Oxford Cambridge and RSA

Wednesday 15 May 2019 – Morning

AS Level Mathematics B (MEI)

H630/01 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

© OCR 2019 [603/0991/X]
DC (LEG/CGW) 174179/4
OCR is an exempt Charity

Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$,

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

Differentiation from first principles

$\mathrm{f}'(x) = \lim\limits_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

© OCR 2019

Answer **all** the questions.

1 **In this question you must show detailed reasoning.**

Show that the equation $x = 7 + 2x^2$ has no real roots. **[3]**

2 **In this question you must show detailed reasoning.**

Fig. 2 shows the graphs of $y = 4\sin x°$ and $y = 3\cos x°$ for $0 \leqslant x \leqslant 360$.

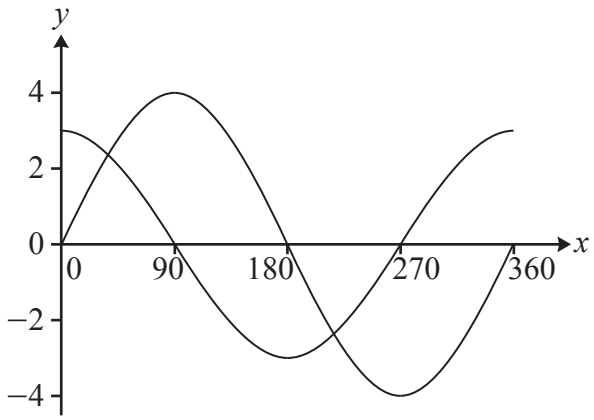

Fig. 2

Find the x-coordinates of the two points of intersection, giving your answers correct to 1 decimal place. **[3]**

3 Given that k is an integer, express $\dfrac{3\sqrt{2} - k}{\sqrt{8} + 1}$ in the form $a + b\sqrt{2}$ where a and b are rational expressions in terms of k. **[4]**

4 A triangle ABC has sides AB = 5 cm, AC = 9 cm and BC = 10 cm.

(a) Find the cosine of angle BAC, giving your answer as a fraction in its lowest terms. **[2]**

(b) Find the exact area of the triangle. **[3]**

5 In this question, the unit vectors **i** and **j** are horizontal and vertically upwards respectively.

A particle has mass 2.5 kg.

(a) Write the weight of the particle as a vector. **[1]**

The particle moves under the action of its weight and two external forces $(3\mathbf{i}-2\mathbf{j})$ N and $(-\mathbf{i}+18\mathbf{j})$ N.

(b) Find the acceleration of the particle, giving your answer in vector form. **[2]**

6 Fig. 6 shows a train consisting of an engine of mass 80 tonnes pulling two trucks each of mass 25 tonnes.

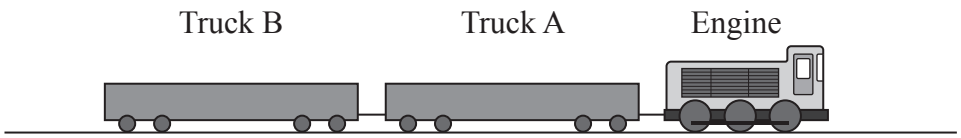

Fig. 6

The engine exerts a driving force of D N and experiences a resistance to motion of 2000 N. Each truck experiences a resistance of 600 N. The train travels in a straight line on a level track with an acceleration of $0.1\,\text{m}\,\text{s}^{-2}$.

(a) Complete the force diagram in the Printed Answer Booklet to show all the forces acting on the engine and each of the trucks. **[3]**

(b) Calculate the value of D. **[2]**

(c) The tension in the coupling between the engine and truck A is larger than that in the coupling between the trucks. Determine how much larger. **[2]**

7 **In this question you must show detailed reasoning.**

(a) Nigel is asked to determine whether $(x+7)$ is a factor of $x^3-37x+84$. He substitutes $x=7$ and calculates $7^3-37\times7+84$. This comes to 168, so Nigel concludes that $(x+7)$ is not a factor.

Nigel's conclusion is wrong.

- Explain why Nigel's argument is not valid.
- Show that $(x+7)$ is a factor of $x^3-37x+84$. **[2]**

(b) Sketch the graph of $y=x^3-37x+84$, indicating the coordinates of the points at which the curve crosses the coordinate axes. **[5]**

(c) The graph in part **(b)** is translated by $\begin{pmatrix}1\\0\end{pmatrix}$. Find the equation of the translated graph, giving your answer in the form $y=x^3+ax^2+bx+c$ where a, b and c are integers. **[4]**

8 In this question you must show detailed reasoning.

Show that the only stationary point on the graph of $y = x^2 - 4\sqrt{x}$ is a minimum point at $(1, -3)$. **[7]**

9 In this question you must show detailed reasoning.

A car accelerates from rest along a straight level road. The velocity of the car after 8 s is $25.6\,\text{m s}^{-1}$.

In one model for the motion, the velocity $v\,\text{m s}^{-1}$ at time t seconds is given by $v = 1.2t^2 - kt^3$, where k is a constant and $0 \leqslant t \leqslant 8$.

(a) The model gives the correct velocity of $25.6\,\text{m s}^{-1}$ at time 8 s. Show that $k = 0.1$. **[2]**

A second model for the motion uses constant acceleration.

(b) Find the value of the acceleration which gives the correct velocity of $25.6\,\text{m s}^{-1}$ at time 8 s. **[2]**

(c) Show that these two models give the same value for the displacement in the first 8 s. **[5]**

10 In this question you must show detailed reasoning.

(a) Sketch the gradient function for the curve $y = 24x - 3x^2 - x^3$. **[5]**

(b) Determine the set of values of x for which $24x - 3x^2 - x^3$ is decreasing. **[2]**

11 David puts a block of ice into a cool-box. He wishes to model the mass m kg of the remaining block of ice at time t hours later. He finds that when $t = 5$, $m = 2.1$, and when $t = 50$, $m = 0.21$.

(a) David at first guesses that the mass may be inversely proportional to time. Show that this model fits his measurements. **[3]**

(b) Explain why this model

(i) is not suitable for small values of t, **[1]**

(ii) cannot be used to find the time for the block to melt completely. **[1]**

David instead proposes a linear model $m = at + b$, where a and b are constants.

(c) Find the values of the constants for which the model fits the mass of the block when $t = 5$ and $t = 50$. **[3]**

(d) Interpret these values of a and b. **[2]**

(e) Find the time according to this model for the block of ice to melt completely. **[1]**

END OF QUESTION PAPER

© OCR 2019

BLANK PAGE

BLANK PAGE

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.