a) Find the times at which the particle is stationary. **[2]**

(b) Find the total distance travelled by the particle in the first 6 seconds. **[3]**

7 **In this question you must show detailed reasoning.**

A curve has equation $y = 4x^3 - 6x^2 - 9x + 4$.

(a) Sketch the gradient function for this curve, clearly indicating the points where the gradient is zero. **[4]**

(b) Find the set of values of x for which the gradient function is decreasing. Give your answer using set notation. **[2]**

8 The point A has coordinates $(-1, -2)$ and the point B has coordinates $(7, 4)$. The perpendicular bisector of AB intersects the line $y + 2x = k$ at P.

Determine the coordinates of P in terms of k. **[7]**

9 A car travelling in a straight line accelerates uniformly from rest to $V\,\mathrm{m\,s^{-1}}$ in T s. It then slows down uniformly, coming to rest after a further $2T$ s.

(a) Sketch a velocity-time graph for the car. **[2]**

The acceleration in the first stage of the motion is $2.5\,\mathrm{m\,s^{-2}}$ and the total distance travelled is 240 m.

(b) Calculate the values of V and T. **[4]**

10 An astronaut on the surface of the moon drops a ball from a point 2 m above the surface.

(a) Without any calculations, explain why a standard model using $g = 9.8\,\text{m}\,\text{s}^{-2}$ will not be appropriate to model the fall of the ball. **[1]**

The ball takes 1.6 s to hit the surface.

(b) Find the acceleration of the ball which best models its motion. Give your answer correct to 2 significant figures. **[2]**

(c) Use this value to predict the maximum height of the ball above the point of projection when thrown vertically upwards with an initial velocity of $15\,\text{m}\,\text{s}^{-1}$. **[2]**

11 In this question you must show detailed reasoning.

(a) A student is asked to solve the inequality $x^{\frac{1}{2}} < 4$.

The student argues that $x^{\frac{1}{2}} < 4 \Leftrightarrow x < 16$, so that the solution is $\{x : x < 16\}$.

Comment on the validity of the student's argument. **[1]**

(b) Solve the inequality $\left(\frac{1}{2}\right)^x < 4$. **[3]**

(c) Show that the equation $2\log_2(x+8) - \log_2(x+6) = 3$ has only one root. **[5]**

12 In this question you must show detailed reasoning.

Fig. 12 shows part of the graph of $y = x^2 + \frac{1}{x^2}$.

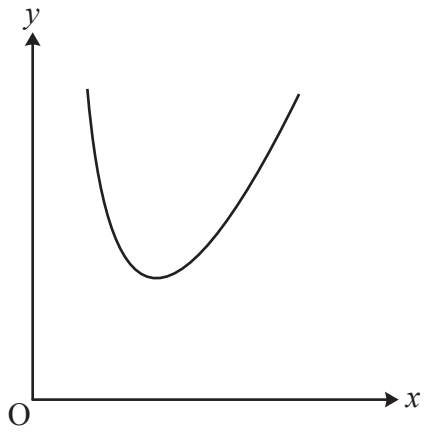

Fig. 12

The tangent to the curve $y = x^2 + \frac{1}{x^2}$ at the point $(2, \frac{17}{4})$ meets the x-axis at A and meets the y-axis at B. O is the origin.

(a) Find the exact area of the triangle OAB. **[6]**

(b) Use calculus to prove that the complete curve has two minimum points and no maximum point. **[6]**

END OF QUESTION PAPER

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

© OCR 2020