OCR
Oxford Cambridge and RSA

Wednesday 15 May 2019 – Morning

AS Level Mathematics B (MEI)

H630/01 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes

7692669314

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

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Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

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Answer **all** the questions.

1 **In this question you must show detailed reasoning.**

Show that the equation $x = 7 + 2x^2$ has no real roots. **[3]**

2 **In this question you must show detailed reasoning.**

Fig. 2 shows the graphs of $y = 4\sin x°$ and $y = 3\cos x°$ for $0 \leqslant x \leqslant 360$.

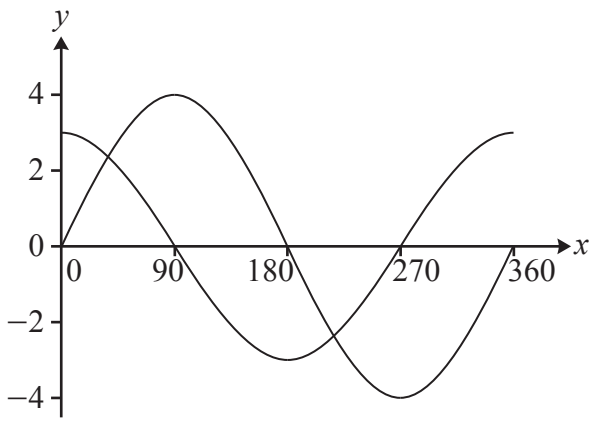


Fig. 2

Find the x-coordinates of the two points of intersection, giving your answers correct to 1 decimal place. **[3]**

3 Given that k is an integer, express $\dfrac{3\sqrt{2}-k}{\sqrt{8}+1}$ in the form $a+b\sqrt{2}$ where a and b are rational expressions in terms of k. **[4]**

4 A triangle ABC has sides AB = 5 cm, AC = 9 cm and BC = 10 cm.

(a) Find the cosine of angle BAC, giving your answer as a fraction in its lowest terms. **[2]**

(b) Find the exact area of the triangle. **[3]**

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5 In this question, the unit vectors **i** and **j** are horizontal and vertically upwards respectively.

A particle has mass 2.5 kg.

(a) Write the weight of the particle as a vector. **[1]**

The particle moves under the action of its weight and two external forces $(3\mathbf{i}-2\mathbf{j})$ N and $(-\mathbf{i}+18\mathbf{j})$ N.

(b) Find the acceleration of the particle, giving your answer in vector form. **[2]**

6 Fig. 6 shows a train consisting of an engine of mass 80 tonnes pulling two trucks each of mass 25 tonnes.

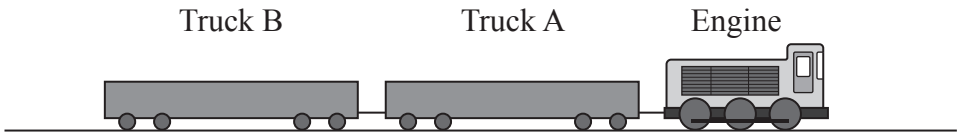


Fig. 6

The engine exerts a driving force of D N and experiences a resistance to motion of 2000 N. Each truck experiences a resistance of 600 N. The train travels in a straight line on a level track with an acceleration of $0.1\,\text{m s}^{-2}$.

(a) Complete the force diagram in the Printed Answer Booklet to show all the forces acting on the engine and each of the trucks. **[3]**

(b) Calculate the value of D. **[2]**

(c) The tension in the coupling between the engine and truck A is larger than that in the coupling between the trucks. Determine how much larger. **[2]**

7 **In this question you must show detailed reasoning.**

(a) Nigel is asked to determine whether $(x+7)$ is a factor of $x^3-37x+84$. He substitutes $x=7$ and calculates $7^3-37\times7+84$. This comes to 168, so Nigel concludes that $(x+7)$ is not a factor.

Nigel's conclusion is wrong.

- Explain why Nigel's argument is not valid.
- Show that $(x+7)$ is a factor of $x^3-37x+84$. **[2]**

(b) Sketch the graph of $y=x^3-37x+84$, indicating the coordinates of the points at which the curve crosses the coordinate axes. **[5]**

(c) The graph in part **(b)** is translated by $\begin{pmatrix}1\\0\end{pmatrix}$. Find the equation of the translated graph, giving your answer in the form $y=x^3+ax^2+bx+c$ where a, b and c are integers. **[4]**

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8 In this question you must show detailed reasoning.

Show that the only stationary point on the graph of $y = x^2 - 4\sqrt{x}$ is a minimum point at $(1, -3)$. **[7]**

9 In this question you must show detailed reasoning.

A car accelerates from rest along a straight level road. The velocity of the car after 8 s is $25.6\,\text{m s}^{-1}$.

In one model for the motion, the velocity $v\,\text{m s}^{-1}$ at time t seconds is given by $v = 1.2t^2 - kt^3$, where k is a constant and $0 \leqslant t \leqslant 8$.

(a) The model gives the correct velocity of $25.6\,\text{m s}^{-1}$ at time 8 s. Show that $k = 0.1$. **[2]**

A second model for the motion uses constant acceleration.

(b) Find the value of the acceleration which gives the correct velocity of $25.6\,\text{m s}^{-1}$ at time 8 s. **[2]**

(c) Show that these two models give the same value for the displacement in the first 8 s. **[5]**

10 In this question you must show detailed reasoning.

(a) Sketch the gradient function for the curve $y = 24x - 3x^2 - x^3$. **[5]**

(b) Determine the set of values of x for which $24x - 3x^2 - x^3$ is decreasing. **[2]**

11 David puts a block of ice into a cool-box. He wishes to model the mass m kg of the remaining block of ice at time t hours later. He finds that when $t = 5$, $m = 2.1$, and when $t = 50$, $m = 0.21$.

(a) David at first guesses that the mass may be inversely proportional to time. Show that this model fits his measurements. **[3]**

(b) Explain why this model

(i) is not suitable for small values of t, **[1]**

(ii) cannot be used to find the time for the block to melt completely. **[1]**

David instead proposes a linear model $m = at + b$, where a and b are constants.

(c) Find the values of the constants for which the model fits the mass of the block when $t = 5$ and $t = 50$. **[3]**

(d) Interpret these values of a and b. **[2]**

(e) Find the time according to this model for the block of ice to melt completely. **[1]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1

$x = 7 + 2x^2 \quad \Rightarrow \quad 2x^2 - x + 7 = 0$

Discriminant $b^2 - 4ac = (-1)^2 - 4 \times 2 \times 7$

$= 1 - 56 = -55$

$-55 < 0$, so the equation has no real roots

2

At intersection: $4 \sin x° = 3 \cos x°$

$\frac{\sin x°}{\cos x°} = \frac{3}{4} \quad \Rightarrow \quad \tan x° = \frac{3}{4}$

$x = \tan^{-1}(0.75) = 36.8698... = 36.9$

or $x = 180 + 36.8698... = 216.8698... = 216.9$

$x = 36.9$ and $x = 216.9$ (both in $0 \leq x \leq 360$)

3

$\sqrt{8} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$

$\frac{3\sqrt{2} - k}{\sqrt{8} + 1} = \frac{3\sqrt{2} - k}{2\sqrt{2} + 1} \times \frac{2\sqrt{2} - 1}{2\sqrt{2} - 1}$

$= \frac{6 \times 2 - 3\sqrt{2} - 2k\sqrt{2} + k}{8 - 2\sqrt{2} + 2\sqrt{2} - 1}$

$= \frac{12 + k - (3 + 2k)\sqrt{2}}{7}$

$= \frac{12 + k}{7} - \frac{3 + 2k}{7}\sqrt{2}$ so $a = \frac{12 + k}{7}$, $b = -\frac{3 + 2k}{7}$

4 (a)

$\cos BAC = \frac{AB^2 + AC^2 - BC^2}{2 \times AB \times AC} = \frac{5^2 + 9^2 - 10^2}{2 \times 5 \times 9}$

$= \frac{25 + 81 - 100}{90} = \frac{6}{90} = \frac{1}{15}$

4 (b)

$\sin BAC = \sqrt{1 - \cos^2 BAC} = \sqrt{1 - 1/225} = \sqrt{224/225} = \frac{4\sqrt{14}}{15}$

Area $= \frac{1}{2} \times AB \times AC \times \sin BAC = \frac{1}{2} \times 5 \times 9 \times \frac{4\sqrt{14}}{15}$

$= 6\sqrt{14}$ cm^2

5 (a)

$W = -2.5g\,j = -24.5j$ N

Question number

5 (b)

N2L: $\Sigma F = ma$

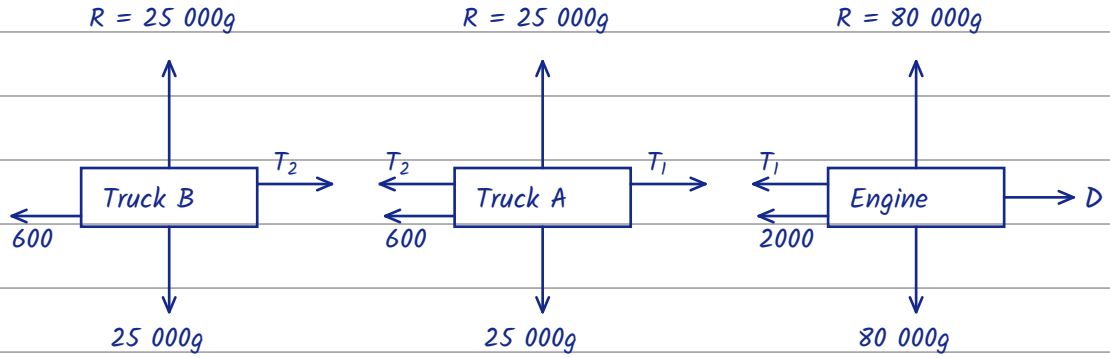
$-24.5j + (3i - 2j) + (-i + 18j) = 2.5a$

$2i - 8.5j = 2.5a$

$a = \frac{1}{2.5}(2i - 8.5j) = (0.8i - 3.4j)\ m\ s^{-2}$

6 (a)

(forces in N)



6 (b)

N2L for the whole train (mass 130 000 kg):

$D - 2000 - 600 - 600 = 130\,000 \times 0.1$

$D - 3200 = 13\,000 \quad \Rightarrow \quad D = 16\,200$

6 (c)

N2L for truck A: $T_1 - T_2 - 600 = 25\,000 \times 0.1$

$T_1 - T_2 = 2500 + 600 = 3100$

(check: engine $16\,200 - 2000 - T_1 = 8000 \Rightarrow T_1 = 6200$;

truck B $T_2 - 600 = 2500 \Rightarrow T_2 = 3100$)

The tension is 3100 N larger

7 (a)

By the factor theorem, $(x + 7)$ is a factor if $f(-7) = 0$, so Nigel should substitute $x = -7$, not $x = 7$. His calculation only shows that $(x - 7)$ is not a factor.

$(-7)^3 - 37 \times (-7) + 84 = -343 + 259 + 84 = 0$

$f(-7) = 0$, so $(x + 7)$ is a factor of $x^3 - 37x + 84$

Question number

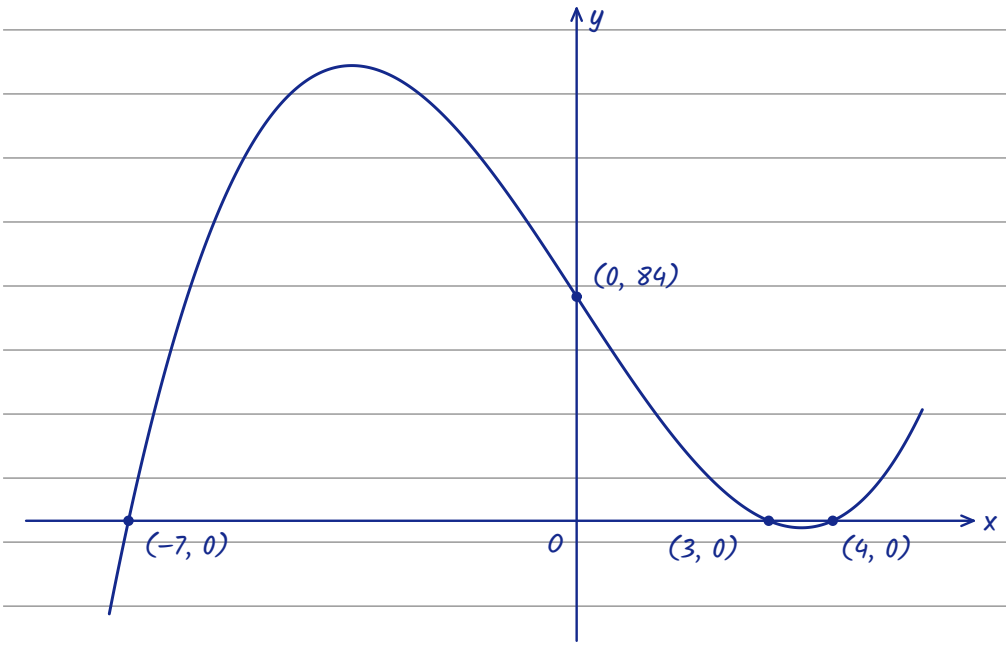
7 (b)

$x^3 - 37x + 84 = (x + 7)(x^2 - 7x + 12)$

$= (x + 7)(x - 3)(x - 4)$

Crosses x-axis at $(-7, 0)$, $(3, 0)$, $(4, 0)$

$x = 0$: $y = 84$, crosses y-axis at $(0, 84)$



7 (c)

Translation by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$: replace x by $(x - 1)$

$y = (x - 1)^3 - 37(x - 1) + 84$

$(x - 1)^3 = x^3 - 3x^2 + 3x - 1$

$y = x^3 - 3x^2 + 3x - 1 - 37x + 37 + 84$

$y = x^3 - 3x^2 - 34x + 120$

8

$y = x^2 - 4x^{1/2} \Rightarrow \frac{dy}{dx} = 2x - 4 \times \frac{1}{2}x^{-1/2} = 2x - 2x^{-1/2}$

$\frac{dy}{dx} = 0$: $2x = 2x^{-1/2} \Rightarrow x^{3/2} = 1 \Rightarrow x = 1$

This is the only solution, so there is only one stationary point.

When $x = 1$: $y = 1^2 - 4\sqrt{1} = 1 - 4 = -3$, so the point is $(1, -3)$

$\frac{d^2y}{dx^2} = 2 - 2 \times \left(-\frac{1}{2}\right)x^{-3/2} = 2 + x^{-3/2}$

When $x = 1$: $\frac{d^2y}{dx^2} = 2 + 1 = 3 > 0$

$\therefore$ the only stationary point is a minimum point at $(1, -3)$

Question number

9 (a) $v = 1.2t^2 - kt^3$, $v = 25.6$ when $t = 8$:

$25.6 = 1.2 \times 8^2 - k \times 8^3 = 76.8 - 512k$

$512k = 76.8 - 25.6 = 51.2 \quad \Rightarrow \quad k = \frac{51.2}{512} = 0.1$

9 (b) $v = u + at$ with $u = 0$, $v = 25.6$, $t = 8$

$25.6 = 0 + 8a \quad \Rightarrow \quad a = 3.2 \text{ m s}^{-2}$

9 (c) First model: $s = \int_0^8 (1.2t^2 - 0.1t^3)\,dt = \left[0.4t^3 - 0.025t^4\right]_0^8$

$= (0.4 \times 8^3 - 0.025 \times 8^4) - (0 - 0)$

$= 204.8 - 102.4 = 102.4 \text{ m}$

Second model: $s = ut + \frac{1}{2}at^2 = 0 \times 8 + \frac{1}{2} \times 3.2 \times 8^2$

$= 102.4 \text{ m}$

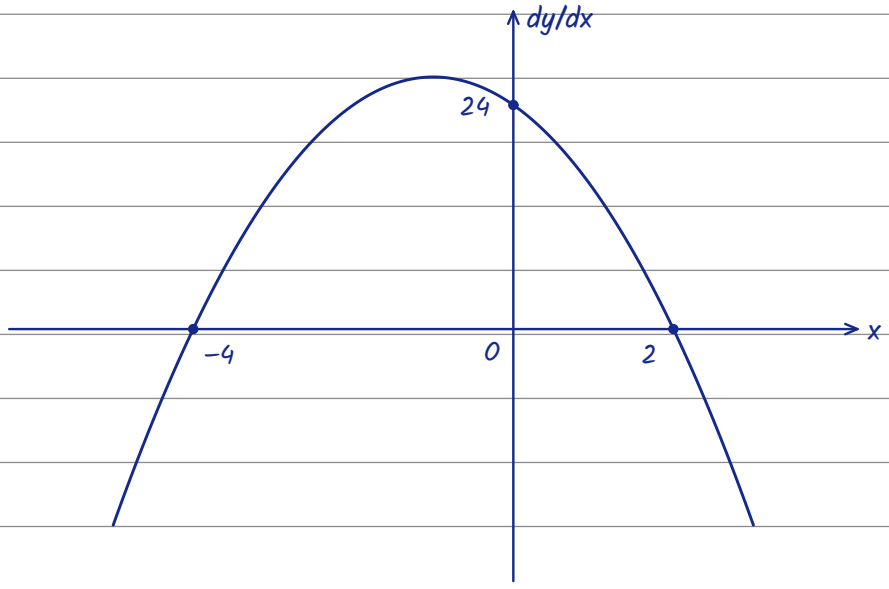
Both models give a displacement of 102.4 m in the first 8 s

10 (a) $y = 24x - 3x^2 - x^3 \quad \Rightarrow \quad \frac{dy}{dx} = 24 - 6x - 3x^2$

When $x = 0$, $\frac{dy}{dx} = 24$

$\frac{dy}{dx} = 0$: $-3(x^2 + 2x - 8) = 0 \quad \Rightarrow \quad (x + 4)(x - 2) = 0$

$x = -4$ or $x = 2$



Question number	
10 (b)	Decreasing when $\frac{dy}{dx} < 0$, i.e. below the x-axis on the sketch:
	$\{x : x < -4\} \cup \{x : x > 2\}$
11 (a)	$m = \frac{k}{t}$: $t = 5$, $m = 2.1$ $\Rightarrow$ $k = 2.1 \times 5 = 10.5$
	When $t = 50$: $m = \frac{10.5}{50} = 0.21$
	This agrees with the measured value, so the model $m = \frac{10.5}{t}$ fits
	both measurements.
11(b)(i)	When t is small, $m = \frac{10.5}{t}$ is very large ($m \to \infty$ as $t \to 0$), so it
	does not give the initial mass of the block correctly.
11(b)(ii)	Melts completely when $m = 0$, but $\frac{10.5}{t} = 0$ has no solution
	($t = \frac{10.5}{0}$ is undefined), so the model never gives $m = 0$.
11 (c)	$t = 5$, $m = 2.1$: $2.1 = 5a + b$
	$t = 50$, $m = 0.21$: $0.21 = 50a + b$
	Subtract: $-1.89 = 45a$ $\Rightarrow$ $a = -0.042$
	$b = 2.1 - 5 \times (-0.042) = 2.1 + 0.21 = 2.31$
	$a = -0.042$, $b = 2.31$
11 (d)	a: the ice melts at a rate of 0.042 kg per hour (mass lost per hour)
	b: 2.31 kg is the initial mass of the block of ice (at $t = 0$)
11 (e)	$m = 0$: $0 = -0.042t + 2.31$ $\Rightarrow$ $t = \frac{2.31}{0.042} = 55$
	The block melts completely after 55 hours