OCR
Oxford Cambridge and RSA

Wednesday 22 May 2019 – Morning

AS Level Mathematics B (MEI)

H630/02 Pure Mathematics and Statistics

Time allowed: 1 hour 30 minutes

7692882149

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

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Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

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Answer **all** the questions.

1 Solve the equation $4x^{-\frac{1}{2}} = 7$, giving your answer as a fraction in its lowest terms. **[3]**

2 Fig. 2 shows a triangle with one angle of 117° given. The lengths are given in centimetres.

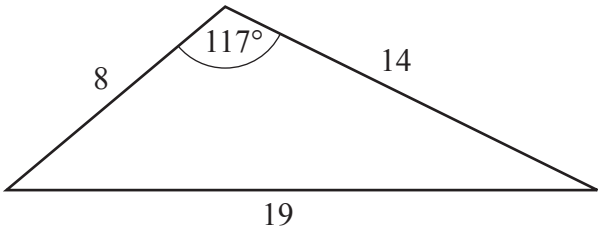


Fig. 2

Calculate the area of the triangle, giving your answer correct to 3 significant figures. **[2]**

3 **Without** using a calculator, prove that $3\sqrt{2} > 2\sqrt{3}$. **[3]**

4 The equation of a circle is $x^2 + y^2 + 8x - 6y - 39 = 0$.

(a) Find the coordinates of the centre of the circle. **[2]**

(b) Find the radius of the circle. **[1]**

5 Each day John either cycles to work or goes on the bus.

- If it is raining when John is ready to set off for work, the probability that he cycles to work is 0.4.
- If it is not raining when John is ready to set off for work, the probability that he cycles to work is 0.9.
- The probability that it is raining when he is ready to set off for work is 0.2.

You should assume that days on which it rains occur randomly and independently.

(a) Draw a tree diagram to show the possible outcomes and their associated probabilities. **[3]**

(b) Calculate the probability that, on a randomly chosen day, John cycles to work. **[3]**

John works 5 days each week.

(c) Calculate the probability that he cycles to work every day in a randomly chosen working week. **[2]**

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6 The large data set gives information about life expectancy at birth for males and females in different London boroughs. Fig. 6.1 shows summary statistics for female life expectancy at birth for the years 2012–2014. Fig. 6.2 shows summary statistics for male life expectancy at birth for the years 2012–2014.

Female Life Expectancy at Birth

n	32
Mean	84.2313
s	1.1563
Σx	2695.4
Σx^2	227078.36
Min	82.1
Q1	83.45
Median	84
Q3	84.9
Max	86.7

Fig. 6.1

Male Life Expectancy at Birth

n	32
Mean	80.2844
s	1.4294
Σx	2569.1
Σx^2	206321.93
Min	77.6
Q1	79
Median	80.25
Q3	81.15
Max	83.3

Fig. 6.2

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(a) Use the information in Fig. 6.1 and Fig. 6.2 to draw two box plots. Draw one box plot for female life expectancy at birth in London boroughs and one box plot for male life expectancy at birth in London boroughs. **[5]**

(b) Compare and contrast the distribution of male life expectancy at birth with the distribution of female life expectancy at birth in London boroughs in 2012–2014. **[2]**

Lorraine, who lives in Lancashire, says she wishes her daughter (who was born in 2013) had been born in the London borough of Barnet, because her daughter would have had a higher life expectancy.

(c) Give **two** reasons why there is no evidence in the large data set to support Lorraine's comment. **[2]**

(d) Use the mean and standard deviation for the summary statistics given in Fig. 6.1 and Fig. 6.2 to show that there is at least one outlier in each set. **[2]**

The scatter diagram in Fig. 6.3 shows male life expectancy at birth plotted against female life expectancy at birth for London boroughs in 2012–14. The outliers have been removed.

Male life expectancy at birth against female life expectancy at birth

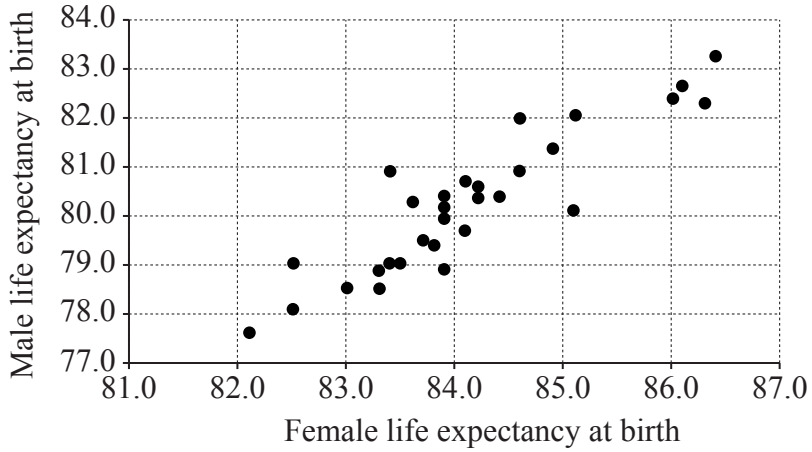


Fig. 6.3

(e) Describe the association between male life expectancy at birth and female life expectancy at birth in London boroughs in 2012–14. **[2]**

7 **(a)** Find $\int x^3\left(15x+\frac{11}{\sqrt[3]{x}}\right)\mathrm{d}x$. **[5]**

(b) Show that $\int_0^8 x^3\left(15x+\frac{11}{\sqrt[3]{x}}\right)\mathrm{d}x = a\times 2^{11}$, where a is a positive integer to be determined. **[3]**

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8 According to the latest research there are 19.8 million male drivers and 16.2 million female drivers on the roads in the UK.

(a) A driver in the UK is selected at random. Find the probability that the driver is male. **[1]**

(b) Calculate the probability that there are 7 female drivers in a random sample of 25 UK drivers. **[1]**

When driving in a built-up area, Rebecca exceeded the speed limit and was obliged to attend a speed awareness course. Her husband said "It's nearly always male drivers who are speeding." When Rebecca attends the course, she finds that there are 25 drivers, 7 of whom are female. You should assume that the drivers on the speed awareness course constitute a random sample of drivers caught speeding.

(c) In this question you must show detailed reasoning.

Conduct a hypothesis test to determine whether there is any evidence at the 5% level to suggest that male drivers are more likely to exceed the speed limit than female drivers. **[7]**

(d) State a modelling assumption that is necessary in order to conduct the hypothesis test in part **(c)**. **[1]**

9 In 2012 Adam bought a second hand car for £8500. Each year Adam has his car valued. He believes that there is a non-linear relationship between t, the time in years since he bought the car, and V, the value of the car in pounds. Fig. 9.1 shows successive values of V and $\log_{10}V$.

t	0	1	2	3	4
V	8500	6970	5720	4690	3840
$\log_{10}V$	3.93	3.84	3.76	3.67	3.58

Fig. 9.1

Adam uses a spreadsheet to plot the points $(t, \log_{10}V)$ shown in Fig. 9.1, and then generates a line of best fit for these points. The line passes through the points (0, 3.93) and (4, 3.58). A copy of his graph is shown in Fig. 9.2.

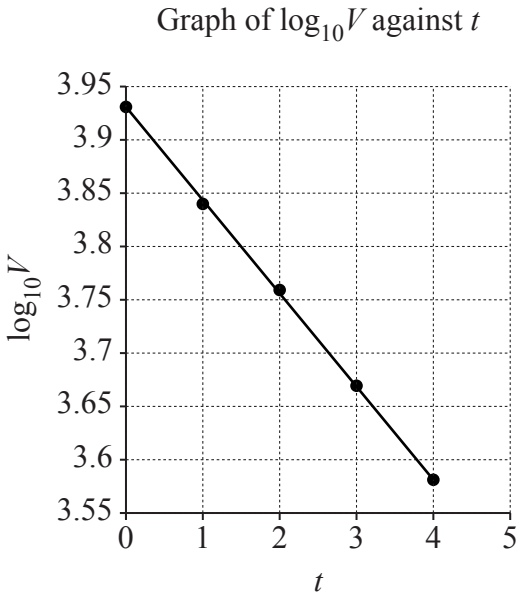


Fig. 9.2

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(a) Find an expression for $\log_{10} V$ in terms of t. **[3]**

(b) Find a model for V in the form $V = A \times b^t$, where A and b are constants to be determined. Give the values of A and b correct to 2 significant figures. **[3]**

In 2017 Adam's car was valued at £3150.

(c) Determine whether the model is a good fit for this data. **[1]**

A company called Webuyoldcars pays £500 for any second hand car. Adam decides that he will sell his car to this company when the annual valuation of his car is less than £500.

(d) According to the model, after how many years will Adam sell his car to Webuyoldcars? **[3]**

10 In this question you must show detailed reasoning.

The equation of a curve is $y = \frac{x^2}{4} + \frac{2}{x} + 1$. A tangent and a normal to the curve are drawn at the point where $x = 2$.

Calculate the area bounded by the tangent, the normal and the x-axis. **[10]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 $4x^{-\frac{1}{2}} = 7 \Rightarrow x^{-\frac{1}{2}} = \frac{7}{4} \Rightarrow x^{\frac{1}{2}} = \frac{4}{7}$

$x = \left(\frac{4}{7}\right)^2 = \frac{16}{49}$

2 Area $= \frac{1}{2} \times 14 \times 8 \times \sin 117°$

$= 49.896... = 49.9 \text{ cm}^2$ (3 s.f.)

3 $3\sqrt{2} = \sqrt{9} \times \sqrt{2} = \sqrt{18}$

$2\sqrt{3} = \sqrt{4} \times \sqrt{3} = \sqrt{12}$

Since $18 > 12$, $\sqrt{18} > \sqrt{12}$, $\therefore 3\sqrt{2} > 2\sqrt{3}$

4 (a) $x^2 + 8x + y^2 - 6y = 39$

$(x + 4)^2 - 16 + (y - 3)^2 - 9 = 39$

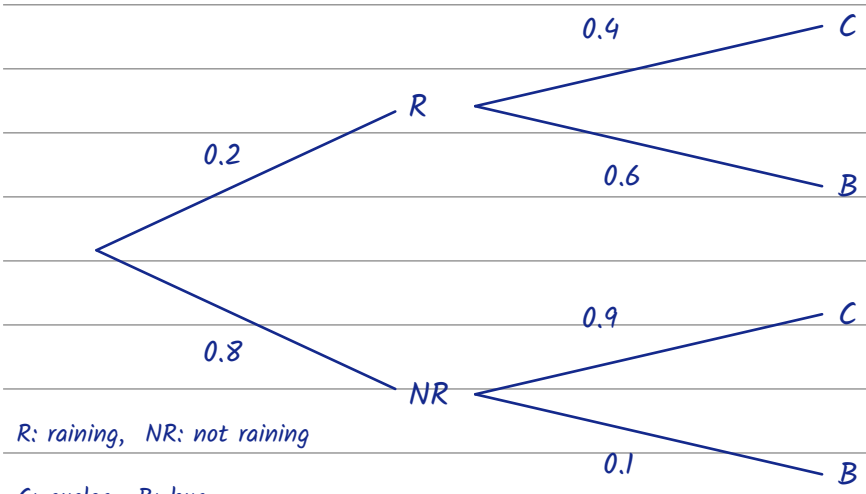
$(x + 4)^2 + (y - 3)^2 = 64$

Centre is $(-4, 3)$

4 (b) $r^2 = 64 \Rightarrow$ radius $= 8$

Question number

5 (a)



R: raining, NR: not raining

C: cycles, B: bus

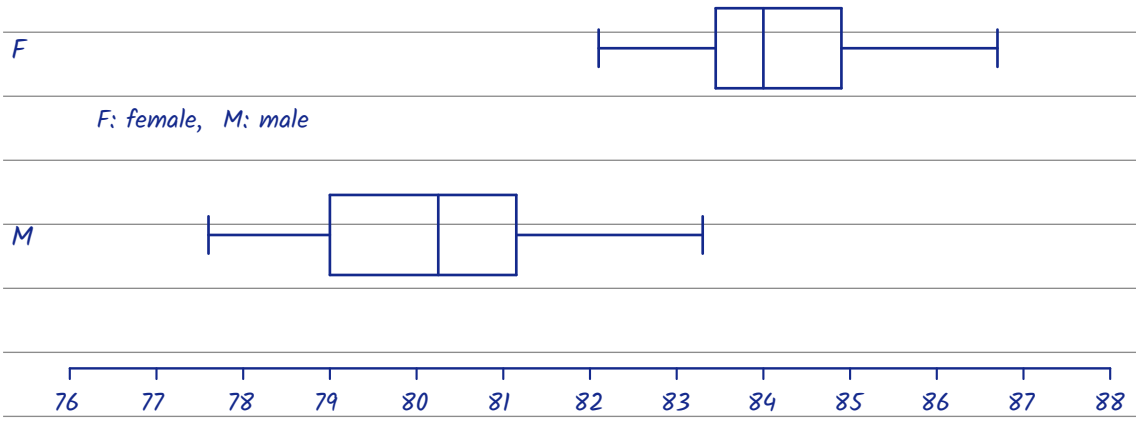
5 (b) $P(C) = P(R \text{ and } C) + P(NR \text{ and } C)$

$= 0.2 \times 0.4 + 0.8 \times 0.9$

$= 0.08 + 0.72 = 0.8$

5 (c) $P(\text{cycles all 5 days}) = 0.8^5 = 0.32768 \approx 0.328$

6 (a)



F: female, M: male

6 (b) Median for males 80.25 < 84 for females, so on average male life expectancy is lower than female life expectancy.

IQR males 81.15 − 79 = 2.15 > females 84.9 − 83.45 = 1.45 (range 5.7 > 4.6), so male life expectancy is more variable than female life expectancy.

Question number

6 (c)

1. The data set has no information about Lancashire, so no comparison can be made.

2. The data are averages for each borough, not individuals: a higher average does not mean her daughter would have lived longer.

6 (d)

Outliers lie more than 2 s.d. from the mean.

F: $84.2313 + 2 \times 1.1563 = 86.5439$, and max $86.7 > 86.5439$

M: $80.2844 + 2 \times 1.4294 = 83.1432$, and max $83.3 > 83.1432$

So there is at least one outlier in each set.

6 (e)

Strong positive (linear) correlation: boroughs with higher female life expectancy tend to have higher male life expectancy.

7 (a)

$$\int x^3\left(15x + \frac{11}{\sqrt[3]{x}}\right) dx = \int \left(15x^4 + 11x^{8/3}\right) dx$$

$$= \frac{15x^5}{5} + \frac{11x^{11/3}}{11/3} + c$$

$$= 3x^5 + 3x^{11/3} + c$$

7 (b)

$$\int_0^8 \ldots dx = \left[3x^5 + 3x^{11/3}\right]_0^8$$

$$= 3 \times 8^5 + 3 \times 8^{11/3} - 0$$

$$= 3 \times 2^{15} + 3 \times 2^{11} \qquad (8^5 = 2^{15},\ 8^{11/3} = 2^{11})$$

$$= (3 \times 2^4 + 3) \times 2^{11} = (48 + 3) \times 2^{11}$$

$$= 51 \times 2^{11}, \quad \text{so } a = 51$$

8 (a)

$$P(\text{male}) = \frac{19.8}{19.8 + 16.2} = \frac{19.8}{36} = 0.55$$

Question number

8 (b) X = number of female drivers, $X \sim B(25, 0.45)$

$P(X = 7) = {}^{25}C_7 \times 0.45^7 \times 0.55^{18} = 0.038096... = 0.0381$

8 (c) Let p = P(a driver caught speeding is female)

H_0: $p = 0.45$ H_1: $p < 0.45$

X = number of female drivers on the course, under H_0 $X \sim B(25, 0.45)$

$P(X \leqslant 7) = 0.063850...$

$0.0639 > 0.05$, so the result is not significant: do not reject H_0.

There is insufficient evidence at the 5% level to suggest that male drivers are more likely to exceed the speed limit than female drivers.

8 (d) Each driver caught speeding has the same probability of being selected for the course, independently of every other driver.

9 (a) Gradient $m = \dfrac{3.58 - 3.93}{4 - 0} = -0.0875$

Intercept = 3.93, so $\log_{10} V = 3.93 - 0.0875t$

9 (b) $V = 10^{3.93 - 0.0875t} = 10^{3.93} \times (10^{-0.0875})^t$

$A = 10^{3.93} = 8511.38... = 8500$ (2 s.f.)

$b = 10^{-0.0875} = 0.81752... = 0.82$ (2 s.f.)

$V = 8500 \times 0.82^t$

9 (c) 2017: $t = 5$, $V = 8500 \times 0.82^5 = 3151.29 \approx 3150$

This is very close to £3150, so the model is a (very) good fit.

Question number

9 (d)

$8500 \times 0.82^t < 500 \quad \Rightarrow \quad 0.82^t < 500 \div 8500$

$t \log 0.82 < \log(500 \div 8500)$

$t > \dfrac{\log(500 \div 8500)}{\log 0.82} = 14.276\ldots$ (log 0.82 < 0, so the inequality reverses)

Valuations are annual: $t = 14$ gives £528, $t = 15$ gives £433

So Adam sells his car after 15 years.

10

$x = 2$: $y = \dfrac{2^2}{4} + \dfrac{2}{2} + 1 = 3$, point $(2, 3)$

$y = \dfrac{1}{4}x^2 + 2x^{-1} + 1 \quad \Rightarrow \quad \dfrac{dy}{dx} = \dfrac{x}{2} - \dfrac{2}{x^2}$

$x = 2$: $\dfrac{dy}{dx} = 1 - \dfrac{2}{4} = \dfrac{1}{2}$

Tangent: $y - 3 = \dfrac{1}{2}(x - 2) \quad \Rightarrow \quad y = \dfrac{1}{2}x + 2$

Normal gradient $= -2$: $y - 3 = -2(x - 2) \quad \Rightarrow \quad y = -2x + 7$

Tangent meets x-axis: $\frac{1}{2}x + 2 = 0 \Rightarrow x = -4$, i.e. $(-4, 0)$

Normal meets x-axis: $-2x + 7 = 0 \Rightarrow x = 3.5$, i.e. $(3.5, 0)$

Triangle has base $3.5 - (-4) = 7.5$ and height 3 (vertex $(2, 3)$)

Area $= \frac{1}{2} \times 7.5 \times 3 = \dfrac{45}{4} = 11.25$ square units