



Oxford Cambridge and RSA

Wednesday 14 October 2020 – Afternoon

A Level Mathematics A

H240/02 Pure Mathematics and Statistics

Time allowed: 2 hours



You must have:

- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **12** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae

A Level Mathematics A (H240)

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2, \quad \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Standard deviation

$$\sqrt{\frac{\sum(x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np , variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Section A: Pure MathematicsAnswer **all** the questions.

- 1 (a) Differentiate the following with respect to x .
- (i) $(2x + 3)^7$ [2]
- (ii) $x^3 \ln x$ [3]
- (b) Find $\int \cos 5x \, dx$. [2]
- (c) Find the equation of the curve through $(1, 3)$ for which $\frac{dy}{dx} = 6x - 5$. [2]
- 2 Simplify fully $\frac{2x^3 + x^2 - 7x - 6}{x^2 - x - 2}$. [4]
- 3 In this question you should assume that $-1 < x < 1$.
- (a) For the binomial expansion of $(1 - x)^{-2}$
- (i) find and simplify the first four terms, [2]
- (ii) write down the term in x^n . [1]
- (b) Write down the sum to infinity of the series $1 + x + x^2 + x^3 + \dots$. [1]
- (c) Hence or otherwise find and simplify an expression for $2 + 3x + 4x^2 + 5x^3 + \dots$ in the form $\frac{a - x}{(b - x)^2}$ where a and b are constants to be determined. [3]
- 4 **In this question you must show detailed reasoning.**
- Solve the equation $3 \sin^4 \phi + \sin^2 \phi = 4$, for $0 \leq \phi < 2\pi$, where ϕ is measured in radians. [5]
- 5 (a) Determine the set of values of n for which $\frac{n^2 - 1}{2}$ and $\frac{n^2 + 1}{2}$ are positive integers. [3]
- A 'Pythagorean triple' is a set of three positive integers a , b and c such that $a^2 + b^2 = c^2$.
- (b) Prove that, for the set of values of n found in part (a), the numbers n , $\frac{n^2 - 1}{2}$ and $\frac{n^2 + 1}{2}$ form a Pythagorean triple. [2]
- 6 Prove that $\sqrt{2} \cos(2\theta + 45^\circ) \equiv \cos^2 \theta - 2 \sin \theta \cos \theta - \sin^2 \theta$, where θ is measured in degrees. [3]

- 7 A and B are fixed points in the x - y plane. The position vectors of A and B are $\mathbf{a}$ and $\mathbf{b}$ respectively.

State, with reference to points A and B , the geometrical significance of

(a) the quantity $|\mathbf{a} - \mathbf{b}|$, [1]

(b) the vector $\frac{1}{2}(\mathbf{a} + \mathbf{b})$. [1]

The circle P is the set of points with position vector $\mathbf{p}$ in the x - y plane which satisfy

$$\left| \mathbf{p} - \frac{1}{2}(\mathbf{a} + \mathbf{b}) \right| = \frac{1}{2}|\mathbf{a} - \mathbf{b}|.$$

(c) State, in terms of $\mathbf{a}$ and $\mathbf{b}$,

(i) the position vector of the centre of P , [1]

(ii) the radius of P . [1]

It is now given that $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ and $\mathbf{p} = \begin{pmatrix} x \\ y \end{pmatrix}$.

(d) Find a cartesian equation of P . [4]

- 8 The rate of change of a certain population P at time t is modelled by the equation $\frac{dP}{dt} = (100 - P)$.

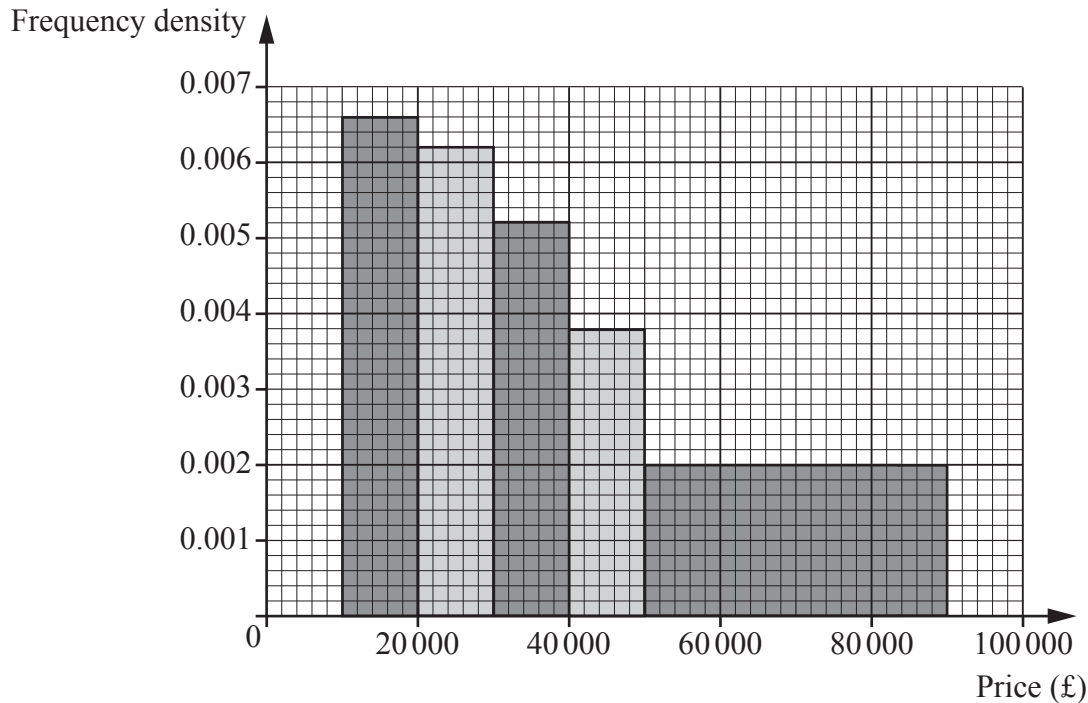
Initially $P = 2000$.

(a) Determine an expression for P in terms of t . [7]

(b) Describe how the population changes over time. [2]

Section B: Statistics
Answer **all** the questions.

- 9 The histogram shows information about the numbers of cars in five different price ranges, sold in one year at a car showroom.



It is given that 66 cars in the price range £10 000 to £20 000 were sold.

- (a) Find the number of cars sold in the price range £50 000 to £90 000. [1]
 - (b) State the units of the frequency density. [1]
 - (c) Suggest one change that the management could make to the diagram so that it would provide more information. [1]
 - (d) Estimate the number of cars sold in the price range £50 000 to £60 000. [1]
- 10 Pierre is a chef. He claims that 90% of his customers are satisfied with his cooking. Yvette suspects that Pierre is over-confident about the level of satisfaction amongst his customers. She talks to a random sample of 15 of Pierre's customers, and finds that 11 customers say that they are satisfied. She then performs a hypothesis test.

Carry out the test at the 5% significance level. [7]

- 11** As part of a research project, the masses, m grams, of a random sample of 1000 pebbles from a certain beach were recorded. The results are summarised in the table.

Mass (g)	$50 \leq m < 150$	$150 \leq m < 200$	$200 \leq m < 250$	$250 \leq m < 350$
Frequency	162	318	355	165

- (a)** Calculate estimates of the mean and standard deviation of these masses. [2]

The masses, x grams, of a random sample of 1000 pebbles on a different beach were also found. It was proposed that the distribution of these masses should be modelled by the random variable $X \sim N(200, 3600)$.

- (b)** Use the model to find $P(150 < X < 210)$. [1]

- (c)** Use the model to determine x_1 such that $P(160 < X < x_1) = 0.6$, giving your answer correct to **five** significant figures. [3]

It was found that the smallest and largest masses of the pebbles in this second sample were 112 g and 288 g respectively.

- (d)** Use these results to show that the model may not be appropriate. [1]

- (e)** Suggest a different value of a parameter of the model in the light of these results. [2]

- 12** In the past, the time for Jeff's journey to work had mean 45.7 minutes and standard deviation 5.6 minutes. This year he is trying a new route. In order to test whether the new route has reduced his journey time, Jeff finds the mean time for a random sample of 30 journeys using the new route. He carries out a hypothesis test at the 2.5% significance level.

Jeff assumes that, for the new route, the journey time has a normal distribution with standard deviation 5.6 minutes.

- (a)** State appropriate null and alternative hypotheses for the test. [2]

- (b)** Determine the rejection region for the test. [4]

13 Andy and Bev are playing a game.

- The game consists of three points.
- On each point, $P(\text{Andy wins}) = 0.4$ and $P(\text{Bev wins}) = 0.6$.
- If one player wins two consecutive points, then they win the game, otherwise neither player wins.

(a) Determine the probability of the following events.

(i) Andy wins the game. [2]

(ii) Neither player wins the game. [3]

Andy and Bev now decide to play a match which consists of a series of games.

- In each game, if a player wins the game then they win the match.
- If neither player wins the game then the players play another game.

(b) Determine the probability that Andy wins the match. [3]

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Turn over for question 14

- 14 Table 1 shows the numbers of usual residents in the age range 0 to 4 in 15 Local Authorities (LAs) in 2001 and 2011. The table also shows the increase in the numbers in this age group, and the same increase as a percentage.

	2001	2011	Increase	% Increase
Bolton	16 779	18 765	1 986	11.84%
Bury	11 117	12 235	1 118	10.06%
Knowsley	9 454	9 121	-333	-3.52%
Liverpool	24 840	26 099	1 259	5.07%
Manchester	24 693	36 413	11 720	47.46%
Oldham	15 196	16 491	1 295	8.52%
Rochdale	13 771	14 754	983	7.14%
Salford	12 529	16 255	3 726	29.74%
Sefton	14 896	14 601	-295	-1.98%
St. Helens	10 083	10 269	186	1.84%
Stockport	16 457	17 342	885	5.38%
Tameside	12 803	14 439	1 636	12.78%
Trafford	11 971	14 870	2 899	24.22%
Wigan	17 561	19 681	2 120	12.07%
Wirral	17 475	18 514	1 039	5.95%

Table 1

Fig. 2 shows the increase in each LA in raw numbers, and Fig. 3 shows the percentage increase in each LA.

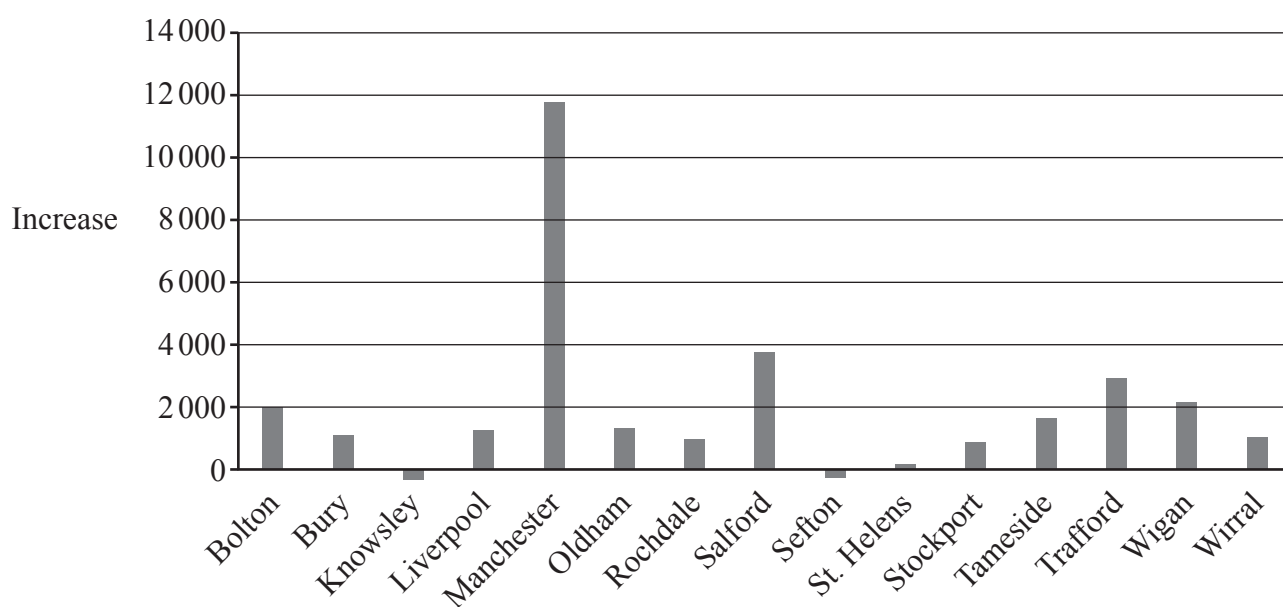


Fig. 2

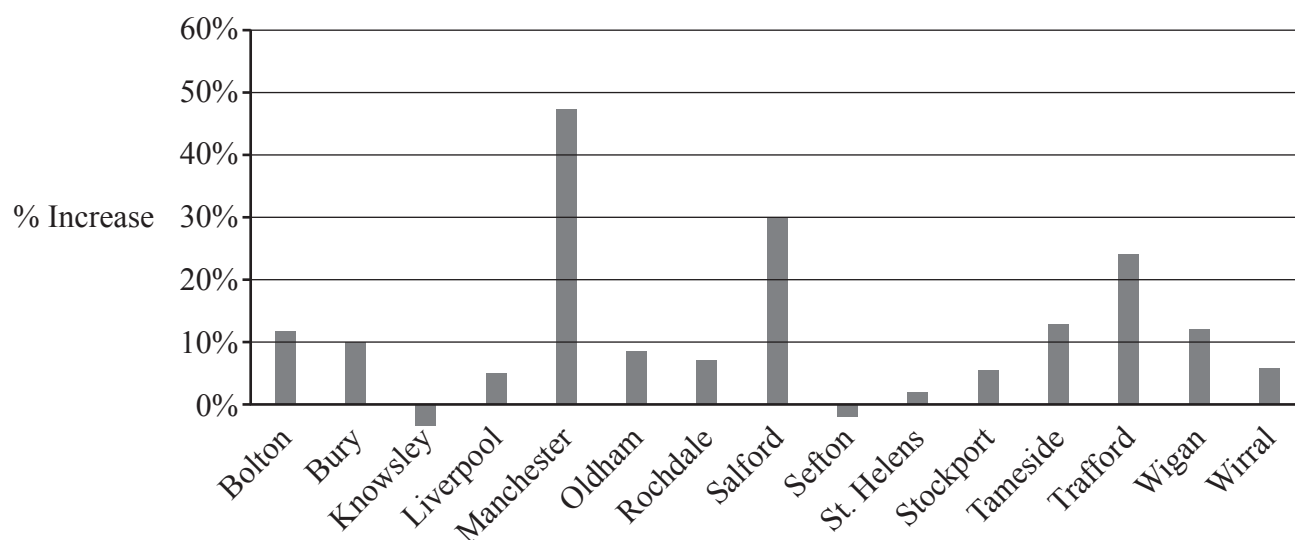


Fig. 3

- (a) The Education Committees in these LAs need to plan for the provision of schools for pupils in their districts.
- (i) Explain why, in this context, the increase is more important than the actual numbers. [1]
 - (ii) In which of the following LAs was there likely to have been the greatest need for extra teachers in the years following 2011: Bolton, Sefton, Tameside or Wigan?
Give a reason for your answer. [2]
 - (iii) State an assumption about the populations needed to make your answer in part (ii) valid. [1]
- (b) In two of the 15 LAs the proportion of young families is greater than in the other 13 LAs. Suggest, using only data from Fig. 2 and Fig. 3 and/or Table 1, which two LAs these are most likely to be. [2]

Turn over for question 15

15 In this question you must show detailed reasoning.

The random variable X has probability distribution defined as follows.

$$P(X = x) = \begin{cases} \frac{15}{64} \times \frac{2^x}{x!} & x = 2, 3, 4, 5, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Show that $P(X = 2) = \frac{15}{32}$. [1]

The values of three independent observations of X are denoted by X_1 , X_2 and X_3 .

- (b) Given that $X_1 + X_2 + X_3 = 9$, determine the probability that at least one of these three values is equal to 2. [6]

Freda chooses values of X at random until she has obtained $X = 2$ exactly three times. She then stops.

- (c) Determine the probability that she chooses exactly 10 values of X . [3]

END OF QUESTION PAPER

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