



Oxford Cambridge and RSA

Monday 19 October 2020 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours



You must have:

- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ ms}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **12** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae

A Level Mathematics A (H240)

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2, \quad \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Standard deviation

$$\sqrt{\frac{\sum(x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np , variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Section A: Pure Mathematics

Answer **all** the questions.

- 1 Triangle ABC has $AB = 8.5$ cm, $BC = 6.2$ cm and angle $B = 35^\circ$.

Calculate the area of the triangle. [2]

- 2 A sequence of transformations maps the curve $y = e^x$ to the curve $y = e^{2x+3}$.

Give details of these transformations. [3]

- 3 The functions f and g are defined for all real values of x by

$$f(x) = 2x^2 + 6x \text{ and } g(x) = 3x + 2.$$

(a) Find the range of f . [3]

(b) Give a reason why f has no inverse. [1]

(c) Given that $fg(-2) = g^{-1}(a)$, where a is a constant, determine the value of a . [4]

(d) Determine the set of values of x for which $f(x) > g(x)$. Give your answer in set notation. [3]

- 4 A curve has equation $y = 2 \ln(k - 3x) + x^2 - 3x$, where k is a positive constant.

(a) Given that the curve has a point of inflection where $x = 1$, show that $k = 6$. [5]

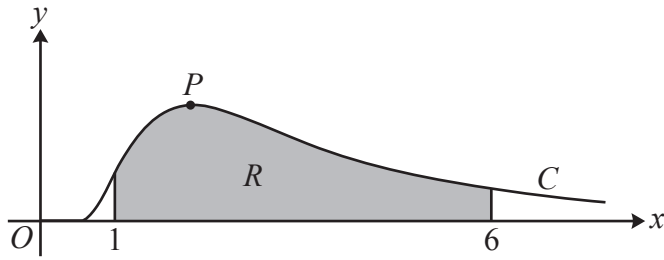
It is also given that the curve intersects the x -axis at exactly one point.

(b) Show by calculation that the x -coordinate of this point lies between 0.5 and 1.5. [2]

(c) Use the Newton-Raphson method, with initial value $x_0 = 1$, to find the x -coordinate of the point where the curve intersects the x -axis, giving your answer correct to 5 decimal places. Show the result of each iteration to 6 decimal places. [3]

(d) By choosing suitable bounds, verify that your answer to part (c) is correct to 5 decimal places. [1]

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The diagram shows the curve C with parametric equations

$$x = \frac{3}{t}, \quad y = t^3 e^{-2t}, \quad \text{where } t > 0.$$

The maximum point on C is denoted by P .

- (a)** Determine the exact coordinates of P . [4]

The shaded region R is enclosed by the curve, the x -axis and the lines $x = 1$ and $x = 6$.

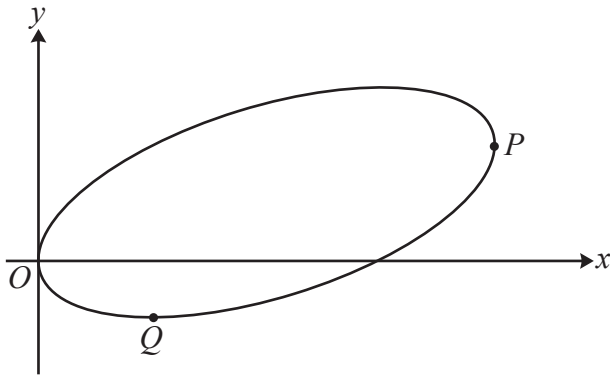
- (b)** Show that the area of R is given by

$$\int_a^b 3te^{-2t} dt,$$

where a and b are constants to be determined. [3]

- (c)** Hence determine the exact area of R . [5]

6 In this question you must show detailed reasoning.



The diagram shows the curve with equation $4xy = 2(x^2 + 4y^2) - 9x$.

- (a)** Show that $\frac{dy}{dx} = \frac{4x - 4y - 9}{4x - 16y}$. **[3]**

At the point P on the curve the tangent to the curve is parallel to the y -axis and at the point Q on the curve the tangent to the curve is parallel to the x -axis.

- (b)** Show that the distance PQ is $k\sqrt{5}$, where k is a rational number to be determined. **[8]**

Section B: Mechanics

Answer **all** the questions.

- 7 A particle P moves with constant acceleration $(-4\mathbf{i} + 2\mathbf{j})\text{ms}^{-2}$. At time $t = 0$ seconds, P is moving with velocity $(7\mathbf{i} + 6\mathbf{j})\text{ms}^{-1}$.

(a) Determine the speed of P when $t = 3$. [4]

(b) Determine the change in displacement of P between $t = 0$ and $t = 3$. [2]

- 8 A car is travelling on a straight horizontal road. The velocity of the car, $v\text{ms}^{-1}$, at time t seconds as it travels past three points, P , Q and R , is modelled by the equation

$$v = at^2 + bt + c,$$

where a , b and c are constants.

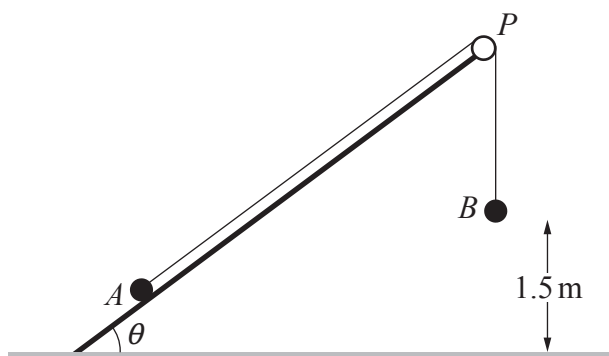
The car passes P at time $t = 0$ with velocity 8ms^{-1} .

(a) State the value of c . [1]

The car passes Q at time $t = 5$ and at that instant its deceleration is 0.12ms^{-2} . The car passes R at time $t = 18$ with velocity 2.96ms^{-1} .

(b) Determine the values of a and b . [4]

(c) Find, to the nearest metre, the distance between points P and R . [2]



One end of a light inextensible string is attached to a particle A of mass 2 kg . The other end of the string is attached to a second particle B of mass 2.5 kg . Particle A is in contact with a rough plane inclined at θ to the horizontal, where $\cos \theta = \frac{4}{5}$. The string is taut and passes over a small smooth pulley P at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane. Particle B hangs freely below P at a distance 1.5 m above horizontal ground, as shown in the diagram.

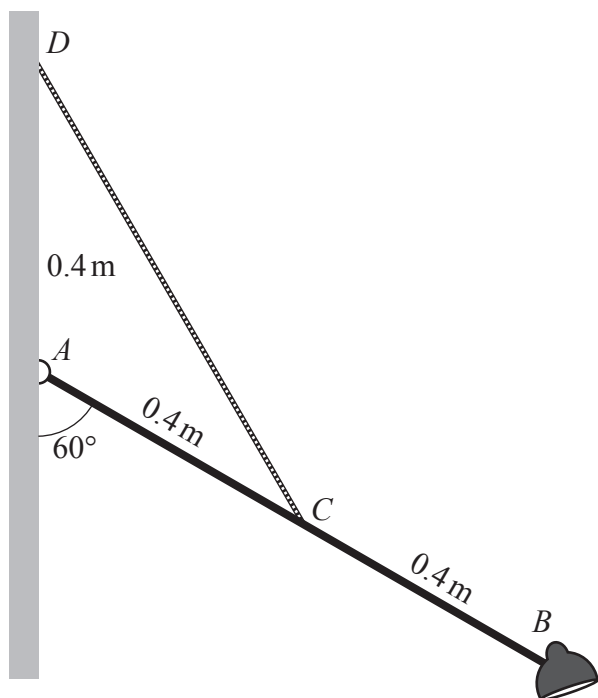
The coefficient of friction between A and the plane is μ . The system is released from rest and in the subsequent motion B hits the ground before A reaches P . The speed of B at the instant that it hits the ground is 1.2 ms^{-1} .

- (a) For the motion before B hits the ground, show that the acceleration of B is 0.48 ms^{-2} . [1]
- (b) For the motion before B hits the ground, show that the tension in the string is 23.3 N . [3]
- (c) Determine the value of μ . [5]

After B hits the ground, A continues to travel up the plane before coming to instantaneous rest before it reaches P .

- (d) Determine the distance that A travels from the instant that B hits the ground until A comes to instantaneous rest. [4]

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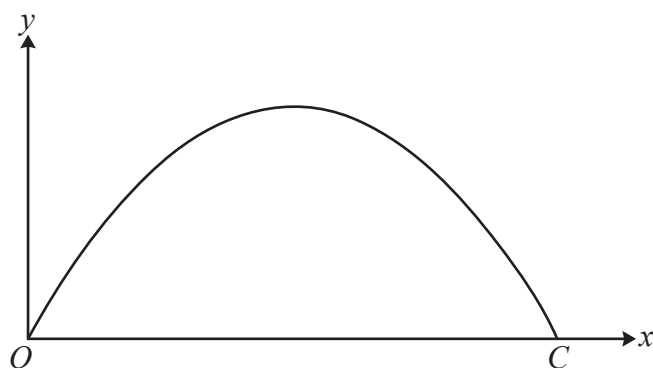


The diagram shows a wall-mounted light. It consists of a rod AB of mass 0.25 kg and length 0.8 m which is freely hinged to a vertical wall at A , and a lamp of mass 0.5 kg fixed at B . The system is held in equilibrium by a chain CD whose end C is attached to the midpoint of AB . The end D is fixed to the wall a distance 0.4 m vertically above A . The rod AB makes an angle of 60° with the downward vertical.

The chain is modelled as a light inextensible string, the rod is modelled as uniform and the lamp is modelled as a particle.

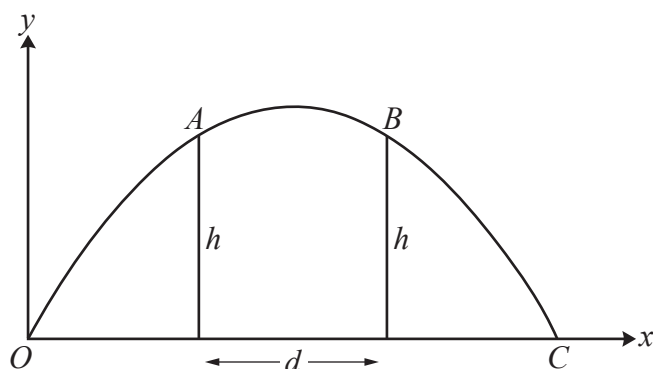
- (a) By taking moments about A , determine the tension in the chain. [4]
- (b) (i) Determine the magnitude of the force exerted on the rod at A . [4]
- (ii) Calculate the direction of the force exerted on the rod at A . [2]
- (c) Suggest one improvement that could be made to the model to make it more realistic. [1]

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A particle P moves freely under gravity in the plane of a fixed horizontal axis Ox , which lies on horizontal ground, and a fixed vertical axis Oy . P is projected from O with a velocity whose components along Ox and Oy are U and V , respectively. P returns to the ground at a point C .

- (a) Determine, in terms of U , V and g , the distance OC . [4]



P passes through two points A and B , each at a height h above the ground and a distance d apart, as shown in the diagram.

- (b) Write down the horizontal and vertical components of the velocity of P at A . [2]
- (c) Hence determine an expression for d in terms of U , V , g and h . [3]
- (d) Given that the direction of motion of P as it passes through A is inclined to the horizontal at an angle θ , where $\tan \theta = \frac{1}{2}$, determine an expression for V in terms of g , d and h . [4]

END OF QUESTION PAPER

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