OCR
Oxford Cambridge and RSA

Wednesday 07 October 2020 – Afternoon

AS Level Mathematics B (MEI)

H630/01 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes

8166139396

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

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Answer **all** the questions.

1 Celia states that $n^2 + 2n + 10$ is always odd when n is a prime number.

Prove that Celia's statement is false. **[2]**

2 Fig. 2 shows a quadrilateral ABCD. The lengths AB and BC are 5 cm and 6 cm respectively. The angles ABC, ACD and DAC are 60°, 60° and 75° respectively.

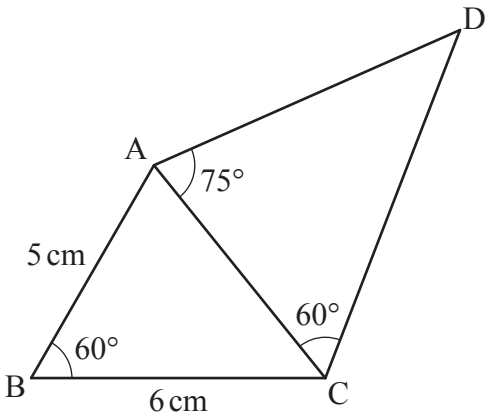


Fig. 2

Calculate the exact value of the length AD. **[4]**

3 Fig. 3 shows a triangle PQR. The vector $\overrightarrow{PQ}$ is $\mathbf{i} + 7\mathbf{j}$ and the vector $\overrightarrow{QR}$ is $4\mathbf{i} - 12\mathbf{j}$.

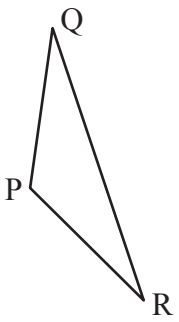


Fig. 3

(a) Show that the triangle PQR is isosceles. **[3]**

The point P has position vector $-3\mathbf{i} - \mathbf{j}$. The point S is added so that PQRS is a parallelogram.

(b) Find the position vector of S. **[2]**

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4 In this question, the x and y directions are horizontal and vertically upwards respectively.

A particle of mass 1.5 kg is in equilibrium under the action of its weight and forces $\mathbf{F}_1 = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$N and $\mathbf{F}_2$.

(a) Find the force $\mathbf{F}_2$. **[3]**

The force $\mathbf{F}_2$ is changed to $\begin{pmatrix} 2 \\ 20 \end{pmatrix}$N.

(b) Find the acceleration of the particle. **[2]**

5 Fig. 5.1 shows part of the curve $y = x^{\frac{1}{2}}$. P is the point (1, 1) and Q is the point on the curve with x-coordinate $1 + h$.

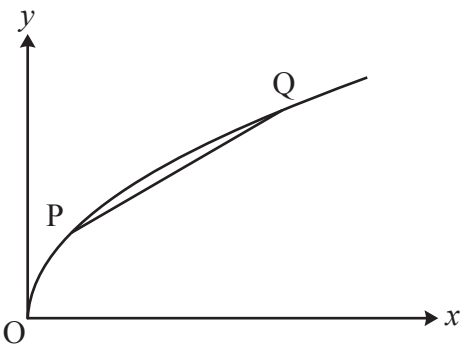


Fig. 5.1

Table 5.2 shows, for different values of h, the coordinates of P, the coordinates of Q, the change in y from P to Q and the gradient of the chord PQ.

x for P	y for P	h	x for Q	y for Q	change in y	gradient PQ
1	1	1				
1	1	0.1	1.1	1.048 809	0.048 809	0.488 088
1	1	0.01	1.01	1.004 988	0.004 988	0.498 756
1	1	0.001	1.001	1.000 500	0.000 500	0.499 875

Table 5.2

(a) Fill in the missing values for the case $h = 1$ in the copy of Table 5.2 in the Printed Answer Booklet. Give your answers correct to 6 decimal places where necessary. **[1]**

(b) Explain how the sequence of values in the last column of Table 5.2 relates to the gradient of the curve $y = x^{\frac{1}{2}}$ at the point P. **[1]**

(c) Use calculus to find the gradient of the curve at the point P. **[2]**

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6 **In this question you must show detailed reasoning.**

A particle moves in a straight line. Its velocity $v\,\text{m s}^{-1}$ after t s is given by $v = t^3 - 5t^2$.

(a) Find the times at which the particle is stationary. **[2]**

(b) Find the total distance travelled by the particle in the first 6 seconds. **[3]**

7 **In this question you must show detailed reasoning.**

A curve has equation $y = 4x^3 - 6x^2 - 9x + 4$.

(a) Sketch the gradient function for this curve, clearly indicating the points where the gradient is zero. **[4]**

(b) Find the set of values of x for which the gradient function is decreasing. Give your answer using set notation. **[2]**

8 The point A has coordinates $(-1, -2)$ and the point B has coordinates (7, 4). The perpendicular bisector of AB intersects the line $y + 2x = k$ at P.

Determine the coordinates of P in terms of k. **[7]**

9 A car travelling in a straight line accelerates uniformly from rest to $V\,\text{m s}^{-1}$ in T s. It then slows down uniformly, coming to rest after a further $2T$ s.

(a) Sketch a velocity-time graph for the car. **[2]**

The acceleration in the first stage of the motion is $2.5\,\text{m s}^{-2}$ and the total distance travelled is 240 m.

(b) Calculate the values of V and T. **[4]**

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10 An astronaut on the surface of the moon drops a ball from a point 2 m above the surface.

(a) Without any calculations, explain why a standard model using $g = 9.8\,\text{m s}^{-2}$ will not be appropriate to model the fall of the ball. **[1]**

The ball takes 1.6 s to hit the surface.

(b) Find the acceleration of the ball which best models its motion. Give your answer correct to 2 significant figures. **[2]**

(c) Use this value to predict the maximum height of the ball above the point of projection when thrown vertically upwards with an initial velocity of $15\,\text{m s}^{-1}$. **[2]**

11 In this question you must show detailed reasoning.

(a) A student is asked to solve the inequality $x^{\frac{1}{2}} < 4$.

The student argues that $x^{\frac{1}{2}} < 4 \Leftrightarrow x < 16$, so that the solution is $\{x : x < 16\}$.

Comment on the validity of the student's argument. **[1]**

(b) Solve the inequality $\left(\frac{1}{2}\right)^x < 4$. **[3]**

(c) Show that the equation $2\log_2(x+8) - \log_2(x+6) = 3$ has only one root. **[5]**

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12 In this question you must show detailed reasoning.

Fig. 12 shows part of the graph of $y = x^2 + \frac{1}{x^2}$.

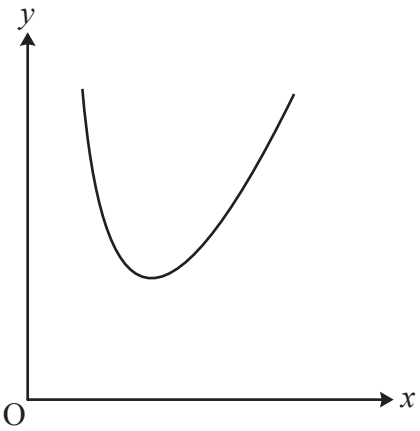


Fig. 12

The tangent to the curve $y = x^2 + \frac{1}{x^2}$ at the point $(2, \frac{17}{4})$ meets the x-axis at A and meets the y-axis at B. O is the origin.

(a) Find the exact area of the triangle OAB. **[6]**

(b) Use calculus to prove that the complete curve has two minimum points and no maximum point. **[6]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1

Take $n = 2$, which is a prime number:

$n^2 + 2n + 10 = 2^2 + 2 \times 2 + 10 = 4 + 4 + 10 = 18$

18 is even, not odd, so $n = 2$ is a counterexample.

$\therefore$ Celia's statement is false.

2

Cosine rule in triangle ABC:

$AC^2 = 5^2 + 6^2 - 2 \times 5 \times 6 \times \cos 60° = 25 + 36 - 30 = 31$

$AC = \sqrt{31}$

In triangle ACD, angle $ADC = 180° - 75° - 60° = 45°$

Sine rule: $\frac{AD}{\sin 60°} = \frac{AC}{\sin 45°} \Rightarrow AD = \frac{\sqrt{31} \times \sin 60°}{\sin 45°}$

$AD = \sqrt{31} \times \frac{\sqrt{3}}{2} \div \frac{\sqrt{2}}{2} = \frac{\sqrt{31} \times \sqrt{3}}{\sqrt{2}} = \frac{\sqrt{93}}{\sqrt{2}} = \frac{\sqrt{93} \times \sqrt{2}}{2}$

$AD = \frac{\sqrt{186}}{2}$ cm

3 (a)

$|\overrightarrow{PQ}| = \sqrt{(1^2 + 7^2)} = \sqrt{50}$

$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = (i + 7j) + (4i - 12j) = 5i - 5j$

$|\overrightarrow{PR}| = \sqrt{(5^2 + (-5)^2)} = \sqrt{50}$

$PQ = PR = \sqrt{50}$, so two sides are equal

$\therefore$ triangle PQR is isosceles.

3 (b)

PQRS is a parallelogram, so $\overrightarrow{PS} = \overrightarrow{QR} = 4i - 12j$

$\overrightarrow{OS} = \overrightarrow{OP} + \overrightarrow{PS} = (-3i - j) + (4i - 12j)$

Position vector of S is $i - 13j$

Question number

4 (a) Weight $= \begin{pmatrix} 0 \\ -1.5g \end{pmatrix} = \begin{pmatrix} 0 \\ -14.7 \end{pmatrix}$ N

Equilibrium: $\begin{pmatrix} 0 \\ -14.7 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \end{pmatrix} + F_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$F_2 = \begin{pmatrix} -4 \\ 16.7 \end{pmatrix}$ N

4 (b) Resultant $= \begin{pmatrix} 0 \\ -14.7 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ 20 \end{pmatrix} = \begin{pmatrix} 6 \\ 3.3 \end{pmatrix}$ N

$F = ma$: $\begin{pmatrix} 6 \\ 3.3 \end{pmatrix} = 1.5a \Rightarrow a = \frac{1}{1.5}\begin{pmatrix} 6 \\ 3.3 \end{pmatrix}$

$a = \begin{pmatrix} 4 \\ 2.2 \end{pmatrix}$ m s^{-2}

5 (a)

x for P	y for P	h	x for Q	y for Q	change in y	gradient PQ
1	1	1	2	1.414214	0.414214	0.414214

(y for Q $= \sqrt{2} = 1.41421356...$, gradient $= 0.414214 \div 1$)

5 (b) As h tends to zero the gradients of the chords PQ tend to a limit (0.5), and this limit is the gradient of the tangent to $y = x^{½}$ at P.

5 (c) $y = x^{½} \Rightarrow \frac{dy}{dx} = \frac{1}{2}x^{-½}$

At P, $x = 1$: $\frac{dy}{dx} = \frac{1}{2} \times 1^{-½} = \frac{1}{2}$

Gradient of the curve at P is 0.5

6 (a) Stationary when $v = 0$: $t^3 - 5t^2 = 0$

Question number

$t^2(t - 5) = 0$

$t = 0$ s or $t = 5$ s

6 (b) $v = t^2(t - 5) < 0$ for $0 < t < 5$ and $v > 0$ for $5 < t \leq 6$,

so the particle changes direction at $t = 5$.

$$\int_0^5 (t^3 - 5t^2)\,dt = \left[\frac{t^4}{4} - \frac{5t^3}{3}\right]_0^5 = \frac{625}{4} - \frac{625}{3} = -\frac{625}{12}$$

$$\int_5^6 (t^3 - 5t^2)\,dt = \left[\frac{t^4}{4} - \frac{5t^3}{3}\right]_5^6 = (324 - 360) - \left(-\frac{625}{12}\right) = \frac{193}{12}$$

Total distance $= \frac{625}{12} + \frac{193}{12} = \frac{818}{12} = \frac{409}{6} = 68.166\ldots$

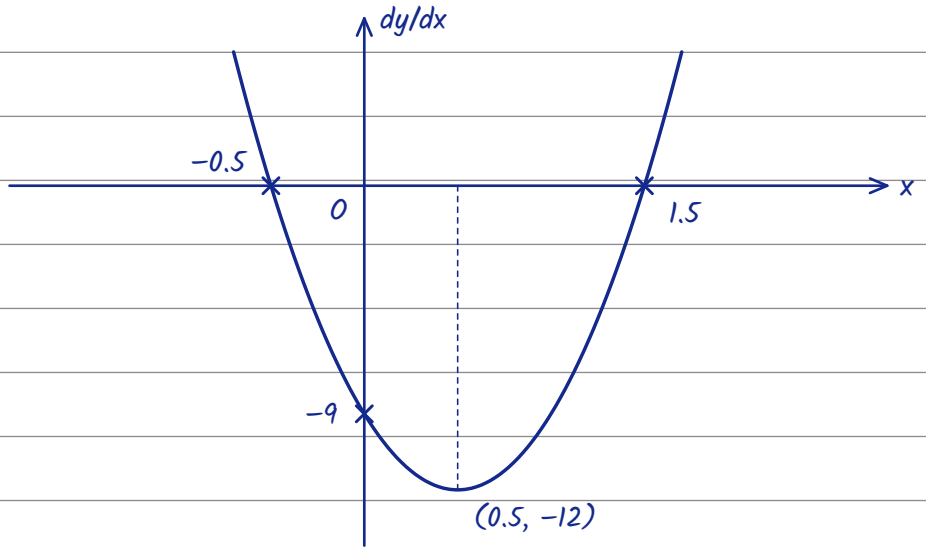
Total distance = 68.2 m (3 s.f.)

7 (a) $\frac{dy}{dx} = 12x^2 - 12x - 9$

Gradient zero when $12x^2 - 12x - 9 = 0 \Rightarrow 3(2x + 1)(2x - 3) = 0$

$x = -\frac{1}{2}$ or $x = \frac{3}{2}$; when $x = 0$, $\frac{dy}{dx} = -9$

Minimum of gradient function at $x = \frac{1}{2}$, where $\frac{dy}{dx} = 3 - 6 - 9 = -12$



Question number

7 (b) $\frac{d^2y}{dx^2} = 24x - 12 = 0 \Rightarrow x = \frac{1}{2}$ (vertex of the gradient function)

Gradient function is decreasing when $24x - 12 < 0$, i.e. $x < \frac{1}{2}$

$\{x : x < \frac{1}{2}\}$

8 Midpoint of AB $= \left(\frac{-1+7}{2}, \frac{-2+4}{2}\right) = (3, 1)$

Gradient of AB $= \frac{4-(-2)}{7-(-1)} = \frac{6}{8} = \frac{3}{4}$

Gradient of perpendicular bisector $= -\frac{4}{3}$

$y - 1 = -\frac{4}{3}(x - 3) \Rightarrow 3y - 3 = -4x + 12 \Rightarrow 4x + 3y = 15$

At P, $y = k - 2x$: $4x + 3(k - 2x) = 15$

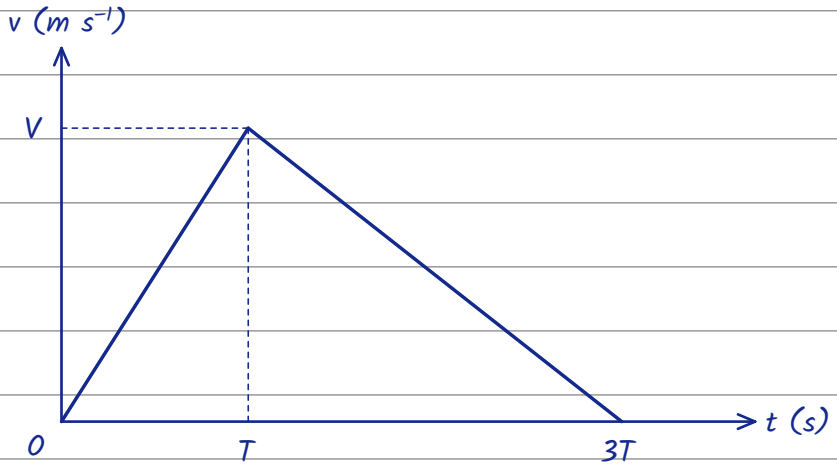
$-2x + 3k = 15 \Rightarrow x = \frac{3k - 15}{2}$

$y = k - 2 \times \frac{3k - 15}{2} = k - 3k + 15 = 15 - 2k$

P is $\left(\frac{3k - 15}{2}, 15 - 2k\right)$

Question number

9 (a)



9 (b) Acceleration = gradient = $\frac{V}{T}$ = 2.5 ⇒ V = 2.5T

Distance = area under graph = $\frac{1}{2}$ × 3T × V = 240 ⇒ VT = 160

Substitute V = 2.5T: $2.5T^2 = 160$ ⇒ $T^2 = 64$

T = 8 , V = 2.5 × 8 = 20

10 (a) 9.8 m s^{-2} is the acceleration due to gravity on Earth; gravity on the moon is much weaker, so g has a different value there.

10 (b) Down positive: s = 2, u = 0, t = 1.6

$s = ut + \frac{1}{2}at^2$ ⇒ $2 = 0 + \frac{1}{2} \times a \times 1.6^2 = 1.28a$

$a = \frac{2}{1.28} = 1.5625$ ⇒ a = 1.6 m s^{-2} (2 s.f.)

10 (c) Up positive: u = 15, v = 0, a = −1.6

$v^2 = u^2 + 2as$ ⇒ $0 = 15^2 - 2 \times 1.6 \times s$

$s = \frac{225}{3.2} = 70.3125$ ⇒ maximum height = 70.3 m

Question number

11 (a) The argument is not valid. $x^{1/2}$ only exists for $x \geq 0$, but $x < 16$ includes negative values of x for which $x^{1/2}$ is not defined.

The correct solution is $\{x : 0 \leq x < 16\}$.

11 (b) $\left(\frac{1}{2}\right)^x < 4 \Rightarrow \log\left(\frac{1}{2}\right)^x < \log 4 \Rightarrow x \log\left(\frac{1}{2}\right) < \log 4$

$\log\left(\frac{1}{2}\right) < 0$, so dividing by it reverses the inequality:

$$x > \frac{\log 4}{\log(\frac{1}{2})} = \frac{2 \log 2}{-\log 2} = -2$$

$x > -2$

11 (c) $2\log_2(x + 8) - \log_2(x + 6) = 3$

$$\log_2(x + 8)^2 - \log_2(x + 6) = 3 \Rightarrow \log_2 \frac{(x + 8)^2}{x + 6} = 3$$

$$\frac{(x + 8)^2}{x + 6} = 2^3 = 8 \Rightarrow (x + 8)^2 = 8(x + 6)$$

$x^2 + 16x + 64 = 8x + 48 \Rightarrow x^2 + 8x + 16 = 0$

Discriminant $b^2 - 4ac = 8^2 - 4 \times 1 \times 16 = 64 - 64 = 0$

Discriminant is zero, so there is one repeated root: $(x + 4)^2 = 0$, $x = -4$

($x = -4$ gives $x + 8 = 4 > 0$ and $x + 6 = 2 > 0$, so the logs exist)

$\therefore$ the equation has only one root.

12 (a) $y = x^2 + x^{-2} \Rightarrow \frac{dy}{dx} = 2x - 2x^{-3}$

At $\left(2, \frac{17}{4}\right)$: gradient $= 2 \times 2 - 2 \times 2^{-3} = 4 - \frac{1}{4} = \frac{15}{4}$

Tangent: $y - \frac{17}{4} = \frac{15}{4}(x - 2) \Rightarrow y = \frac{15}{4}x - \frac{13}{4}$

Question number

A: $y = 0 \Rightarrow \frac{15}{4}x = \frac{13}{4} \Rightarrow x = \frac{13}{15}$, so A is $(\frac{13}{15}, 0)$

B: $x = 0 \Rightarrow y = -\frac{13}{4}$, so B is $(0, -\frac{13}{4})$

Area of $OAB = \frac{1}{2} \times \frac{13}{15} \times \frac{13}{4} = \frac{169}{120}$

12 (b) Stationary points: $\frac{dy}{dx} = 2x - 2x^{-3} = 0$

$2x = \frac{2}{x^3} \Rightarrow x^4 = 1 \Rightarrow x = 1$ or $x = -1$ (only real roots)

So there are exactly two stationary points, (1, 2) and (−1, 2).

$\frac{d^2y}{dx^2} = 2 + 6x^{-4} = 2 + \frac{6}{x^4}$

When $x = 1$: $\frac{d^2y}{dx^2} = 2 + 6 = 8 > 0$, so (1, 2) is a minimum point

When $x = -1$: $\frac{d^2y}{dx^2} = 2 + 6 = 8 > 0$, so (−1, 2) is a minimum point

Both stationary points are minimum points, so the curve has two minimum points and no maximum point.