OCR
Oxford Cambridge and RSA

Thursday 25 May 2023 – Afternoon

AS Level Mathematics B (MEI)

H630/02 Pure Mathematics and Statistics

Time allowed: 1 hour 30 minutes

9917247140

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

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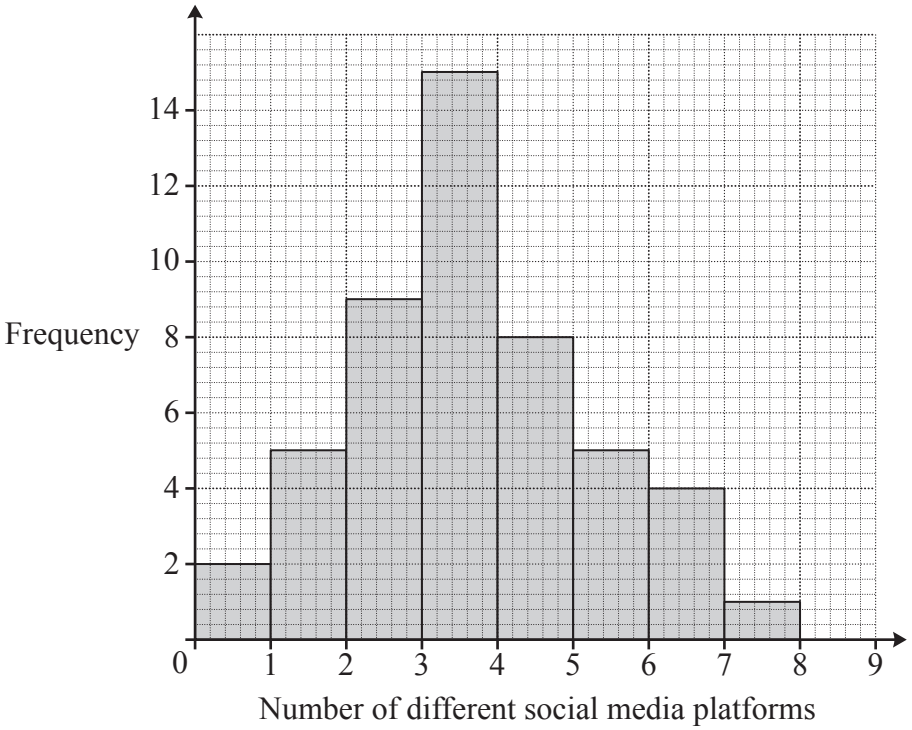
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1 A researcher collects data concerning the number of different social media platforms used by school pupils on a typical weekday.

The frequency table for the data is shown below.

Number of different social media platforms	0	1	2	3	4	5	6	7
Frequency	2	5	9	15	8	5	4	1

The researcher uses software to represent the results in this diagram.



(a) Explain why this diagram is inappropriate. **[1]**

(b) Calculate the following for the number of social media platforms used:

(i) the mean, **[1]**

(ii) the standard deviation. **[1]**

2 **(a)** Express $x^2 - 6x + 1$ in the form $(x-a)^2 - b$, where a and b are integers to be determined. **[2]**

(b) **Hence** state the coordinates of the turning point on the graph of $y = x^2 - 6x + 1$. **[1]**

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3 A student makes the following conjecture.

For all positive integers n, $6n-1$ is always prime.

Use a counter example to disprove this conjecture. **[2]**

4 The equation of a curve is $y = \frac{k}{x^2}$, where k is a constant.

The curve passes through the point (2, 1).

(a) Find the value of k. **[1]**

(b) Sketch the curve. **[2]**

5 Show that the distance between the points (5, 2) and (11, –1) is $a\sqrt{b}$, where a and b are integers to be determined. **[3]**

6 An app on my new smartphone records the number of times in a day I use the phone. The data for each day since I bought the phone are shown in the stem and leaf diagram.

1	9
2	6
3	8 9
4	0 1 2 2 3 5 6 7 9 9
5	1 2 2 2 3 4 5 5 7 8 9 9
6	0 1 1 3 9

Key: 3|1 means 31

(a) Explain whether these data are a sample or a population. **[1]**

(b) Describe the shape of the distribution. **[1]**

(c) Determine the interquartile range. **[2]**

(d) Use your answer to part **(c)** to determine whether there are any outliers in the lower tail. **[2]**

7 **(a)** Use the factor theorem to show that $(x-2)$ is a factor of x^3+6x^2-x-30. **[1]**

(b) Factorise x^3+6x^2-x-30 completely. **[3]**

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8 The pre-release material contains information on Pulse Rate and Body Mass Index (BMI).
A student is investigating whether there is a relationship between pulse rate and BMI. A **section** of the available data is shown in the table.

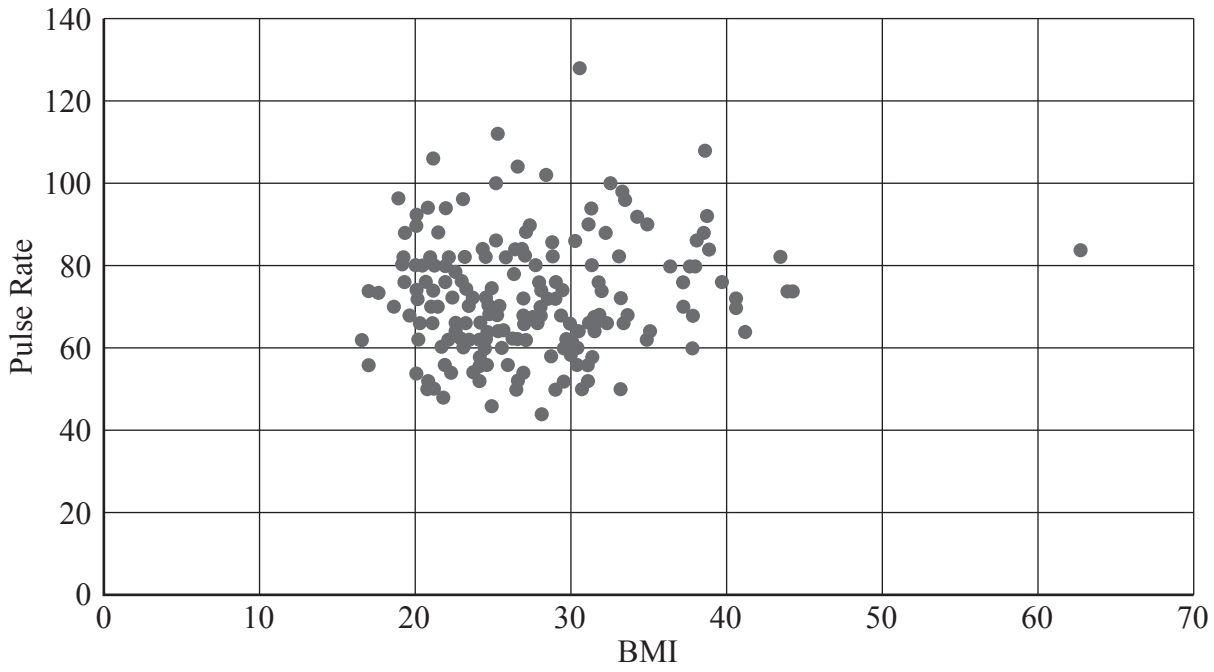
Sex	Age	BMI	Pulse
Male	62	29.54	60
Female	20	23.68	#N/A
Male	17	26.97	72
Male	35	24.7	64
Male	17	20.09	54
Male	85	23.86	54
Female	81	24.04	#N/A

The student decides to draw a scatter diagram.

(a) With reference to the table, explain which data should be cleaned before any analysis takes place. **[1]**

The student cleans the data for BMI and Pulse Rate in the pre-release material and draws a scatter diagram.

Scatter diagram of Pulse Rate against BMI



The student identifies **one** outlier.

(b) On the copy of the scatter diagram in the Printed Answer Booklet, circle this outlier. **[1]**

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The student decides to remove this outlier from the data. They then use the LINEST function in the spreadsheet to obtain the following formula for the line of best fit.

$P = 0.29Q + 64.2$,

where P = Pulse Rate and Q = BMI.

They use this to estimate the Pulse Rate of a person with BMI 23.68.
They obtain a value of 71 correct to the nearest whole number.

(c) With reference to the scatter diagram, explain whether it is appropriate to use the formula for the line of best fit. **[1]**

It is suggested that all pairs of values where the pulse rate is above 100 should also be cleaned from the data, as they must be incorrect.

(d) Use your knowledge of the pre-release material to explain whether or not all pairs of values with a pulse rate of more than 100 should be cleaned from the data. **[1]**

9 The table shows the probability distribution for the discrete random variable X.

x	1	2	3	4	5
$\mathrm{P}(X = x)$	0.1	0.3	q	$2q$	$3q$

You are given that q is a positive constant.

(a) Determine the value of q. **[2]**

(b) Calculate $\mathrm{P}(X \leqslant 4)$. **[1]**

Two independent values of X are taken.

(c) Determine the probability that the sum of the two values is 3. **[2]**

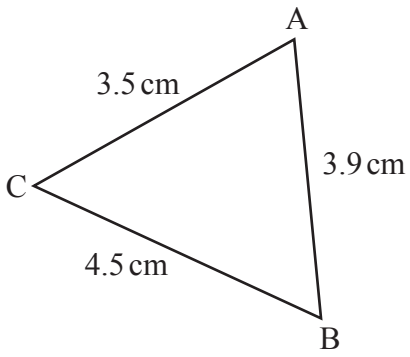
Fifty independent values of X are taken.

(d) Find the probability that a value of 2 occurs exactly 17 times. **[1]**

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10 In this question you must show detailed reasoning.

The diagram shows triangle ABC, where $AB = 3.9\,\text{cm}$, $BC = 4.5\,\text{cm}$ and $AC = 3.5\,\text{cm}$.



Determine the area of triangle ABC. **[5]**

11 In this question you must show detailed reasoning.

The equation of a curve is $y = 2x^3 + 9x^2 + 24x - 8$.

Show that there are no stationary points on this curve. **[5]**

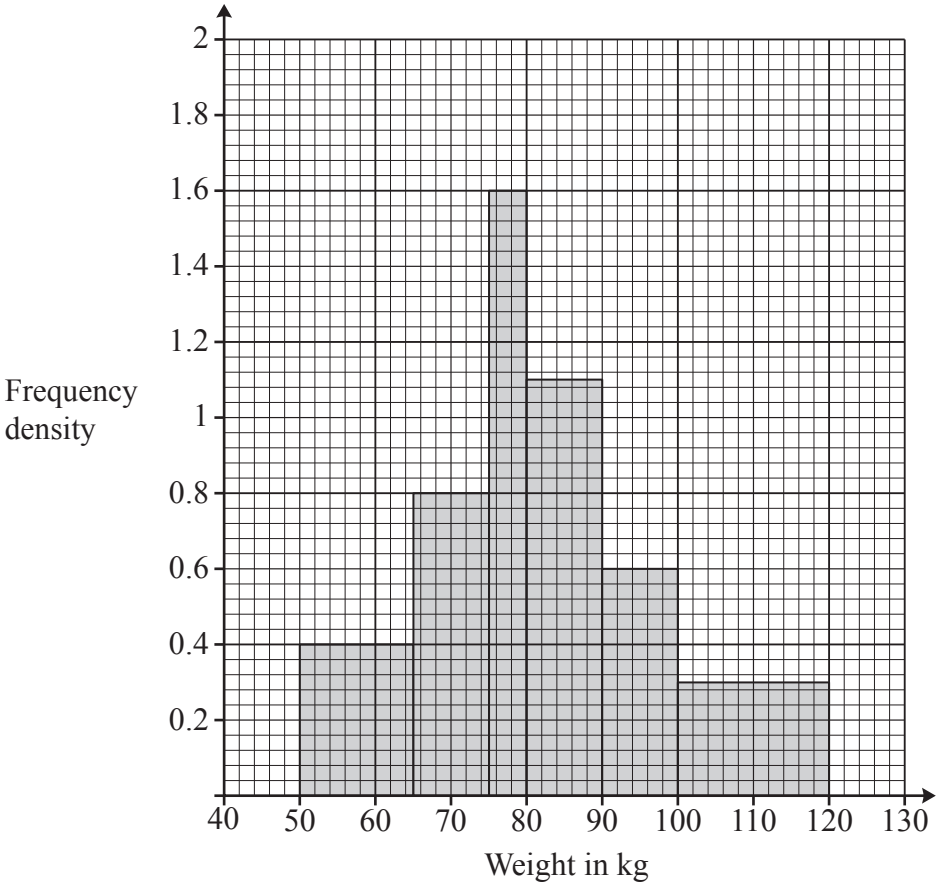
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12 Doctors are investigating the weights of adult males registered at their surgery. One week they collect a sample by noting the weight in kilograms of all the adult males who have an appointment at their surgery.

(a) State the sampling method they use. **[1]**

(b) Explain why this method will **not** generate a simple random sample of all the adult males registered at their surgery. **[1]**

They represent the data using a histogram.



An incomplete frequency table for the data is shown below.

Weight in kg	50–	65–	75–	80–	90–	100–120
Frequency		8				

(c) Complete the copy of the frequency table in the Printed Answer Booklet. **[1]**

One of these patients is selected at random.

(d) Determine an estimate of the probability that he weighs either less than 60 kg or more than 110 kg. **[2]**

(e) Explain why your answer to part **(d)** is an estimate and not exact. **[1]**

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13 In a report published in October 2021 it is stated that 37% of adults in the United Kingdom never exercise or play sport. A researcher believes that the true percentage is less than this. They decide to carry out a hypothesis test at the 5% level to investigate the claim.

(a) State the null and alternative hypotheses for their test. **[1]**

(b) Define the parameter for their test. **[1]**

In a random sample of 118 adults, they find that 35 of them never exercise or play sport.

(c) Carry out the test. **[4]**

14 In this question you must show detailed reasoning.

The equation of a curve is $y = 16\sqrt{x} + \frac{8}{x}$.

Determine the equation of the tangent to the curve at the point where $x = 4$. **[7]**

15 A family is planning a holiday in Europe. They need to buy some euros before they go. The exchange rate, y, is the number of euros they can buy per pound. They believe that the exchange rate may be modelled by the formula

$$y = at^2 + bt + c,$$

where t is the time in days from when they first check the exchange rate.

Initially, when $t = 0$, the exchange rate is 1.14.

(a) Write down the value of c. **[1]**

When $t = 2$, $y = 1.20$ and when $t = 4$, $y = 1.25$.

(b) Calculate the values of a and b. **[2]**

The family will only buy their euros when their model predicts an exchange rate of at least 1.29.

(c) Determine the range of values of t for which, according to their model, they will buy their euros. **[3]**

(d) Explain why the family's model is not viable in the long run. **[1]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a) The number of platforms is discrete data, but this diagram (a histogram with no gaps, bars from 0 to 1, 1 to 2, ...) represents continuous data, so it is inappropriate.

1 (b)(i) $n = 49$, $\Sigma fx = 156$, $\text{mean} = \frac{156}{49} = 3.183673... = 3.18$

1 (b)(ii) $\Sigma fx^2 = 622$

$S_{xx} = 622 - \frac{156^2}{49} = 125.346...$

$s = \sqrt{\left(\frac{125.346...}{48}\right)} = \sqrt{2.611394...} = 1.615980... = 1.62$

2 (a) $x^2 - 6x + 1 = (x - 3)^2 - 9 + 1$

$= (x - 3)^2 - 8 \qquad (a = 3, b = 8)$

2 (b) Turning point $(3, -8)$

3 $n = 1$: $6 \times 1 - 1 = 5$ (prime)

$n = 6$: $6 \times 6 - 1 = 35 = 5 \times 7$

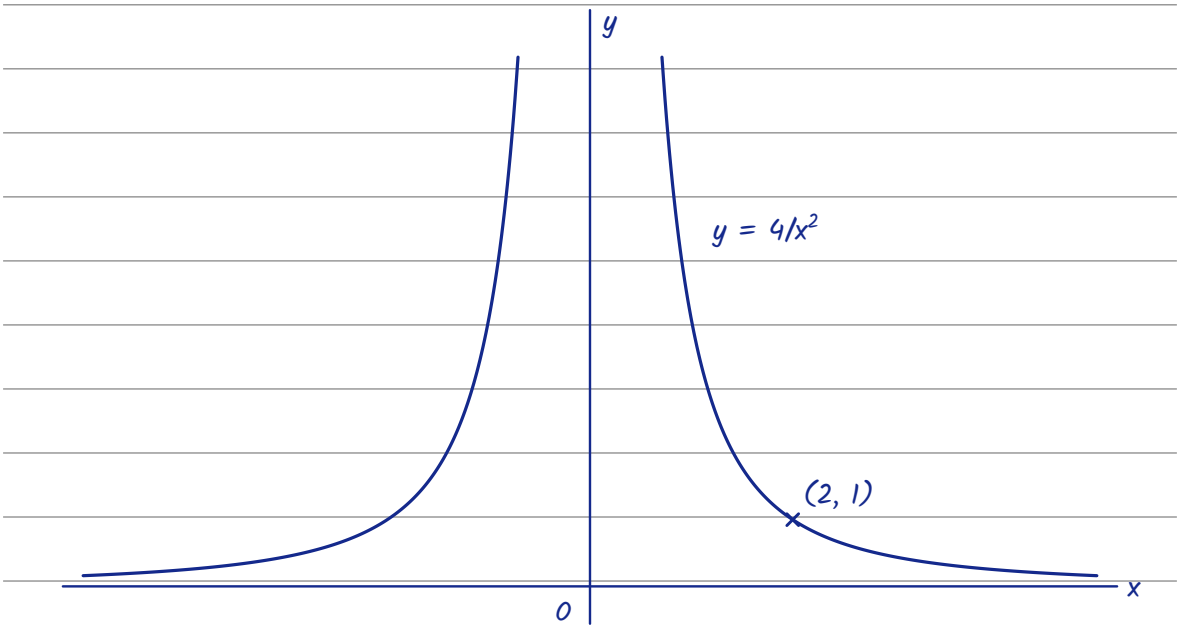
35 has factors 5 and 7, so 35 is not prime.

So the conjecture is false (counter example $n = 6$).

4 (a) $1 = \frac{k}{2^2} \Rightarrow k = 4$

Question number

4 (b)



5 $d = \sqrt{((11 - 5)^2 + (-1 - 2)^2)}$

$= \sqrt{(6^2 + (-3)^2)} = \sqrt{(36 + 9)}$

$= \sqrt{45} = \sqrt{9} \times \sqrt{5}$

$= 3\sqrt{5}$ so $a = 3$, $b = 5$

6 (a) Population: the data include every day since I bought the phone, so all the available data are used.

6 (b) Negative skew

6 (c) $n = 31$

Q_1 = 8th value = 42, Q_3 = 24th value = 58

$IQR = 58 - 42 = 16$

6 (d) Lower fence = $Q_1 - 1.5 \times IQR = 42 - 1.5 \times 16 = 18$

Smallest value is 19, and $19 > 18$,

so there are no outliers in the lower tail.

Question number

7 (a)

$f(x) = x^3 + 6x^2 - x - 30$

$f(2) = 2^3 + 6 \times 2^2 - 2 - 30 = 8 + 24 - 2 - 30 = 0$

$f(2) = 0$, so by the factor theorem $(x - 2)$ is a factor.

7 (b)

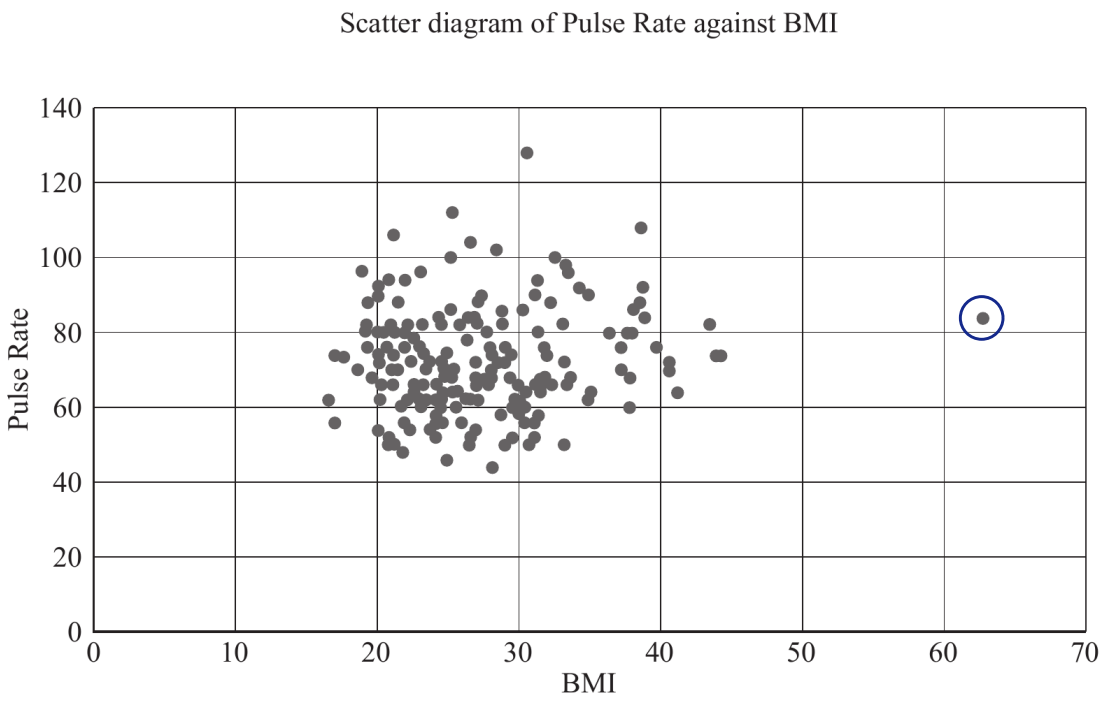
$x^3 + 6x^2 - x - 30 = (x - 2)(x^2 + 8x + 15)$

$= (x - 2)(x + 3)(x + 5)$

8 (a)

Remove any rows with #N/A in the Pulse column (e.g. the females aged 20 and 81), as there is no data available for pulse rate.

8 (b)



8 (c)

Not appropriate: the scatter diagram shows no evidence of a linear relationship (very little correlation) between pulse rate and BMI, so the estimate is unlikely to be reliable.

8 (d)

They should not all be removed: pulse rates above 100 are not that unusual (e.g. after exercise or for some people), so these values are not necessarily

Question number

8 cont. incorrect and should be kept.

9 (a) $0.1 + 0.3 + q + 2q + 3q = 1$

$0.4 + 6q = 1 \Rightarrow q = 0.1$

9 (b) $P(X \leq 4) = 0.1 + 0.3 + 0.1 + 0.2 = 0.7$

9 (c) Sum = 3: (1, 2) or (2, 1)

$P = 0.1 \times 0.3 + 0.3 \times 0.1 = 0.03 + 0.03 = 0.06$

9 (d) $Y \sim B(50, 0.3)$, Y = number of times a 2 occurs

$P(Y = 17) = {}^{50}C_{17} \times 0.3^{17} \times 0.7^{33} = 0.098314... = 0.0983$

10 Using the cosine rule for angle C:

$$\cos C = \frac{3.5^2 + 4.5^2 - 3.9^2}{2 \times 3.5 \times 4.5} = \frac{12.25 + 20.25 - 15.21}{31.5} = \frac{17.29}{31.5}$$

$\cos C = 0.548888...$

$C = 56.709...°$

$$\text{Area} = \frac{1}{2} \times 3.5 \times 4.5 \times \sin 56.709...°$$

$= 7.875 \times 0.835895...$

$= 6.582675... = 6.58 \text{ cm}^2$

11 $y = 2x^3 + 9x^2 + 24x - 8$

$dy/dx = 6x^2 + 18x + 24$

At a stationary point $dy/dx = 0$: $6x^2 + 18x + 24 = 0$

Discriminant $b^2 - 4ac = 18^2 - 4 \times 6 \times 24 = 324 - 576$

$= -252 < 0$

so $dy/dx = 0$ has no real solutions,

Question number

11 cont. therefore there are no stationary points on the curve.

12 (a) Opportunity (convenience) sampling

12 (b) Not every adult male registered at the surgery has an equal chance of being selected: only those with an appointment that week can be chosen, so the sampling frame is incomplete.

12 (c) Frequency = frequency density × class width

e.g. 50 – 65: 0.4 × 15 = 6; 80 – 90: 1.1 × 10 = 11

Weight in kg	50 –	65 –	75 –	80 –	90 –	100 – 120
Frequency	6	8	8	11	6	6

12 (d) Total frequency = 6 + 8 + 8 + 11 + 6 + 6 = 45

Less than 60 kg: 0.4 × 10 = 4 (⅔ × 6)

More than 110 kg: 0.3 × 10 = 3 (½ × 6)

$$P = \frac{4 + 3}{45} = \frac{7}{45} = 0.1555\ldots = 0.156$$

12 (e) The individual weights are not known: we assume the weights are evenly distributed within each class, but the distribution within each class is unknown.

13 (a) H_0: $p = 0.37$ H_1: $p < 0.37$

13 (b) p is the probability that an adult selected at random in the United Kingdom never exercises or plays sport.

13 (c) Let X = number in the sample who never exercise or play sport.

Question number

13 cont. Under H_0, $X \sim B(118, 0.37)$

$P(X \leqslant 35) = 0.058220... = 0.0582$

$0.0582 > 0.05$, so the result is not significant: do not reject H_0

There is insufficient evidence at the 5% level to suggest that the percentage of adults in the United Kingdom who never exercise or play sport is less than 37%.

14 $y = 16\sqrt{x} + \frac{8}{x}$

$y = 16x^{1/2} + 8x^{-1}$

$dy/dx = 16 \times \frac{1}{2} x^{-1/2} - 8x^{-2} = 8x^{-1/2} - 8x^{-2}$

$x = 4$: $dy/dx = \frac{8}{\sqrt{4}} - \frac{8}{4^2} = 4 - \frac{1}{2} = \frac{7}{2}$

$x = 4$: $y = 16\sqrt{4} + \frac{8}{4} = 32 + 2 = 34$

Tangent: $y - 34 = \frac{7}{2}(x - 4)$

$y = \frac{7}{2}x - 14 + 34$

$y = \frac{7}{2}x + 20$

15 (a) $c = 1.14$

15 (b) $t = 2$: $1.20 = 4a + 2b + 1.14 \Rightarrow 4a + 2b = 0.06$

$t = 4$: $1.25 = 16a + 4b + 1.14 \Rightarrow 16a + 4b = 0.11$

Doubling the first: $8a + 4b = 0.12$; subtract: $8a = -0.01$

$a = -0.00125$, $b = (0.06 + 0.005) \div 2 = 0.0325$

Question number	
15 (c)	$-0.00125t^2 + 0.0325t + 1.14 \geq 1.29$
	$-0.00125t^2 + 0.0325t - 0.15 \geq 0$
	$\times (-800)$: $t^2 - 26t + 120 \leq 0$
	$(t - 6)(t - 20) \leq 0 \quad \Rightarrow$ critical values $t = 6$ and $t = 20$
	$6 \leq t \leq 20$
15 (d)	As $a < 0$, for large t the model will eventually predict a negative exchange rate, which is impossible.