OCR
Oxford Cambridge and RSA

Monday 18 October 2021 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours

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You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient rule $y = \frac{u}{v},\ \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta,\ \cos\theta \approx 1 - \frac{1}{2}\theta^2,\ \tan\theta \approx \theta$ where θ is measured in radians

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Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

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Section A: Pure Mathematics

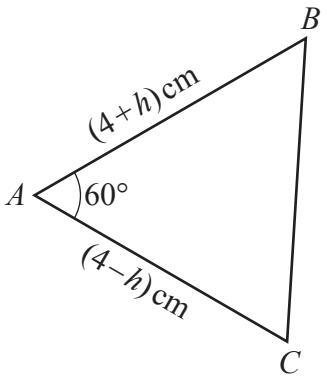
Answer **all** the questions.

1 Show in a sketch the region of the x-y plane within which all three of the following inequalities hold.

$y \geqslant x^2, \quad x + y \leqslant 2, \quad x \geqslant 0.$

You should indicate the region for which the inequalities hold by labelling the region R. **[3]**

2



The diagram shows triangle ABC in which angle A is 60° and the lengths of AB and AC are $(4+h)$ cm and $(4-h)$ cm respectively.

(a) Show that the length of BC is p cm where

$p^2 = 16 + 3h^2.$ **[2]**

(b) Hence show that, when h is small, $p \approx 4 + \lambda h^2 + \mu h^4$, where λ and μ are rational numbers whose values are to be determined. **[4]**

3 An arithmetic progression has first term 2 and common difference d, where $d \neq 0$. The first, third and thirteenth terms of this progression are also the first, second and third terms, respectively, of a geometric progression.

By determining d, show that the arithmetic progression is an increasing sequence. **[5]**

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4 **(a)** Sketch, on a single diagram, the following graphs.

- $y = |x - 1|$
- $y = \frac{k}{x}$, where k is a negative constant **[2]**

(b) Hence explain why the equation $x|x - 1| = k$ has exactly one real root for any negative value of k. **[1]**

(c) Determine the real root of the equation $x|x - 1| = -6$. **[2]**

5 A particle P moves along a straight line in such a way that at time t seconds P has velocity $v\,\text{m s}^{-1}$, where

$v = 12\cos t + 5\sin t$.

(a) Express v in the form $R\cos(t - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the value of α correct to **4** significant figures. **[3]**

(b) Hence find the two smallest positive values of t for which P is moving, in either direction, with a speed of $3\,\text{m s}^{-1}$. **[3]**

6 The equation $6\arcsin(2x - 1) - x^2 = 0$ has exactly one real root.

(a) Show by calculation that the root lies between 0.5 and 0.6. **[2]**

In order to find the root, the iterative formula

$x_{n+1} = p + q\sin(rx_n^2)$,

with initial value $x_0 = 0.5$, is to be used.

(b) Determine the values of the constants p, q and r. **[2]**

(c) Hence find the root correct to **4** significant figures. Show the result of each step of the iteration process. **[2]**

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7 A curve C in the x-y plane has the property that the gradient of the tangent at the point $P(x, y)$ is three times the gradient of the line joining the point (3, 2) to P.

(a) Express this property in the form of a differential equation. **[2]**

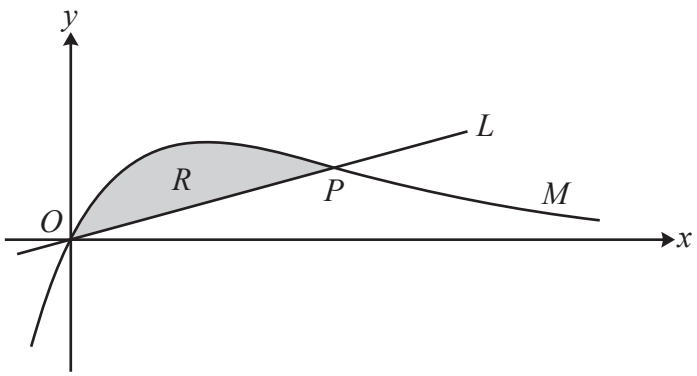
It is given that C passes through the point (4, 3) and that $x > 3$ and $y > 2$ at all points on C.

(b) Determine the equation of C giving your answer in the form $y = \mathrm{f}(x)$. **[4]**

The curve C may be obtained by a transformation of part of the curve $y = x^3$.

(c) Describe fully this transformation. **[2]**

8



The diagram shows the curve M with equation $y = x\mathrm{e}^{-2x}$.

(a) Show that M has a point of inflection at the point P where $x = 1$. **[5]**

The line L passes through the origin O and the point P. The shaded region R is enclosed by the curve M and the line L.

(b) Show that the area of R is given by

$\frac{1}{4}(a + b\mathrm{e}^{-2})$,

where a and b are integers to be determined. **[6]**

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Section B: Mechanics

Answer **all** the questions.

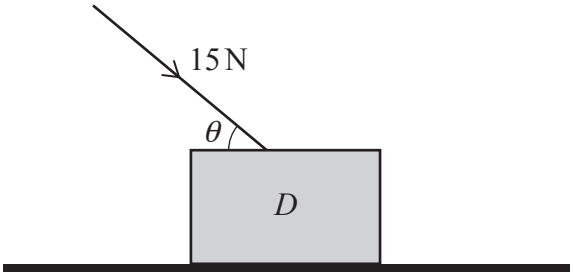
9 There are three checkpoints, A, B and C, in that order, on a straight horizontal road. A car travels along the road, in the direction from A to C, with constant acceleration. The car takes 20 s to travel from B to C. The speed of the car at B is $14\,\text{m s}^{-1}$ and the speed of the car at C is $18\,\text{m s}^{-1}$.

(a) Find the acceleration of the car. **[1]**

It is given that the distance between A and B is 330 m.

(b) Determine the speed of the car at A. **[2]**

10



A block D of weight 50 N lies at rest in equilibrium on a fixed rough horizontal surface. A force of magnitude 15 N is applied to D at an angle θ to the horizontal (see diagram).

(a) Complete the diagram in the Printed Answer Booklet showing all the forces acting on D. **[1]**

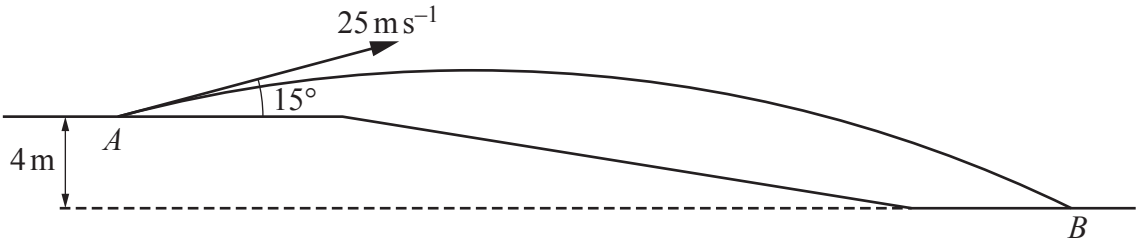
It is given that D remains at rest and the coefficient of friction between D and the surface is 0.2.

(b) Show that

$$15\cos\theta - 3\sin\theta \leqslant 10.$$ **[5]**

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11



A golfer hits a ball from a point A with a speed of 25 m s^{-1} at an angle of 15° above the horizontal. While the ball is in the air, it is modelled as a particle moving under the influence of gravity. Take the acceleration due to gravity to be 10 m s^{-2}.

The ball first lands at a point B which is 4 m below the level of A (see diagram).

(a) Determine the time taken for the ball to travel from A to B. **[3]**

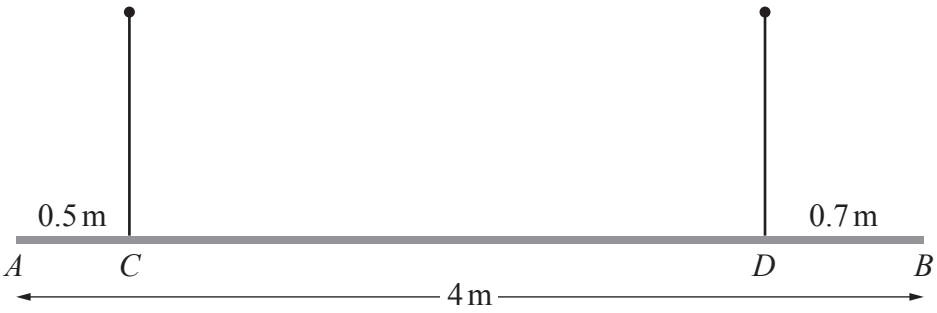
(b) Determine the horizontal distance of B from A. **[2]**

(c) Determine the direction of motion of the ball 1.5 seconds after the golfer hits the ball. **[4]**

The horizontal distance from A to B is found to be greater than the answer to part **(b)**.

(d) State one factor that could account for this difference. **[1]**

12



A beam, AB, has length 4 m and mass 20 kg. The beam is suspended horizontally by two vertical ropes. One rope is attached to the beam at C, where $AC = 0.5$ m. The other rope is attached to the beam at D, where $DB = 0.7$ m (see diagram).

The beam is modelled as a non-uniform rod and the ropes as light inextensible strings.

It is given that the tension in the rope at C is three times the tension in the rope at D.

(a) Determine the distance of the centre of mass of the beam from A. **[5]**

A particle of mass m kg is now placed on the beam at a point where the magnitude of the moment of the particle's weight about C is $3.5mg$ N m. The beam remains horizontal and in equilibrium.

(b) Determine the largest possible value of m. **[2]**

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13 In this question the unit vectors **i** and **j** are in the directions east and north respectively.

At time t seconds, where $t \geqslant 0$, a particle P of mass 2 kg is moving on a smooth horizontal surface under the action of a constant horizontal force $(-8\mathbf{i} - 54\mathbf{j})$N and a variable horizontal force $\left(4t\mathbf{i} + 6(2t-1)^2\mathbf{j}\right)$N.

(a) Determine the value of t when the forces acting on P are in equilibrium. **[2]**

It is given that P is at rest when $t = 0$.

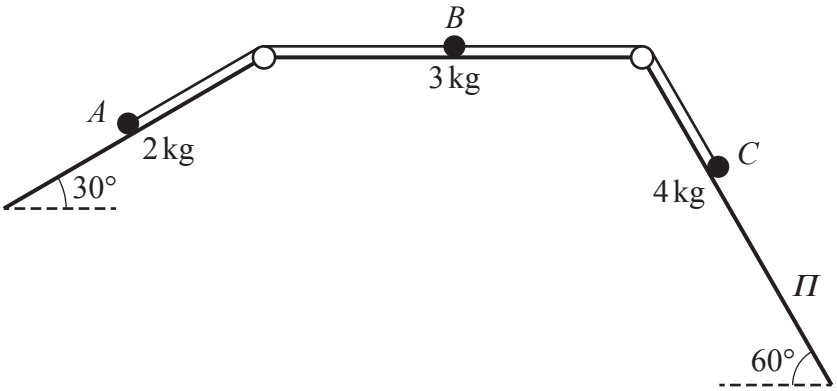
(b) Determine the speed of P at the instant when P is moving due north. **[6]**

(c) Determine the distance between the positions of P when $t = 0$ and $t = 3$. **[5]**

Turn over for question 14

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14



One end of a light inextensible string is attached to a particle A of mass 2 kg. The other end of the string is attached to a second particle B of mass 3 kg. Particle A is in contact with a smooth plane inclined at 30° to the horizontal and particle B is in contact with a rough horizontal plane.

A second light inextensible string is attached to B. The other end of this second string is attached to a third particle C of mass 4 kg. Particle C is in contact with a smooth plane Π inclined at an angle of 60° to the horizontal.

Both strings are taut and pass over small smooth pulleys that are at the tops of the inclined planes. The parts of the strings from A to the pulley, and from C to the pulley, are parallel to lines of greatest slope of the corresponding planes (see diagram).

The coefficient of friction between B and the horizontal plane is μ. The system is released from rest and in the subsequent motion C moves down Π with acceleration $a\,\text{m s}^{-2}$.

(a) By considering an equation involving μ, a and g show that $a < \frac{1}{9}g(2\sqrt{3}-1)$. **[7]**

(b) Given that $a = \frac{1}{9}g$, determine the magnitude of the contact force between B and the horizontal plane. Give your answer correct to **3** significant figures. **[4]**

END OF QUESTION PAPER

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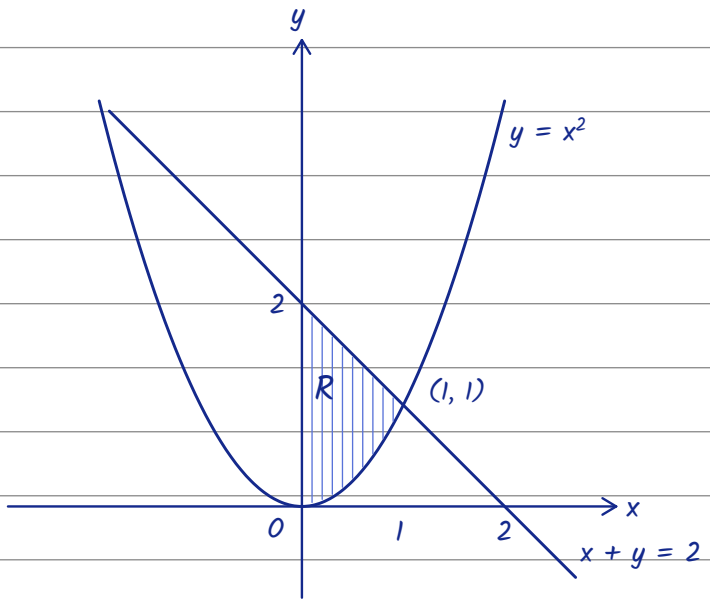
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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1



$y \geq x^2$: on or above the parabola; $x + y \leq 2$: on or below the line; $x \geq 0$

R is the region bounded by the y-axis, the curve $y = x^2$ and the line $x + y = 2$

2 (a)

Cosine rule: $p^2 = (4 + h)^2 + (4 - h)^2 - 2(4 + h)(4 - h)\cos 60°$

$p^2 = (16 + 8h + h^2) + (16 - 8h + h^2) - 2(16 - h^2)(\frac{1}{2})$

$p^2 = 32 + 2h^2 - 16 + h^2$

$\therefore \quad p^2 = 16 + 3h^2$ as required

2 (b)

$p = (16 + 3h^2)^{1/2} = 16^{1/2}(1 + \frac{3}{16}h^2)^{1/2} = 4(1 + \frac{3}{16}h^2)^{1/2}$

$(1 + \frac{3}{16}h^2)^{1/2} \approx 1 + (\frac{1}{2})(\frac{3}{16}h^2) + \frac{(1/2)(-1/2)}{2!}(\frac{3}{16}h^2)^2$

$= 1 + \frac{3}{32}h^2 - \frac{1}{8} \times \frac{9}{256}h^4 = 1 + \frac{3}{32}h^2 - \frac{9}{2048}h^4$

$p \approx 4(1 + \frac{3}{32}h^2 - \frac{9}{2048}h^4)$

$p \approx 4 + \frac{3}{8}h^2 - \frac{9}{512}h^4$

$\lambda = \frac{3}{8}, \quad \mu = -\frac{9}{512}$

Question number

3

AP: $u_1 = 2$, $u_3 = 2 + 2d$, $u_{13} = 2 + 12d$. GP: first term 2, ratio r

$2 + 2d = 2r \Rightarrow r = 1 + d$

$2 + 12d = 2r^2$

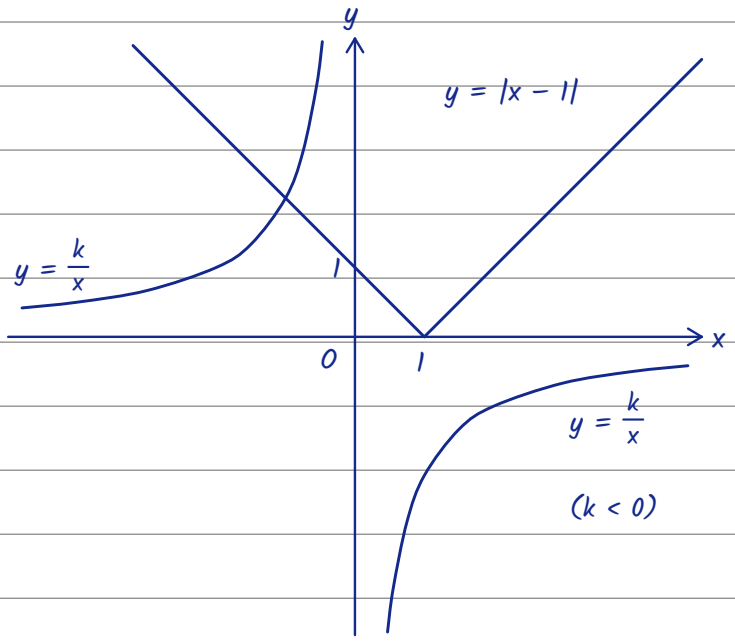
$\Rightarrow 2 + 12d = 2(1 + d)^2 \Rightarrow 1 + 6d = 1 + 2d + d^2$

$d^2 - 4d = 0 \Rightarrow d(d - 4) = 0$

$d \neq 0$, so $d = 4$ (AP: 2, 6, 10, 14, ...)

The common difference $d = 4$ is positive, so each term is 4 more than the previous one: the arithmetic progression is an increasing sequence.

4 (a)



4 (b)

$x|x - 1| = k \Leftrightarrow |x - 1| = \frac{k}{x}$ $(x \neq 0)$, so the roots of the equation are the x-coordinates of the intersections of the two graphs in (a).

For any negative k, $y = \frac{k}{x}$ (2nd and 4th quadrants) meets $y = |x - 1|$ at exactly one point (for $x < 0$), so the equation has exactly one real root.

4 (c)

From the graph the root is negative, so for $x < 0$: $|x - 1| = 1 - x$

$x(1 - x) = -6 \Rightarrow x^2 - x - 6 = 0 \Rightarrow (x - 3)(x + 2) = 0$

$x = 3$ (rejected, since $x < 0$) or $x = -2$

Root: $x = -2$

Question number	
5 (a)	$12\cos t + 5\sin t = R\cos(t - \alpha) = R\cos t\cos\alpha + R\sin t\sin\alpha$
	$R\cos\alpha = 12, \quad R\sin\alpha = 5$
	$R = \sqrt{(12^2 + 5^2)} = \sqrt{169} = 13$
	$\tan\alpha = \frac{5}{12} \quad \Rightarrow \quad \alpha = 0.394791\ldots = 0.3948 \quad (4\text{ s.f.})$
	$v = 13\cos(t - 0.3948)$
5 (b)	Speed 3 $\Rightarrow$ $v = 3$ or $v = -3$ $\Rightarrow$ $13\cos(t - 0.3948) = \pm 3$
	$\cos(t - 0.3948) = \frac{3}{13}$: $t - 0.3948 = 1.337928\ldots \Rightarrow t = 1.732719\ldots$
	$\cos(t - 0.3948) = -\frac{3}{13}$: $t - 0.3948 = \pi - 1.337928\ldots = 1.803664\ldots$
	$\Rightarrow \quad t = 2.198455\ldots$ (other solutions give $t < 0$ or $t > 2.20$)
	$t = 1.73$ s and $t = 2.20$ s
6 (a)	Let $f(x) = 6\arcsin(2x - 1) - x^2$ (radians)
	$f(0.5) = 6\arcsin(0) - 0.25 = -0.25 < 0$
	$f(0.6) = 6\arcsin(0.2) - 0.36 = 0.848147\ldots > 0$
	f is continuous and there is a change of sign,
	so the root lies between 0.5 and 0.6.
6 (b)	$6\arcsin(2x - 1) = x^2 \quad \Rightarrow \quad \arcsin(2x - 1) = \frac{1}{6}x^2$
	$2x - 1 = \sin(\frac{1}{6}x^2) \quad \Rightarrow \quad x = \frac{1}{2} + \frac{1}{2}\sin(\frac{1}{6}x^2)$
	$x_{n+1} = \frac{1}{2} + \frac{1}{2}\sin(\frac{1}{6}x_n^2)$: $p = \frac{1}{2}, \quad q = \frac{1}{2}, \quad r = \frac{1}{6}$
6 (c)	$x_0 = 0.5$
	$x_1 = 0.5208273057\ldots$
	$x_2 = 0.5225973903\ldots$
	$x_3 = 0.5227511445\ldots$
	$x_4 = 0.5227645245\ldots$
	$x_5 = 0.5227656890\ldots$
	Root = 0.5228 (4 s.f.)

Question number	
7 (a)	Gradient of line from (3, 2) to P(x, y) = $\frac{y-2}{x-3}$
	$\frac{dy}{dx} = 3\left(\frac{y-2}{x-3}\right)$
7 (b)	Separate variables: $\int \frac{1}{y-2}\,dy = \int \frac{3}{x-3}\,dx$
	$\ln(y-2) = 3\ln(x-3) + c$ (x > 3, y > 2 so no modulus needed)
	At (4, 3): $\ln 1 = 3\ln 1 + c \Rightarrow c = 0$
	$\ln(y-2) = \ln(x-3)^3 \Rightarrow y - 2 = (x-3)^3$
	$y = (x-3)^3 + 2$
7 (c)	Translation by vector $(3, 2)^T$: 3 units in the positive x-direction and
	2 units in the positive y-direction.
8 (a)	$y = xe^{-2x}$
	$\frac{dy}{dx} = e^{-2x} - 2xe^{-2x} = e^{-2x}(1-2x)$
	$\frac{d^2y}{dx^2} = -2e^{-2x}(1-2x) + e^{-2x}(-2) = e^{-2x}(4x-4)$
	At x = 1: $\frac{d^2y}{dx^2} = e^{-2}(4-4) = 0$
	x = 0.5: $\frac{d^2y}{dx^2} = e^{-1}(2-4) = -2e^{-1} < 0$
	x = 1.5: $\frac{d^2y}{dx^2} = e^{-3}(6-4) = 2e^{-3} > 0$
	$\frac{d^2y}{dx^2} = 0$ at x = 1 and changes sign either side, so M has a point of
	inflection at P where x = 1.
8 (b)	P = $(1, e^{-2})$. By parts with u = x, $\frac{dv}{dx} = e^{-2x}$ $\left(v = -\frac{1}{2}e^{-2x}\right)$:
	$\int xe^{-2x}\,dx = -\frac{1}{2}xe^{-2x} + \int \frac{1}{2}e^{-2x}\,dx = -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$
	$\int_0^1 xe^{-2x}\,dx = \left[-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}\right]_0^1$
	$= \left(-\frac{1}{2}e^{-2} - \frac{1}{4}e^{-2}\right) - \left(0 - \frac{1}{4}\right) = \frac{1}{4} - \frac{3}{4}e^{-2}$
	Area of triangle under L from O to P = $\frac{1}{2} \times 1 \times e^{-2} = \frac{1}{2}e^{-2}$
	Area of R = $\frac{1}{4} - \frac{3}{4}e^{-2} - \frac{1}{2}e^{-2} = \frac{1}{4} - \frac{5}{4}e^{-2}$
	$= \frac{1}{4}(1 - 5e^{-2})$, a = 1, b = −5

Question number

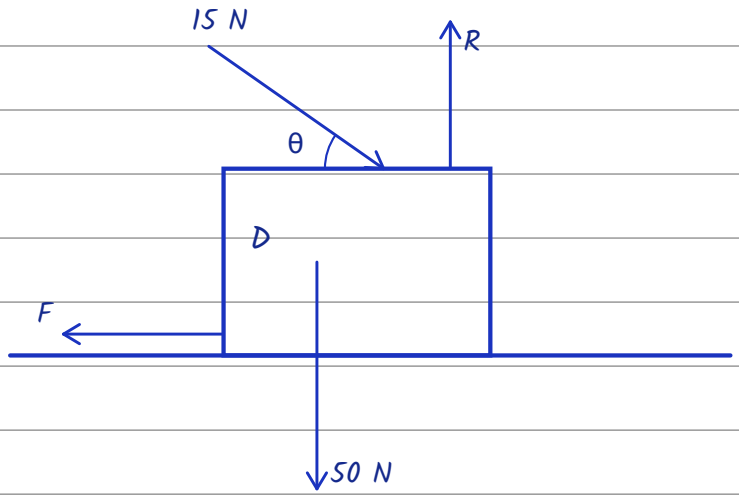
9 (a) $v = u + at$: $18 = 14 + 20a \Rightarrow a = 0.2 \text{ m s}^{-2}$

9 (b) From A to B: $v^2 = u^2 + 2as \Rightarrow 14^2 = u^2 + 2(0.2)(330)$

$196 = u^2 + 132 \Rightarrow u^2 = 64$

Speed at A = 8 m s^{-1}

10 (a)



Forces: weight 50 N, normal reaction R, friction F and the 15 N force

10 (b) Resolve horizontally (at rest): $F = 15\cos\theta$

Resolve vertically: $R = 50 + 15\sin\theta$

Equilibrium (no slipping): $F \leq \mu R = 0.2R$

$15\cos\theta \leq 0.2(50 + 15\sin\theta) = 10 + 3\sin\theta$

$\therefore \quad 15\cos\theta - 3\sin\theta \leq 10$ as required

11 (a) Vertically (up +ve): $u = 25\sin 15° = 6.470476\ldots$, $a = -10$, $s = -4$

$s = ut + \frac{1}{2}at^2$: $-4 = (25\sin 15°)t - 5t^2$

$5t^2 - 6.470476\ldots t - 4 = 0$

$t = \frac{6.470476\ldots + \sqrt{(6.470476\ldots^2 + 80)}}{10} = 1.750981\ldots$ (taking $t > 0$)

Time = 1.75 s

Question number	
11 (b)	Horizontally: $x = (25\cos 15°)t = 24.148145... \times 1.750981...$
	$= 42.282962...$ Horizontal distance = 42.3 m
11 (c)	At $t = 1.5$: horizontal velocity $v_h = 25\cos 15° = 24.148145...$
	vertical velocity $v_v = 25\sin 15° - 10(1.5) = -8.529523...$ (i.e. downwards)
	$\tan\theta = \frac{8.529523...}{24.148145...} = 0.353216... \Rightarrow \theta = 19.454...°$
	Direction of motion: 19.5° below the horizontal
11 (d)	The value $g = 10$ used is larger than the true value (about 9.8), so in
	the model the ball falls faster and lands sooner. (Or: backspin / the wind.)
12 (a)	Let $T_D = T$, $T_C = 3T$; centre of mass G at distance x m from A.
	Resolve vertically: $3T + T = 20g \Rightarrow T = 5g = 49$ N, $T_C = 15g = 147$ N
	Moments about A: $0.5T_C + (4 - 0.7)T_D = 20g\,x$
	$0.5(147) + 3.3(49) = 196x \Rightarrow 73.5 + 161.7 = 196x$
	$235.2 = 196x \Rightarrow x = 1.2$
	Centre of mass is 1.2 m from A
12 (b)	Moment 3.5mg about C ⇒ particle 3.5 m from C, i.e. at B (beyond D).
	Largest m when beam is about to turn about D: $T_C = 0$
	Moments about D: $mg(0.7) = 20g(3.3 - 1.2) = 20g(2.1)$
	$0.7m = 42 \Rightarrow m = 60$
13 (a)	Resultant $F = (4t - 8)i + (6(2t - 1)^2 - 54)j$
	Equilibrium: $4t - 8 = 0 \Rightarrow t = 2$; check j: $6(3)^2 - 54 = 0$ ✓
	$t = 2$
13 (b)	$F = ma$: $a = \frac{1}{2}F = (2t - 4)i + (3(2t - 1)^2 - 27)j$
	$= (2t - 4)i + (12t^2 - 12t - 24)j$

Question number	
	$v = \int a\,dt = (t^2 - 4t)i + (4t^3 - 6t^2 - 24t)j + c$
	$t = 0,\ v = 0 \Rightarrow c = 0,$ so $v = (t^2 - 4t)i + (4t^3 - 6t^2 - 24t)j$
	Due north $\Rightarrow$ i-component $= 0$: $t^2 - 4t = 0 \Rightarrow t(t - 4) = 0$
	$t = 0$ (at rest) or $t = 4$, so $t = 4$
	$t = 4$: $v = (16 - 16)i + (256 - 96 - 96)j = 64j$
	Speed $= 64\ m\ s^{-1}$
13 (c)	$r = \int v\,dt = (\frac{1}{3}t^3 - 2t^2)i + (t^4 - 2t^3 - 12t^2)j$ (taking $r = 0$ at $t = 0$)
	$t = 3$: $r = (9 - 18)i + (81 - 54 - 108)j = -9i - 81j$
	Distance $= \sqrt{((-9)^2 + (-81)^2)} = \sqrt{6642} = 81.498466...$
	Distance $= 81.5\ m$
14 (a)	T_1 = tension in string AB, T_2 = tension in string BC.
	A (N2L up the plane): $T_1 - 2g \sin 30° = 2a$
	C (N2L down the plane): $4g \sin 60° - T_2 = 4a$
	B (N2L towards C): $T_2 - T_1 - F = 3a$, where $R = 3g$, $F = \mu R = 3\mu g$
	Add the three equations: $4g \sin 60° - 2g \sin 30° - 3\mu g = 9a$
	$9a = g(4 \times \frac{\sqrt{3}}{2} - 2 \times \frac{1}{2} - 3\mu) = g(2\sqrt{3} - 1 - 3\mu)$
	B's plane is rough so $\mu > 0 \Rightarrow 3\mu g > 0$
	$\therefore\ 9a < g(2\sqrt{3} - 1) \Rightarrow a < \frac{1}{9}g(2\sqrt{3} - 1)$ as required
14 (b)	$a = \frac{1}{9}g$: $g = g(2\sqrt{3} - 1 - 3\mu) \Rightarrow 3\mu = 2\sqrt{3} - 2$
	$\mu = \frac{2}{3}(\sqrt{3} - 1) = 0.488033...$
	Normal reaction $R = 3g = 29.4\ N$
	Friction $F = \mu R = 3\mu g = 2(\sqrt{3} - 1)g = 14.348195...\ N$
	Contact force $= \sqrt{(R^2 + F^2)} = \sqrt{(29.4^2 + 14.348195...^2)} = 32.714380...$
	Magnitude of contact force $= 32.7\ N$