OCR
Oxford Cambridge and RSA

Monday 19 October 2020 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours

8233206614

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient rule $y = \frac{u}{v},\ \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta,\ \cos\theta \approx 1 - \frac{1}{2}\theta^2,\ \tan\theta \approx \theta$ where θ is measured in radians

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Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

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Section A: Pure Mathematics

Answer **all** the questions.

1 Triangle ABC has $AB = 8.5\,\text{cm}$, $BC = 6.2\,\text{cm}$ and angle $B = 35°$.

Calculate the area of the triangle. **[2]**

2 A sequence of transformations maps the curve $y = \text{e}^x$ to the curve $y = \text{e}^{2x+3}$.

Give details of these transformations. **[3]**

3 The functions f and g are defined for all real values of x by

$\text{f}(x) = 2x^2 + 6x$ and $\text{g}(x) = 3x + 2$.

(a) Find the range of f. **[3]**

(b) Give a reason why f has no inverse. **[1]**

(c) Given that $\text{fg}(-2) = \text{g}^{-1}(a)$, where a is a constant, determine the value of a. **[4]**

(d) Determine the set of values of x for which $\text{f}(x) > \text{g}(x)$. Give your answer in set notation. **[3]**

4 A curve has equation $y = 2\ln(k - 3x) + x^2 - 3x$, where k is a positive constant.

(a) Given that the curve has a point of inflection where $x = 1$, show that $k = 6$. **[5]**

It is also given that the curve intersects the x-axis at exactly one point.

(b) Show by calculation that the x-coordinate of this point lies between 0.5 and 1.5. **[2]**

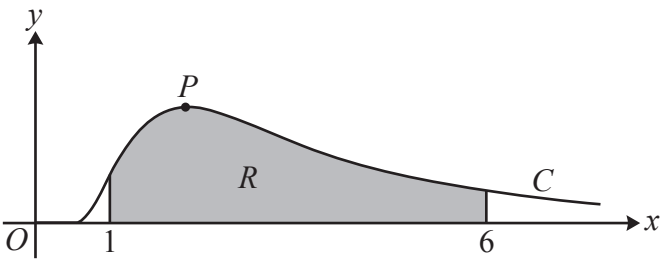
(c) Use the Newton-Raphson method, with initial value $x_0 = 1$, to find the x-coordinate of the point where the curve intersects the x-axis, giving your answer correct to 5 decimal places. Show the result of each iteration to 6 decimal places. **[3]**

(d) By choosing suitable bounds, verify that your answer to part **(c)** is correct to 5 decimal places. **[1]**

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The diagram shows the curve C with parametric equations

$x = \frac{3}{t}$, $y = t^3 e^{-2t}$, where $t > 0$.

The maximum point on C is denoted by P.

(a) Determine the exact coordinates of P. **[4]**

The shaded region R is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 6$.

(b) Show that the area of R is given by

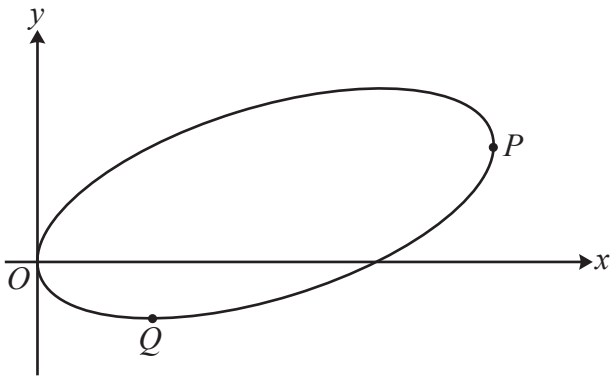
$$\int_a^b 3te^{-2t}\,dt,$$

where a and b are constants to be determined. **[3]**

(c) Hence determine the exact area of R. **[5]**

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6 In this question you must show detailed reasoning.



The diagram shows the curve with equation $4xy = 2(x^2 + 4y^2) - 9x$.

(a) Show that $\frac{dy}{dx} = \frac{4x - 4y - 9}{4x - 16y}$. **[3]**

At the point P on the curve the tangent to the curve is parallel to the y-axis and at the point Q on the curve the tangent to the curve is parallel to the x-axis.

(b) Show that the distance PQ is $k\sqrt{5}$, where k is a rational number to be determined. **[8]**

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Section B: Mechanics

Answer **all** the questions.

7 A particle P moves with constant acceleration $(-4\mathbf{i}+2\mathbf{j})\,\mathrm{m\,s^{-2}}$. At time $t = 0$ seconds, P is moving with velocity $(7\mathbf{i}+6\mathbf{j})\,\mathrm{m\,s^{-1}}$.

(a) Determine the speed of P when $t = 3$. **[4]**

(b) Determine the change in displacement of P between $t = 0$ and $t = 3$. **[2]**

8 A car is travelling on a straight horizontal road. The velocity of the car, $v\,\mathrm{m\,s^{-1}}$, at time t seconds as it travels past three points, P, Q and R, is modelled by the equation

$$v = at^2 + bt + c,$$

where a, b and c are constants.

The car passes P at time $t = 0$ with velocity $8\,\mathrm{m\,s^{-1}}$.

(a) State the value of c. **[1]**

The car passes Q at time $t = 5$ and at that instant its deceleration is $0.12\,\mathrm{m\,s^{-2}}$. The car passes R at time $t = 18$ with velocity $2.96\,\mathrm{m\,s^{-1}}$.

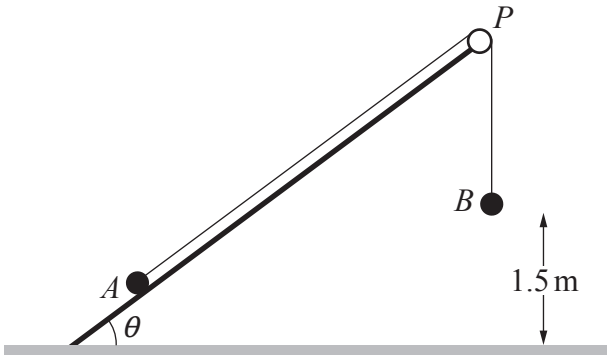
(b) Determine the values of a and b. **[4]**

(c) Find, to the nearest metre, the distance between points P and R. **[2]**

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One end of a light inextensible string is attached to a particle A of mass 2 kg. The other end of the string is attached to a second particle B of mass 2.5 kg. Particle A is in contact with a rough plane inclined at θ to the horizontal, where $\cos\theta = \frac{4}{5}$. The string is taut and passes over a small smooth pulley P at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane. Particle B hangs freely below P at a distance 1.5 m above horizontal ground, as shown in the diagram.

The coefficient of friction between A and the plane is μ. The system is released from rest and in the subsequent motion B hits the ground before A reaches P. The speed of B at the instant that it hits the ground is $1.2\,\text{m s}^{-1}$.

(a) For the motion before B hits the ground, show that the acceleration of B is $0.48\,\text{m s}^{-2}$. **[1]**

(b) For the motion before B hits the ground, show that the tension in the string is 23.3 N. **[3]**

(c) Determine the value of μ. **[5]**

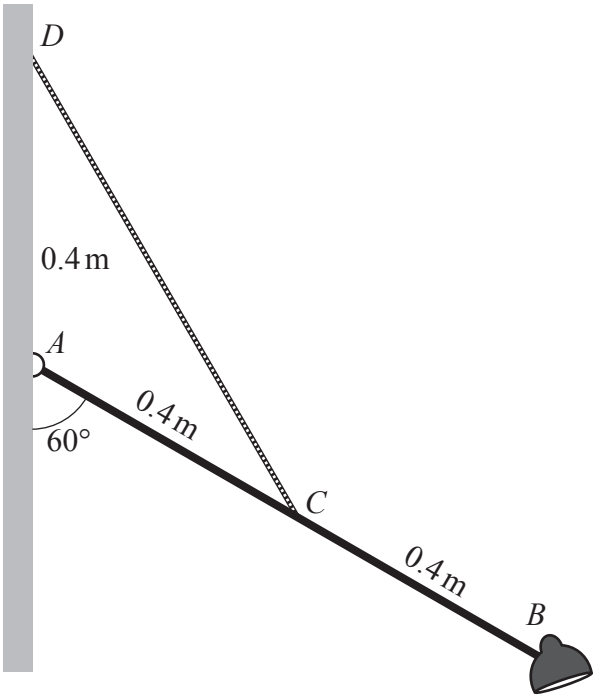
After B hits the ground, A continues to travel up the plane before coming to instantaneous rest before it reaches P.

(d) Determine the distance that A travels from the instant that B hits the ground until A comes to instantaneous rest. **[4]**

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The diagram shows a wall-mounted light. It consists of a rod AB of mass 0.25 kg and length 0.8 m which is freely hinged to a vertical wall at A, and a lamp of mass 0.5 kg fixed at B. The system is held in equilibrium by a chain CD whose end C is attached to the midpoint of AB. The end D is fixed to the wall a distance 0.4 m vertically above A. The rod AB makes an angle of 60° with the downward vertical.

The chain is modelled as a light inextensible string, the rod is modelled as uniform and the lamp is modelled as a particle.

(a) By taking moments about A, determine the tension in the chain. **[4]**

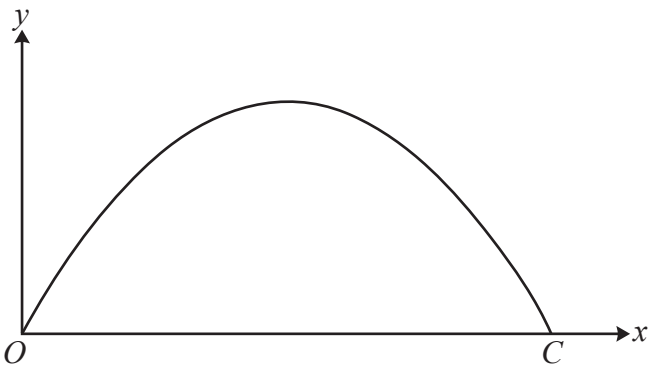
(b) (i) Determine the magnitude of the force exerted on the rod at A. **[4]**

(ii) Calculate the direction of the force exerted on the rod at A. **[2]**

(c) Suggest one improvement that could be made to the model to make it more realistic. **[1]**

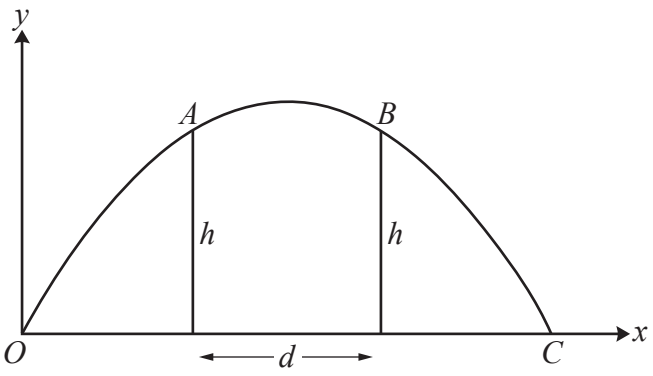
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A particle P moves freely under gravity in the plane of a fixed horizontal axis Ox, which lies on horizontal ground, and a fixed vertical axis Oy. P is projected from O with a velocity whose components along Ox and Oy are U and V, respectively. P returns to the ground at a point C.

(a) Determine, in terms of U, V and g, the distance OC. **[4]**



P passes through two points A and B, each at a height h above the ground and a distance d apart, as shown in the diagram.

(b) Write down the horizontal and vertical components of the velocity of P at A. **[2]**

(c) Hence determine an expression for d in terms of U, V, g and h. **[3]**

(d) Given that the direction of motion of P as it passes through A is inclined to the horizontal at an angle θ, where $\tan\theta = \frac{1}{2}$, determine an expression for V in terms of g, d and h. **[4]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1

Area $= \frac{1}{2}ab\sin C = \frac{1}{2} \times 8.5 \times 6.2 \times \sin 35°$

$= 15.1137\ldots = 15.1\ \text{cm}^2$

2

Translation by the vector $(-3, 0)$, i.e. 3 units in the negative x-direction,

giving $y = e^{x+3}$;

followed by a stretch parallel to the x-axis, scale factor 0.5, giving $y = e^{2x+3}$.

3 (a)

$f(x) = 2[x^2 + 3x] = 2[(x + 3/2)^2 - (3/2)^2]$

$f(x) = 2(x + \frac{3}{2})^2 - \frac{9}{2}$ so minimum value is $-\frac{9}{2}$

Range: $f(x) \geq -\frac{9}{2}$

3 (b)

f is not one-one (it is many-one): e.g. $f(0) = f(-3) = 0$.

3 (c)

$g^{-1}(x)$: $x = 3y + 2 \Rightarrow y = \frac{x-2}{3}$, so $g^{-1}(a) = \frac{a-2}{3}$

$fg(-2) = f(3(-2) + 2) = f(-4) = 2(-4)^2 + 6(-4) = 32 - 24 = 8$

$8 = \frac{a-2}{3} \Rightarrow a - 2 = 24$

$a = 26$

Question number

3 (d)

$2x^2 + 6x > 3x + 2 \quad \Rightarrow \quad 2x^2 + 3x - 2 > 0$

$(2x - 1)(x + 2) > 0$, critical values $x = \frac{1}{2}$, $x = -2$

Positive quadratic is > 0 outside the roots:

$\{x : x < -2\} \cup \{x : x > \frac{1}{2}\}$

4 (a)

$y = 2\ln(k - 3x) + x^2 - 3x \quad \Rightarrow \quad \frac{dy}{dx} = \frac{2(-3)}{k - 3x} + 2x - 3 = \frac{6}{3x - k} + 2x - 3$

$\frac{d^2y}{dx^2} = -\frac{18}{(3x - k)^2} + 2$

Point of inflection at $x = 1$: $\frac{d^2y}{dx^2} = 0 \quad \Rightarrow \quad \frac{18}{(3 - k)^2} = 2$

$(3 - k)^2 = 9 \quad \Rightarrow \quad 3 - k = \pm 3 \quad \Rightarrow \quad k = 0 \text{ or } k = 6$

k is positive, so $k = 6$

4 (b)

$f(x) = 2\ln(6 - 3x) + x^2 - 3x$

$f(0.5) = 2\ln 4.5 + 0.25 - 1.5 = 1.758\ldots > 0$

$f(1.5) = 2\ln 1.5 + 2.25 - 4.5 = -1.439\ldots < 0$

Change of sign (f continuous for $x < 2$), so the root lies between 0.5 and 1.5.

4 (c)

$x_{n+1} = x_n - \frac{2\ln(6 - 3x_n) + x_n^2 - 3x_n}{6(3x_n - 6)^{-1} + 2x_n - 3}$

$x_0 = 1$

$x_1 = 1.065742 \quad (1.0657415\ldots)$

$x_2 = 1.065675 \quad (1.0656753\ldots)$

$x_3 = 1.065675$

x-coordinate = 1.06568 (5 d.p.)

Question number

4 (d)

$f(1.065675) = 0.0000009\ldots > 0$

$f(1.065685) = -0.000029\ldots < 0$

Change of sign, so the root lies in $[1.065675, 1.065685]$

$\therefore$ 1.06568 is correct to 5 decimal places.

5 (a)

$\frac{dy}{dt} = 3t^2e^{-2t} + t^3(-2e^{-2t}) = t^2e^{-2t}(3 - 2t)$

At P, $\frac{dy}{dt} = 0$: $t^2e^{-2t}(3 - 2t) = 0$, $t > 0$ and $e^{-2t} \neq 0 \Rightarrow t = \frac{3}{2}$

$x = 3 \div \frac{3}{2} = 2$, $y = (\frac{3}{2})^3e^{-3} = \frac{27}{8}e^{-3}$

P $(2, \frac{27}{8}e^{-3})$

5 (b)

Area $= \int y\,dx = \int y\,\frac{dx}{dt}\,dt$, $x = 3t^{-1} \Rightarrow \frac{dx}{dt} = -3t^{-2}$

$x = 1 \Rightarrow t = 3$; $x = 6 \Rightarrow t = 0.5$

Area $= \int_3^{0.5} t^3e^{-2t} \times (-\frac{3}{t^2})\,dt = \int_3^{0.5} -3te^{-2t}\,dt = \int_{0.5}^{3} 3te^{-2t}\,dt$

so $a = 0.5$ and $b = 3$

5 (c)

By parts: $u = 3t$, $du/dt = 3$; $dv/dt = e^{-2t}$, $v = -\frac{1}{2}e^{-2t}$

$\int 3te^{-2t}\,dt = -(3/2)te^{-2t} + \int (3/2)e^{-2t}\,dt$

$= -(3/2)te^{-2t} - (3/4)e^{-2t} + c$

Area $= [\,-(3/2)te^{-2t} - (3/4)e^{-2t}\,]_{0.5}^{3}$

$= (\,-(3/2)(3)e^{-6} - (3/4)e^{-6}\,) - (\,-(3/2)(0.5)e^{-1} - (3/4)e^{-1}\,)$

$= -(21/4)e^{-6} + (3/2)e^{-1}$

Question number

6 (a)

$4xy = 2(x^2 + 4y^2) - 9x = 2x^2 + 8y^2 - 9x$

Differentiate implicitly: $4y + 4x\frac{dy}{dx} = 4x + 16y\frac{dy}{dx} - 9$

$4x\frac{dy}{dx} - 16y\frac{dy}{dx} = 4x - 4y - 9$

$\frac{dy}{dx}(4x - 16y) = 4x - 4y - 9 \Rightarrow \frac{dy}{dx} = \frac{4x - 4y - 9}{4x - 16y}$ as required

6 (b)

At P, tangent parallel to the y-axis: denominator $4x - 16y = 0 \Rightarrow x = 4y$

Substitute: $4(4y)y = 2(16y^2 + 4y^2) - 9(4y)$

$16y^2 = 40y^2 - 36y \Rightarrow 24y^2 - 36y = 0$

$12y(2y - 3) = 0 \Rightarrow y = 3/2$ ($y = 0$ gives O, where the tangent is not at P)

$x = 4 \times 3/2 = 6$, P (6, 3/2)

At Q, tangent parallel to the x-axis: numerator $4x - 4y - 9 = 0 \Rightarrow y = x - 9/4$

Substitute: $4x(x - 9/4) = 2x^2 + 8(x - 9/4)^2 - 9x$

$4x^2 - 9x = 2x^2 + 8x^2 - 36x + 81/2 - 9x$

$0 = 6x^2 - 36x + 81/2 \Rightarrow 4x^2 - 24x + 27 = 0$

$(2x - 3)(2x - 9) = 0 \Rightarrow x = 3/2$ or $x = 9/2$

$x = 3/2 \Rightarrow y = -3/4$; $x = 9/2 \Rightarrow y = 9/4$ (above the x-axis, not Q)

Q is below the x-axis, so Q (3/2, −3/4)

$PQ^2 = (6 - 3/2)^2 + (3/2 - (-3/4))^2 = (9/2)^2 + (9/4)^2 = 81/4 + 81/16 = 405/16$

$PQ = \sqrt{405}/4 = 9\sqrt{5}/4$, so $k = 9/4$

7 (a)

$v = u + at = (7i + 6j) + 3(-4i + 2j)$

$v = -5i + 12j$

$|v| = \sqrt{((-5)^2 + 12^2)} = \sqrt{169}$

Speed = 13 m s^{-1}

Question number	
7 (b)	$s = ut + \frac{1}{2}at^2 = 3(7i + 6j) + \frac{1}{2}(-4i + 2j)(3)^2$
	$= 21i + 18j - 18i + 9j = (3i + 27j)$ m
8 (a)	$t = 0$, $v = 8 \Rightarrow c = 8$
8 (b)	$a = dv/dt = 2at + b$
	$t = 5$, deceleration 0.12: $10a + b = -0.12$ ...(1)
	$t = 18$, $v = 2.96$: $a(18)^2 + 18b + 8 = 2.96 \Rightarrow 324a + 18b = -5.04$
	$\Rightarrow 18a + b = -0.28$ ...(2)
	(2) − (1): $8a = -0.16 \Rightarrow a = -0.02$
	$b = -0.12 - 10(-0.02) = 0.08$
8 (c)	$v > 0$ for $0 \leq t \leq 18$, so distance $= \int_0^{18} (-0.02t^2 + 0.08t + 8)\,dt$
	$= \left[-(0.02/3)t^3 + 0.04t^2 + 8t \right]_0^{18} = -38.88 + 12.96 + 144 = 118.08$
	Distance PR = 118 m
9 (a)	For B: $v^2 = u^2 + 2as$: $1.2^2 = 0^2 + 2a(1.5)$
	$1.44 = 3a \Rightarrow a = 0.48$ m s^{-2} as required
9 (b)	N2L for B (downwards): $2.5g - T = 2.5a$
	$T = 2.5(9.8) - 2.5(0.48) = 24.5 - 1.2$
	$T = 23.3$ N as required
9 (c)	$\sin\theta = 3/5$. Perpendicular to plane: $R = 2g\cos\theta = 2(9.8)(4/5) = 15.68$ N
	N2L for A up the plane: $T - F - 2g\sin\theta = 2a$
	Moving, so friction limiting: $F = \mu R = 15.68\mu$
	$23.3 - 15.68\mu - 2(9.8)(3/5) = 2(0.48)$
	$23.3 - 15.68\mu - 11.76 = 0.96 \Rightarrow 15.68\mu = 10.58$
	$\mu = 0.674744... = 0.675$ (3 s.f.)

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9 (d)

String slack, $T = 0$. N2L for A up the plane: $-\mu R - 2g \sin\theta = 2a$

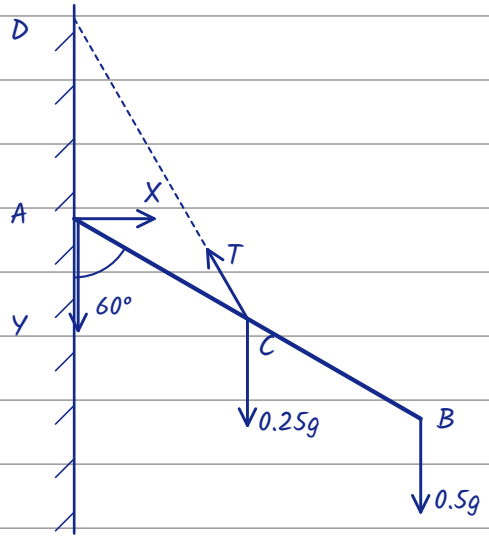
$-10.58 - 11.76 = 2a \Rightarrow a = -11.17\ m\ s^{-2}$

$v^2 = u^2 + 2as$: $0^2 = 1.2^2 + 2(-11.17)s$

$s = 1.44 / 22.34 = 0.064458...$

Distance = 0.0645 m (3 s.f.)

10 (a)



(force at A has components X and Y)

Triangle ACD: AC = AD = 0.4 m,

angle DAC = 120° ⇒ angle ACD = 30°

Moments about A (perpendicular distances from the vertical through A):

$0.25g \times 0.4 \sin 60° + 0.5g \times 0.8 \sin 60° = T \times 0.4 \sin 30°$

$0.5g \sin 60° = 0.2T \Rightarrow T = 2.5g \sin 60° = (49/4)\sqrt{3} = 21.2176...$

Tension = 21.2 N

10 (b)(i)

CD makes 30° with the vertical, so T has components T cos 60° horizontally (towards the wall) and T sin 60° vertically (upwards).

Horizontally: $X = T \cos 60° = 10.608...$

Vertically: $T \sin 60° = Y + 0.25g + 0.5g \Rightarrow Y = 18.375 - 7.35 = 11.025$

$|force| = \sqrt{(10.608...^2 + 11.025^2)} = 15.3002...$

Magnitude = 15.3 N

Question number	
10 (b)(ii)	$\tan\alpha = Y / X = 11.025 / 10.608... = 1.0392...$
	$\alpha = 46.102...°$
	Force at A acts at 46.1° below the horizontal (directed away from the wall).
10 (c)	Include the weight of the chain (model the chain as having mass).
11 (a)	Horizontally: $OC = Ut$
	Vertically: $s = ut + \frac{1}{2}at^2$: $0 = Vt - \frac{1}{2}gt^2 \Rightarrow t(V - \frac{1}{2}gt) = 0$
	$t \neq 0 \Rightarrow t = 2V/g$
	$OC = U \times 2V/g = 2UV/g$
11 (b)	Horizontal component at A: U
	Vertical: $v^2 = u^2 + 2as = V^2 - 2gh$, vertical component at A: $\sqrt{(V^2 - 2gh)}$
11 (c)	From A to B horizontal speed is constant, so time from A to B is $T = d/U$
	Vertically, A to B ($s = 0$): $0 = \sqrt{(V^2 - 2gh)}\,T - \frac{1}{2}gT^2 \Rightarrow \sqrt{(V^2 - 2gh)} - \frac{1}{2}gT = 0$
	$\sqrt{(V^2 - 2gh)} - \frac{1}{2}g(d/U) = 0$
	$d = (2U/g)\sqrt{(V^2 - 2gh)}$
11 (d)	At A: $\tan\theta$ = (vertical component)/(horizontal component): $\frac{1}{2} = \sqrt{(V^2 - 2gh)} / U$
	so $\sqrt{(V^2 - 2gh)} = U/2$. Then from (c): $d = (2U/g)(U/2) = U^2/g \Rightarrow U = \sqrt{(gd)}$
	$\sqrt{(gd)} = 2\sqrt{(V^2 - 2gh)} \Rightarrow gd = 4(V^2 - 2gh) = 4V^2 - 8gh$
	$V^2 = (gd + 8gh)/4$
	$V = \frac{1}{2}\sqrt{(\,g(8h + d)\,)}$