OCR
Oxford Cambridge and RSA

Thursday 18 May 2023 – Afternoon

AS Level Mathematics B (MEI)

H630/01 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes

9917034691

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

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1 A particle moves along a straight line. Its velocity $v\,\text{m}\,\text{s}^{-1}$ at time $t\,\text{s}$ is given by $v = 2t + 0.6t^2$.

Find an expression for the acceleration of the particle at time t. **[2]**

2 The height of the first part of a rollercoaster track is $h\,\text{m}$ at a horizontal distance of $x\,\text{m}$ from the start. A student models this using the equation $h = 17 + 15\cos 6x$, for $0 \leqslant x \leqslant 40$, using the values of h given when their calculator is set to work in degrees.

(a) Find the height that the student's model predicts when the horizontal distance from the start is 40 m. **[1]**

(b) The student argues that the model predicts that the rollercoaster track will achieve a maximum height of 32 m more than once because the cosine function is periodic.

Comment on the validity of the student's argument. **[2]**

3 The points A and B have position vectors $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$ respectively. The vector $\overrightarrow{\text{AC}}$ is $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$.

(a) Write down the position vector of C as a column vector. **[1]**

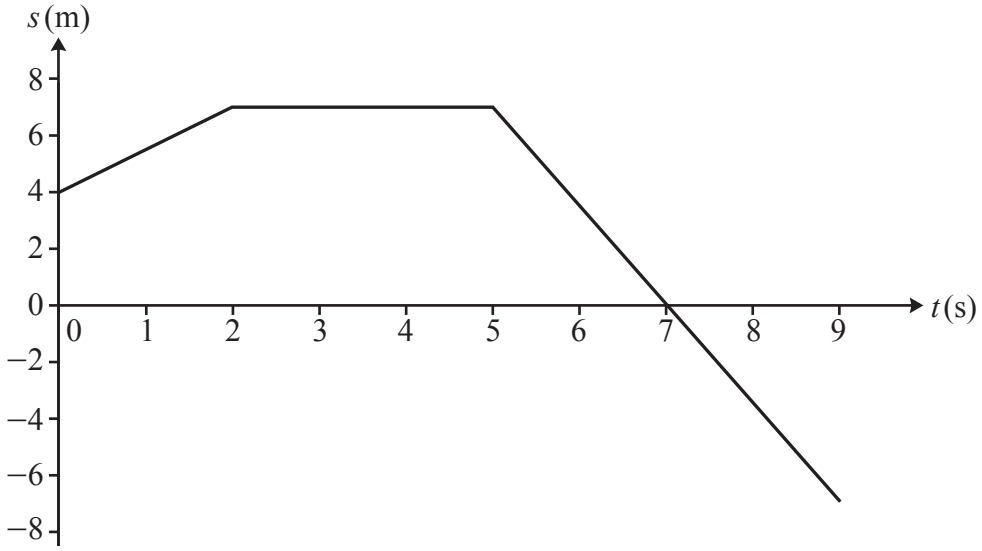
(b) Show that B is equidistant from A and C. **[3]**

4 **In this question you must show detailed reasoning.**

Solve the equation $6\cos^2 x + \sin x = 5$, giving all the roots in the interval $-180° \leqslant x \leqslant 180°$. **[5]**

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5 The graph shows displacement s m against time t s for a model of the motion of a bead moving along a straight wire. The points (0, 4), (2, 7), (5, 7) and (9, –7) are the endpoints of the line segments.



(a) Find an expression for the displacement of the bead for $0 \leqslant t \leqslant 2$. **[2]**

(b) Sketch the velocity-time graph for this model. **[2]**

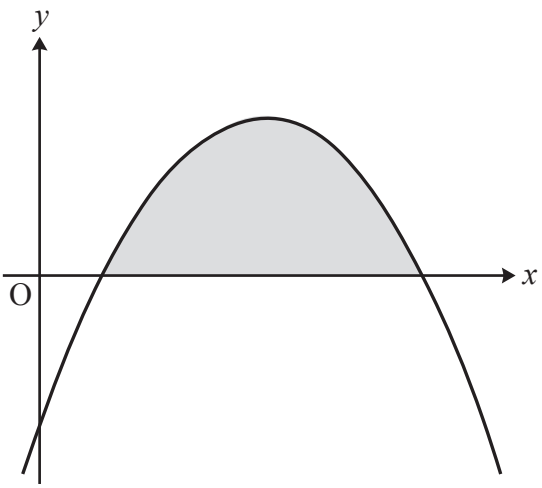
(c) Explain why the model may not be suitable at $t = 2$ and $t = 5$. **[1]**

6 Show that the expression $3x^3 + x^2 - 6x - 5$ can be written in the form $(x+2)(ax^2 + bx + c) + d$ where a, b, c and d are constants to be determined. **[5]**

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7 In this question you must show detailed reasoning.



Find the exact area of the shaded region shown in the diagram, enclosed by the x-axis and the curve $y = -3x^2 + 7x - 2$. **[6]**

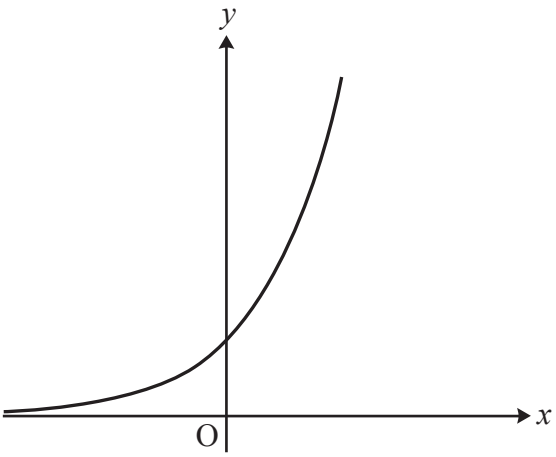
8 In this question you must show detailed reasoning.

(a) Find the centre and radius of the circle with equation $x^2 + y^2 - 2x + 4y - 20 = 0$. **[4]**

(b) Find the points of intersection of the circle with the line $x + 3y - 10 = 0$. **[5]**

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9 The graph shows the function $y = e^{2x}$.



(a) Describe the transformation of the graph of $y = e^{x}$ that gives the graph of $y = e^{2x}$. **[2]**

A second function is defined by $y = k + e^{x}$.

(b) A copy of the graph of $y = e^{2x}$ is given in the Printed Answer Booklet.

Add a sketch of the graph of $y = k + e^{x}$ in a case where k is a positive constant. **[2]**

(c) Show that the two graphs do not intersect for values of k less than $-\frac{1}{4}$. **[3]**

(d) In the case where $k = 2$, show that the only point of intersection occurs when $x = \ln 2$. **[2]**

10 Layla invests money in the bank and receives compound interest. The amount £L that she has after t years is given by the equation $L = 2800 \times 1.023^{t}$.

(a) (i) State the amount she invests. **[1]**

(ii) State the annual rate of interest. **[1]**

Amit invests £3000 and receives 2% compound interest per year. The amount £A that he has after t years is given by the equation $A = ab^{t}$.

(b) Determine the values of the constants a and b. **[2]**

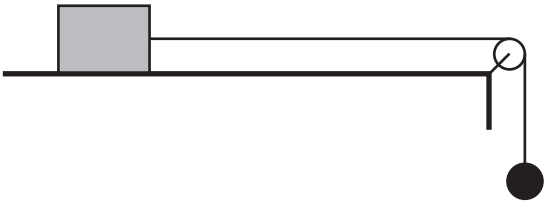
(c) Layla and Amit invest their money in the bank at the same time.

Determine the value of t for which Layla and Amit have equal amounts in the bank. Give your answer correct to **1** decimal place. **[3]**

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11 A block of mass 3 kg is at rest on a smooth horizontal table. It is attached to a light inextensible string which passes over a smooth pulley. This part of the string is horizontal. A sphere of mass 1.2 kg is attached to the other end of the string. The sphere hangs with this part of the string vertical as shown in the diagram. A horizontal force of magnitude F N is applied to the block to prevent motion.



(a) Complete the copy of the diagram in the Printed Answer Booklet to show all the forces acting on the block and the sphere. **[2]**

(b) Find the value of F. **[2]**

The force F N is removed, and the system begins to move.

(c) The equation of motion of the block is $T = 3a$, where T N is the tension in the string and $a\,\text{m}\,\text{s}^{-2}$ is the acceleration of the block.

Write down the equation of motion of the sphere. **[1]**

(d) Find the value of T. **[2]**

12 Points A, B and C lie in a straight line in that order on horizontal ground. A box of mass 5 kg is pushed from A to C by a horizontal force of magnitude 8 N. The box is at rest at A and takes 3 seconds to reach B. The ground is smooth between A and B. Between B and C the ground is rough and the resistance to motion is 28 N. The box comes to rest at C.

Determine the distance AC. **[8]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 $a = \frac{dv}{dt} = 2 + 1.2t \quad (\text{m s}^{-2})$

2 (a) $x = 40$: $h = 17 + 15\cos(6 \times 40) = 17 + 15\cos 240°$

$h = 17 + 15 \times (-0.5) = 9.5$ m

2 (b) Maximum height 32 m when $\cos 6x = 1$, i.e. $6x = 0°, 360°, \ldots$

so $x = 0, 60, 120, \ldots$ (period of $\cos 6x$ is 60°)

But the model is only valid for $0 \leq x \leq 40$, so the only maximum in this range is at $x = 0$. The track reaches 32 m only once, so the student's argument is not valid.

3 (a) $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

3 (b) $\overrightarrow{AB} = \begin{pmatrix} 5-2 \\ 4-(-1) \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \qquad \overrightarrow{BC} = \begin{pmatrix} 0-5 \\ 1-4 \end{pmatrix} = \begin{pmatrix} -5 \\ -3 \end{pmatrix}$

$|AB| = \sqrt{(3^2 + 5^2)} = \sqrt{34}$

$|BC| = \sqrt{((-5)^2 + (-3)^2)} = \sqrt{34}$

$AB = BC = \sqrt{34}$, so B is equidistant from A and C.

4 $6\cos^2 x + \sin x = 5$, using $\cos^2 x = 1 - \sin^2 x$:

$6(1 - \sin^2 x) + \sin x = 5$

$6 - 6\sin^2 x + \sin x - 5 = 0 \Rightarrow 6\sin^2 x - \sin x - 1 = 0$

$(2\sin x - 1)(3\sin x + 1) = 0$

Question number

$\sin x = \frac{1}{2}$ or $\sin x = -\frac{1}{3}$

$\sin x = \frac{1}{2}$: $x = 30°$ or $x = 180° - 30° = 150°$

$\sin x = -\frac{1}{3}$: $x = \sin^{-1}(-\frac{1}{3}) = -19.47...°$

or $x = -180° + 19.47...° = -160.52...°$

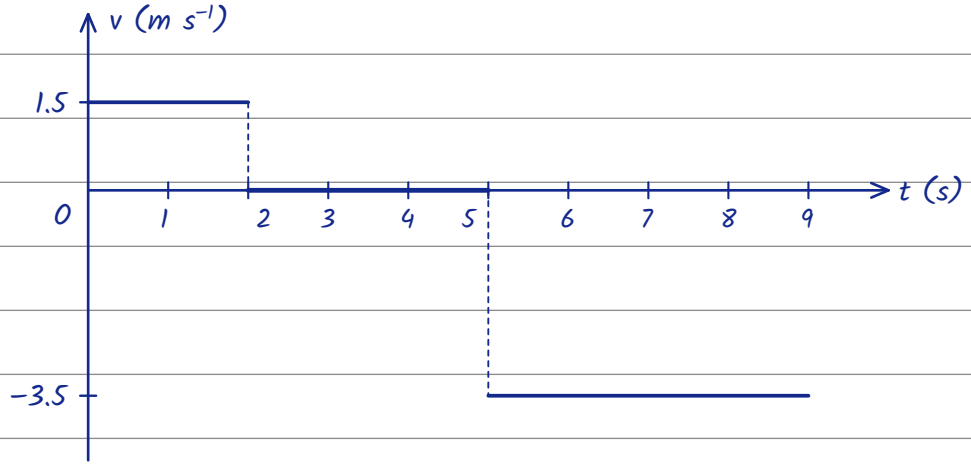
$x = -160.5°, -19.5°, 30°, 150°$

5 (a) Gradient from (0, 4) to (2, 7) $= \frac{7-4}{2-0} = 1.5$

$s = 1.5t + 4$

5 (b) Velocities: $0 < t < 2$: $v = 1.5$; $2 < t < 5$: $v = 0$;

$5 < t < 9$: $v = \frac{-7-7}{9-5} = -3.5$



5 (c) The model has the velocity changing instantaneously at $t = 2$ and $t = 5$ (infinite acceleration). In reality the velocity changes over a period of time, so the graph would be curved.

Question number

6

Comparing x^3 terms: $a = 3$

Divide $3x^3 + x^2 - 6x - 5$ by $(x + 2)$:

$3x^3 + x^2 - 6x - 5 = 3x^2(x + 2) - 5x^2 - 6x - 5$

$= 3x^2(x + 2) - 5x(x + 2) + 4x - 5$

$= 3x^2(x + 2) - 5x(x + 2) + 4(x + 2) - 13$

$= (x + 2)(3x^2 - 5x + 4) - 13$

Check: $f(-2) = -24 + 4 + 12 - 5 = -13 = d$ ✓

$a = 3,\ b = -5,\ c = 4,\ d = -13$

7

Curve meets x-axis when $-3x^2 + 7x - 2 = 0$

$3x^2 - 7x + 2 = 0 \Rightarrow (3x - 1)(x - 2) = 0 \Rightarrow x = \frac{1}{3}, 2$

$\text{Area} = \int_{\frac{1}{3}}^{2} (-3x^2 + 7x - 2)\,dx$

$= \left[-x^3 + \frac{7}{2}x^2 - 2x\right]_{\frac{1}{3}}^{2}$

$= (-8 + 14 - 4) - \left(-\frac{1}{27} + \frac{7}{18} - \frac{2}{3}\right)$

$= 2 - \left(-\frac{17}{54}\right) = 2 + \frac{17}{54}$

$\text{Area} = \frac{125}{54}$ square units

8 (a)

$x^2 - 2x + y^2 + 4y = 20$

$(x - 1)^2 - 1 + (y + 2)^2 - 4 = 20$

$(x - 1)^2 + (y + 2)^2 = 25$

Centre $(1, -2)$, radius $= \sqrt{25} = 5$

8 (b)

Line: $x + 3y - 10 = 0 \Rightarrow x = 10 - 3y$

Question number

Substitute into $(x-1)^2 + (y+2)^2 = 25$:

$(9-3y)^2 + (y+2)^2 = 25$

$81 - 54y + 9y^2 + y^2 + 4y + 4 = 25$

$10y^2 - 50y + 60 = 0 \Rightarrow y^2 - 5y + 6 = 0$

$(y-2)(y-3) = 0 \Rightarrow y = 2$ or $y = 3$

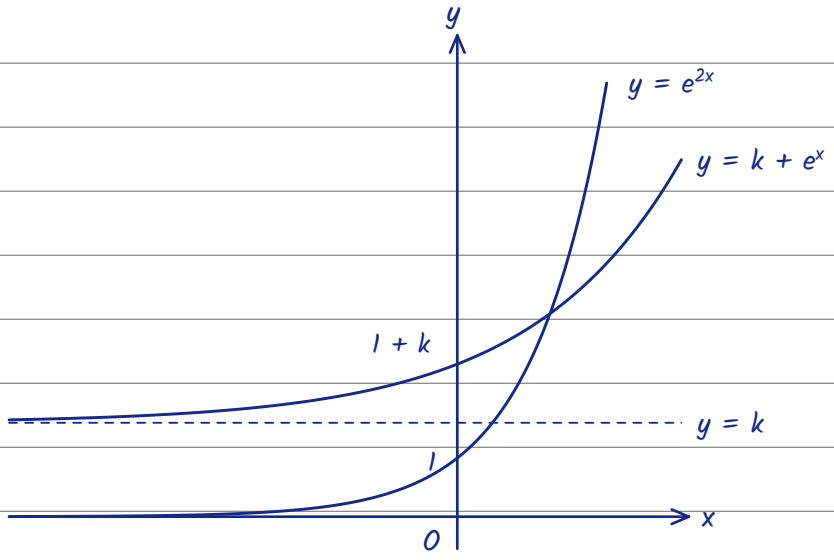
$y = 2$: $x = 10 - 6 = 4$; $y = 3$: $x = 10 - 9 = 1$

Points of intersection: $(4, 2)$ and $(1, 3)$

9 (a) Stretch parallel to the x-axis (in the x-direction),

scale factor $\frac{1}{2}$.

9 (b) Sketch drawn with $k > 0$: horizontal asymptote $y = k$, y-intercept $1 + k$



9 (c) Intersect when $e^{2x} = k + e^x \Rightarrow (e^x)^2 - e^x - k = 0$

This is a quadratic in e^x, with discriminant

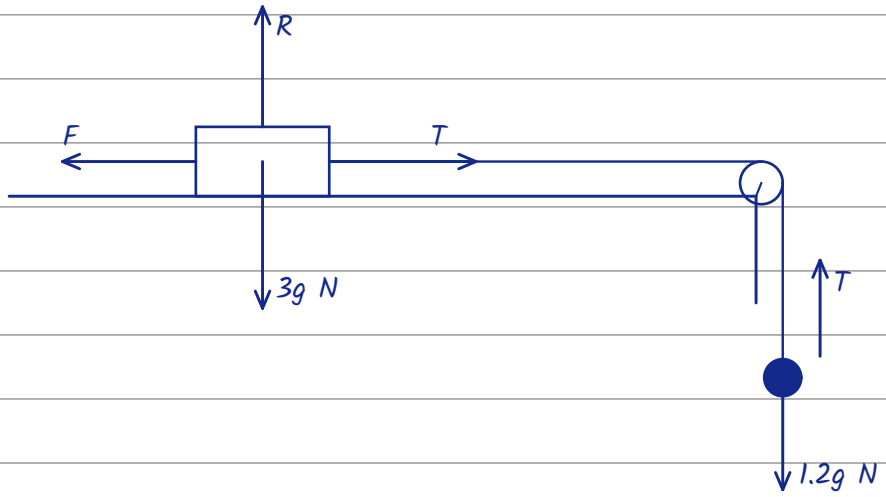
$b^2 - 4ac = (-1)^2 - 4(1)(-k) = 1 + 4k$

If $k < -\frac{1}{4}$ then $4k < -1$, so $1 + 4k < 0$

Question number	
	so there are no real roots for e^x, hence the two graphs
	do not intersect for $k < -\frac{1}{4}$.
9 (d)	$k = 2$: $e^{2x} = 2 + e^x \Rightarrow (e^x)^2 - e^x - 2 = 0$
	$(e^x - 2)(e^x + 1) = 0 \Rightarrow e^x = 2$ or $e^x = -1$
	$e^x = -1$ is not possible since $e^x > 0$ for all x
	so $e^x = 2 \Rightarrow x = \ln 2$ is the only point of intersection.
10 (a)(i)	When $t = 0$, $L = 2800 \times 1 = 2800$. She invests £2800.
10 (a)(ii)	Each year the amount is multiplied by 1.023,
	so the annual rate of interest is 2.3%.
10 (b)	$A = 3000 \times 1.02^t$ so $a = 3000$, $b = 1.02$
10 (c)	Equal amounts when $3000 \times 1.02^t = 2800 \times 1.023^t$
	$\ln 3000 + t \ln 1.02 = \ln 2800 + t \ln 1.023$
	$t(\ln 1.023 - \ln 1.02) = \ln 3000 - \ln 2800$
	$t = \frac{\ln 3000 - \ln 2800}{\ln 1.023 - \ln 1.02} = 23.49...$
	They have equal amounts after $t = 23.5$ years.

Question number

11 (a)



11 (b) Sphere in equilibrium: $T = 1.2g$

Block in equilibrium: $F = T = 1.2g = 1.2 \times 9.8$

$F = 11.76$

11 (c) $1.2g - T = 1.2a$

11 (d) Block: $T = 3a$ Sphere: $1.2g - T = 1.2a$

Add: $1.2g = 4.2a \Rightarrow a = \frac{1.2 \times 9.8}{4.2} = 2.8 \text{ m s}^{-2}$

$T = 3a = 3 \times 2.8 = 8.4$

$T = 8.4$

12 A to B (smooth), N2L: $8 = 5a \Rightarrow a = 1.6 \text{ m s}^{-2}$

$u = 0, a = 1.6, t = 3$

$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 1.6 \times 3^2 = 7.2 \text{ m} \Rightarrow AB = 7.2 \text{ m}$

$v = u + at = 0 + 1.6 \times 3 = 4.8 \text{ m s}^{-1}$ (speed at B)

B to C (rough), N2L in direction of motion: $8 - 28 = 5a$

$a = -4 \text{ m s}^{-2}$

Question number

$u = 4.8,\ v = 0,\ a = -4$: $v^2 = u^2 + 2as$

$0 = 4.8^2 + 2(-4)s \Rightarrow 8s = 23.04 \Rightarrow s = 2.88\ m \Rightarrow BC = 2.88\ m$

$AC = AB + BC = 7.2 + 2.88 = 10.08\ m$

$AC = 10.08\ m$ (10.1 m to 3 s.f.)