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# Wednesday 14 October 2020 – Afternoon

## A Level Mathematics A

### H240/02 Pure Mathematics and Statistics

**Time allowed: 2 hours**

*8232755751*

**You must have:**
- the Printed Answer Booklet
- a scientific or graphical calculator

**INSTRUCTIONS**
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

**ADVICE**
- Read each question carefully before you start your answer.

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# Formulae
# A Level Mathematics A (H240)

## Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

## Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

## Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

## Differentiation

| $\mathrm{f}(x)$ | $\mathrm{f}'(x)$ |
|---|---|
| $\tan kx$ | $k\sec^2 kx$ |
| $\sec x$ | $\sec x \tan x$ |
| $\cot x$ | $-\mathrm{cosec}^2 x$ |
| $\mathrm{cosec}\, x$ | $-\mathrm{cosec}\, x \cot x$ |

Quotient rule $y = \frac{u}{v}, \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

## Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

## Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

## Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where $\theta$ is measured in radians

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**Trigonometric identities**

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

**Numerical methods**

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

**Probability**

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

**Standard deviation**

$\sqrt{\dfrac{\Sigma(x-\overline{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \overline{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\overline{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \overline{x}^2}$

**The binomial distribution**

If $X \sim B(n, p)$ then $P(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, mean of $X$ is $np$, variance of $X$ is $np(1-p)$

**Hypothesis test for the mean of a normal distribution**

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0,1)$

**Percentage points of the normal distribution**

If $Z$ has a normal distribution with mean 0 and variance 1 then, for each value of $p$, the table gives the value of $z$ such that $P(Z \leqslant z) = p$.

| $p$ | 0.75 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 | 0.9975 | 0.999 | 0.9995 |
|---|---|---|---|---|---|---|---|---|---|
| $z$ | 0.674 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 2.807 | 3.090 | 3.291 |

**Kinematics**

| Motion in a straight line | Motion in two dimensions |
|---|---|
| $v = u + at$ | $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ |
| $s = ut + \frac{1}{2}at^2$ | $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ |
| $s = \frac{1}{2}(u+v)t$ | $\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$ |
| $v^2 = u^2 + 2as$ | |
| $s = vt - \frac{1}{2}at^2$ | $\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$ |

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## Section A: Pure Mathematics

Answer **all** the questions.

**1** **(a)** Differentiate the following with respect to $x$.

**(i)** $(2x+3)^7$ **[2]**

**(ii)** $x^3 \ln x$ **[3]**

**(b)** Find $\int \cos 5x \, dx$. **[2]**

**(c)** Find the equation of the curve through (1, 3) for which $\frac{dy}{dx} = 6x - 5$. **[2]**

**2** Simplify fully $\frac{2x^3 + x^2 - 7x - 6}{x^2 - x - 2}$. **[4]**

**3** In this question you should assume that $-1 < x < 1$.

**(a)** For the binomial expansion of $(1-x)^{-2}$

**(i)** find and simplify the first four terms, **[2]**

**(ii)** write down the term in $x^n$. **[1]**

**(b)** Write down the sum to infinity of the series $1 + x + x^2 + x^3 + \ldots$. **[1]**

**(c)** Hence or otherwise find and simplify an expression for $2 + 3x + 4x^2 + 5x^3 + \ldots$ in the form $\frac{a-x}{(b-x)^2}$ where $a$ and $b$ are constants to be determined. **[3]**

**4** **In this question you must show detailed reasoning.**

Solve the equation $3\sin^4\phi + \sin^2\phi = 4$, for $0 \leqslant \phi < 2\pi$, where $\phi$ is measured in radians. **[5]**

**5** **(a)** Determine the set of values of $n$ for which $\frac{n^2-1}{2}$ and $\frac{n^2+1}{2}$ are positive integers. **[3]**

A 'Pythagorean triple' is a set of three positive integers $a$, $b$ and $c$ such that $a^2 + b^2 = c^2$.

**(b)** Prove that, for the set of values of $n$ found in part **(a)**, the numbers $n$, $\frac{n^2-1}{2}$ and $\frac{n^2+1}{2}$ form a Pythagorean triple. **[2]**

**6** Prove that $\sqrt{2}\cos(2\theta + 45°) \equiv \cos^2\theta - 2\sin\theta\cos\theta - \sin^2\theta$, where $\theta$ is measured in degrees. **[3]**

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**7** $A$ and $B$ are fixed points in the $x$-$y$ plane. The position vectors of $A$ and $B$ are $\mathbf{a}$ and $\mathbf{b}$ respectively.

State, with reference to points $A$ and $B$, the geometrical significance of

**(a)** the quantity $|\mathbf{a}-\mathbf{b}|$, **[1]**

**(b)** the vector $\frac{1}{2}(\mathbf{a}+\mathbf{b})$. **[1]**

The circle $P$ is the set of points with position vector $\mathbf{p}$ in the $x$-$y$ plane which satisfy

$$\left|\mathbf{p}-\frac{1}{2}(\mathbf{a}+\mathbf{b})\right|=\frac{1}{2}|\mathbf{a}-\mathbf{b}|.$$

**(c)** State, in terms of $\mathbf{a}$ and $\mathbf{b}$,

**(i)** the position vector of the centre of $P$, **[1]**

**(ii)** the radius of $P$. **[1]**

It is now given that $\mathbf{a}=\begin{pmatrix}2\\-1\end{pmatrix}$, $\mathbf{b}=\begin{pmatrix}4\\5\end{pmatrix}$ and $\mathbf{p}=\begin{pmatrix}x\\y\end{pmatrix}$.

**(d)** Find a cartesian equation of $P$. **[4]**

**8** The rate of change of a certain population $P$ at time $t$ is modelled by the equation $\frac{\mathrm{d}P}{\mathrm{d}t}=(100-P)$.

Initially $P=2000$.

**(a)** Determine an expression for $P$ in terms of $t$. **[7]**

**(b)** Describe how the population changes over time. **[2]**

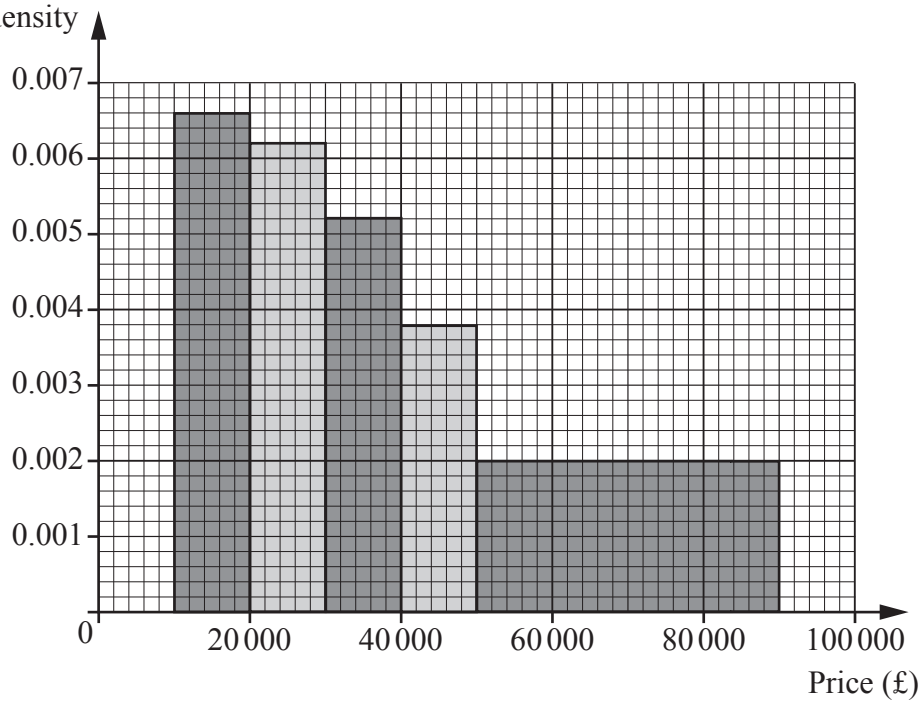


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## Section B: Statistics

Answer **all** the questions.

**9** The histogram shows information about the numbers of cars in five different price ranges, sold in one year at a car showroom.




It is given that 66 cars in the price range £10 000 to £20 000 were sold.

**(a)** Find the number of cars sold in the price range £50 000 to £90 000. **[1]**

**(b)** State the units of the frequency density. **[1]**

**(c)** Suggest one change that the management could make to the diagram so that it would provide more information. **[1]**

**(d)** Estimate the number of cars sold in the price range £50 000 to £60 000. **[1]**

**10** Pierre is a chef. He claims that 90% of his customers are satisfied with his cooking. Yvette suspects that Pierre is over-confident about the level of satisfaction amongst his customers. She talks to a random sample of 15 of Pierre's customers, and finds that 11 customers say that they are satisfied. She then performs a hypothesis test.

Carry out the test at the 5% significance level. **[7]**

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**11** As part of a research project, the masses, $m$ grams, of a random sample of 1000 pebbles from a certain beach were recorded. The results are summarised in the table.

| Mass (g) | $50 \leqslant m < 150$ | $150 \leqslant m < 200$ | $200 \leqslant m < 250$ | $250 \leqslant m < 350$ |
|---|---|---|---|---|
| Frequency | 162 | 318 | 355 | 165 |

**(a)** Calculate estimates of the mean and standard deviation of these masses. **[2]**

The masses, $x$ grams, of a random sample of 1000 pebbles on a different beach were also found. It was proposed that the distribution of these masses should be modelled by the random variable $X \sim \text{N}(200, 3600)$.

**(b)** Use the model to find $\text{P}(150 < X < 210)$. **[1]**

**(c)** Use the model to determine $x_1$ such that $\text{P}(160 < X < x_1) = 0.6$, giving your answer correct to **five** significant figures. **[3]**

It was found that the smallest and largest masses of the pebbles in this second sample were 112 g and 288 g respectively.

**(d)** Use these results to show that the model may not be appropriate. **[1]**

**(e)** Suggest a different value of a parameter of the model in the light of these results. **[2]**

**12** In the past, the time for Jeff's journey to work had mean 45.7 minutes and standard deviation 5.6 minutes. This year he is trying a new route. In order to test whether the new route has reduced his journey time, Jeff finds the mean time for a random sample of 30 journeys using the new route. He carries out a hypothesis test at the 2.5% significance level.

Jeff assumes that, for the new route, the journey time has a normal distribution with standard deviation 5.6 minutes.

**(a)** State appropriate null and alternative hypotheses for the test. **[2]**

**(b)** Determine the rejection region for the test. **[4]**



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**13** Andy and Bev are playing a game.

- The game consists of three points.
- On each point, P(Andy wins) = 0.4 and P(Bev wins) = 0.6.
- If one player wins two consecutive points, then they win the game, otherwise neither player wins.

**(a)** Determine the probability of the following events.

**(i)** Andy wins the game. **[2]**

**(ii)** Neither player wins the game. **[3]**

Andy and Bev now decide to play a match which consists of a series of games.

- In each game, if a player wins the game then they win the match.
- If neither player wins the game then the players play another game.

**(b)** Determine the probability that Andy wins the match. **[3]**

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**Turn over for question 14**



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**14** Table 1 shows the numbers of usual residents in the age range 0 to 4 in 15 Local Authorities (LAs) in 2001 and 2011. The table also shows the increase in the numbers in this age group, and the same increase as a percentage.

| | 2001 | 2011 | Increase | % Increase |
|---|---|---|---|---|
| **Bolton** | 16 779 | 18 765 | 1 986 | 11.84% |
| **Bury** | 11 117 | 12 235 | 1 118 | 10.06% |
| **Knowsley** | 9 454 | 9 121 | −333 | −3.52% |
| **Liverpool** | 24 840 | 26 099 | 1 259 | 5.07% |
| **Manchester** | 24 693 | 36 413 | 11 720 | 47.46% |
| **Oldham** | 15 196 | 16 491 | 1 295 | 8.52% |
| **Rochdale** | 13 771 | 14 754 | 983 | 7.14% |
| **Salford** | 12 529 | 16 255 | 3 726 | 29.74% |
| **Sefton** | 14 896 | 14 601 | −295 | −1.98% |
| **St. Helens** | 10 083 | 10 269 | 186 | 1.84% |
| **Stockport** | 16 457 | 17 342 | 885 | 5.38% |
| **Tameside** | 12 803 | 14 439 | 1 636 | 12.78% |
| **Trafford** | 11 971 | 14 870 | 2 899 | 24.22% |
| **Wigan** | 17 561 | 19 681 | 2 120 | 12.07% |
| **Wirral** | 17 475 | 18 514 | 1 039 | 5.95% |

**Table 1**

Fig. 2 shows the increase in each LA in raw numbers, and Fig. 3 shows the percentage increase in each LA.

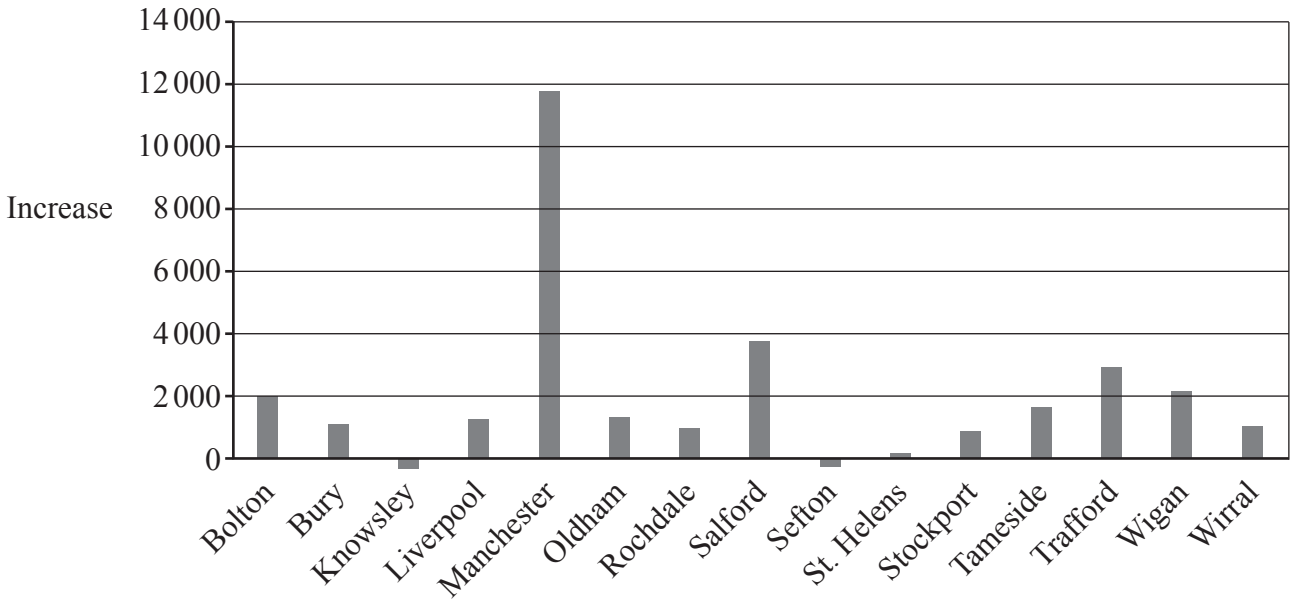



**Fig. 2**



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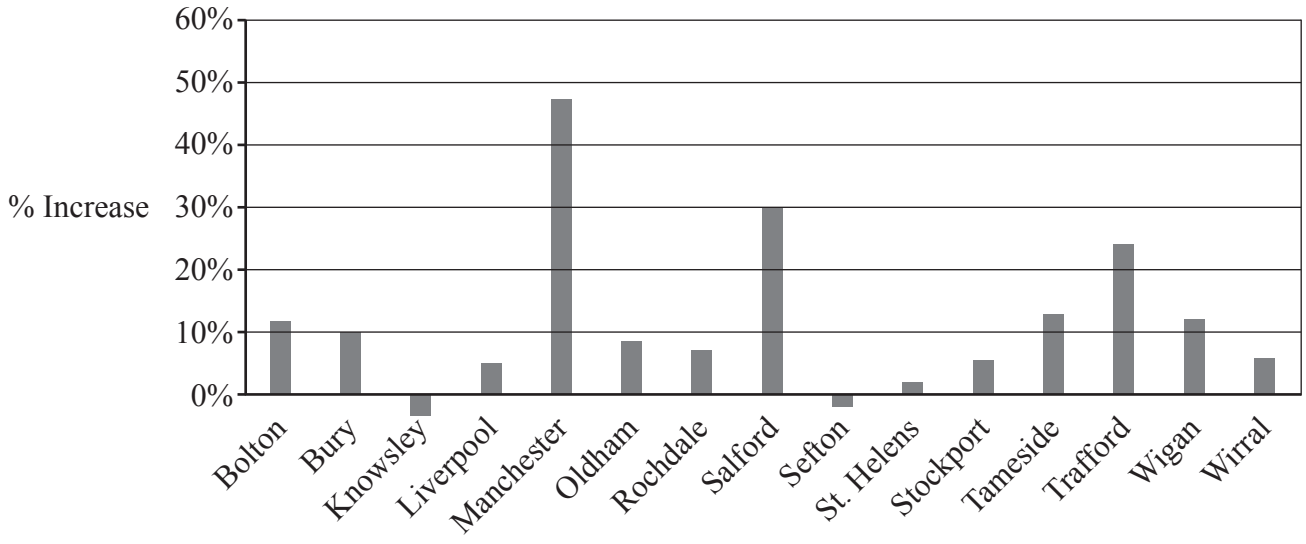



**Fig. 3**

**(a)** The Education Committees in these LAs need to plan for the provision of schools for pupils in their districts.

**(i)** Explain why, in this context, the increase is more important than the actual numbers. **[1]**

**(ii)** In which of the following LAs was there likely to have been the greatest need for extra teachers in the years following 2011: Bolton, Sefton, Tameside or Wigan?
Give a reason for your answer. **[2]**

**(iii)** State an assumption about the populations needed to make your answer in part **(ii)** valid. **[1]**

**(b)** In two of the 15 LAs the proportion of young families is greater than in the other 13 LAs.
Suggest, using only data from Fig. 2 and Fig. 3 and/or Table 1, which two LAs these are most likely to be. **[2]**

**Turn over for question 15**



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**15 In this question you must show detailed reasoning.**

The random variable $X$ has probability distribution defined as follows.

$$\mathrm{P}(X = x) = \begin{cases} \frac{15}{64} \times \frac{2^x}{x!} & x = 2, 3, 4, 5, \\ 0 & \text{otherwise.} \end{cases}$$

**(a)** Show that $\mathrm{P}(X = 2) = \frac{15}{32}$. **[1]**

The values of three independent observations of $X$ are denoted by $X_1$, $X_2$ and $X_3$.

**(b)** Given that $X_1 + X_2 + X_3 = 9$, determine the probability that at least one of these three values is equal to 2. **[6]**

Freda chooses values of $X$ at random until she has obtained $X = 2$ exactly three times. She then stops.

**(c)** Determine the probability that she chooses exactly 10 values of $X$. **[3]**

**END OF QUESTION PAPER**

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| Surname | StemUp Model Solution |
|---|---|
| Answer booklet | Write the question number in the left-hand margin at the start of each answer. |

Question number

**1 (a)(i)** $y = (2x + 3)^7 \Rightarrow \frac{dy}{dx} = 7(2x + 3)^6 \times 2 = 14(2x + 3)^6$

**1 (a)(ii)** $u = x^3$, $u' = 3x^2$; $v = \ln x$, $v' = \frac{1}{x}$

$\frac{dy}{dx} = 3x^2 \ln x + x^3 \times \frac{1}{x} = 3x^2 \ln x + x^2$

**1 (b)** $\int \cos 5x \, dx = \frac{1}{5}\sin 5x + c$

**1 (c)** $y = \int (6x - 5) \, dx = 3x^2 - 5x + c$

At $(1, 3)$: $3 = 3 - 5 + c \Rightarrow c = 5$

$y = 3x^2 - 5x + 5$

**2** Numerator: $f(x) = 2x^3 + x^2 - 7x - 6$, $f(-1) = -2 + 1 + 7 - 6 = 0$

so $(x + 1)$ is a factor: $2x^3 + x^2 - 7x - 6 = (x + 1)(2x^2 - x - 6)$

$= (x + 1)(x - 2)(2x + 3)$

Denominator: $x^2 - x - 2 = (x + 1)(x - 2)$

$\frac{2x^3 + x^2 - 7x - 6}{x^2 - x - 2} = \frac{(x + 1)(x - 2)(2x + 3)}{(x + 1)(x - 2)} = 2x + 3$

**3 (a)(i)** $(1 - x)^{-2} = 1 + (-2)(-x) + \frac{(-2)(-3)}{2!}(-x)^2 + \frac{(-2)(-3)(-4)}{3!}(-x)^3 + \ldots$

$= 1 + 2x + 3x^2 + 4x^3 + \ldots$

**3 (a)(ii)** Term in $x^n$: $(n + 1)x^n$

**3 (b)** $1 + x + x^2 + x^3 + \ldots = \frac{1}{1 - x}$ (geometric series, $r = x$, $|x| < 1$)

| Question number | |
|---|---|
| 3 (c) | $2 + 3x + 4x^2 + 5x^3 + \ldots = (1 + x + x^2 + x^3 + \ldots) + (1 + 2x + 3x^2 + 4x^3 + \ldots)$ |
| | $= \frac{1}{1-x} + \frac{1}{(1-x)^2} = \frac{(1-x)+1}{(1-x)^2} = \frac{2-x}{(1-x)^2}$ |
| | So $a = 2$ and $b = 1$ |
| 4 | Let $u = \sin^2 \varphi$: $3u^2 + u - 4 = 0$ |
| | $(3u + 4)(u - 1) = 0$ |
| | $\sin^2 \varphi = -\frac{4}{3}$ or $\sin^2 \varphi = 1$ |
| | $\sin^2 \varphi = -\frac{4}{3}$ is impossible since $\sin^2 \varphi \geq 0$ for all real $\varphi$ |
| | $\sin^2 \varphi = 1 \Rightarrow \sin \varphi = 1$ or $\sin \varphi = -1$ |
| | $\sin \varphi = 1 \Rightarrow \varphi = \frac{\pi}{2}$; $\sin \varphi = -1 \Rightarrow \varphi = \frac{3\pi}{2}$ |
| | $\varphi = \frac{\pi}{2}, \frac{3\pi}{2}$ |
| 5 (a) | For $\frac{n^2-1}{2}$ and $\frac{n^2+1}{2}$ to be integers, $n^2 \pm 1$ must be even, so $n^2$ is odd, |
| | i.e. $n$ is odd (taking $n$ to be a positive integer). |
| | For them to be positive we need $\frac{n^2-1}{2} > 0$, so $n^2 > 1$, i.e. $n > 1$. |
| | Set of values: $n$ is an odd integer with $n \geq 3$, i.e. $n = 3, 5, 7, \ldots$ |
| 5 (b) | $\frac{n^2-1}{2}$ and $\frac{n^2+1}{2}$ are positive integers from (a), and $n$ is a positive integer. |
| | $n^2 + \left(\frac{n^2-1}{2}\right)^2 = n^2 + \frac{n^4 - 2n^2 + 1}{4} = \frac{4n^2 + n^4 - 2n^2 + 1}{4}$ |
| | $= \frac{n^4 + 2n^2 + 1}{4} = \left(\frac{n^2+1}{2}\right)^2$ |
| | So $a^2 + b^2 = c^2$ with $a = n$, $b = \frac{n^2-1}{2}$, $c = \frac{n^2+1}{2}$, |
| | so they form a Pythagorean triple. |
| 6 | LHS $= \sqrt{2}\cos(2\theta + 45°) = \sqrt{2}(\cos 2\theta \cos 45° - \sin 2\theta \sin 45°)$ |
| | $= \sqrt{2} \times \frac{1}{\sqrt{2}}(\cos 2\theta - \sin 2\theta)$ since $\cos 45° = \sin 45° = \frac{1}{\sqrt{2}}$ |
| | $= \cos 2\theta - \sin 2\theta$ |
| | $= (\cos^2 \theta - \sin^2 \theta) - 2 \sin \theta \cos \theta$ |
| | $= \cos^2 \theta - 2 \sin \theta \cos \theta - \sin^2 \theta$ = RHS as required |

| Question number | |
|---|---|
| 7 (a) | $\lvert a - b \rvert$ is the length of AB (the distance from A to B). |
| 7 (b) | The vector $\frac{1}{2}(a + b)$ is the position vector of the midpoint of AB. |
| 7 (c)(i) | Centre: $\frac{1}{2}(a + b)$ |
| 7 (c)(ii) | Radius: $\frac{1}{2}\lvert a - b \rvert$ |
| 7 (d) | Centre $= \frac{1}{2}(a + b) = \frac{1}{2}\begin{pmatrix} 6 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ so centre is (3, 2) |
| | $a - b = \begin{pmatrix} -2 \\ -6 \end{pmatrix}$, $\lvert a - b \rvert = \sqrt{(4 + 36)} = \sqrt{40} = 2\sqrt{10}$ |
| | radius $r = \frac{1}{2} \times 2\sqrt{10} = \sqrt{10}$, so $r^2 = 10$ |
| | $(x - 3)^2 + (y - 2)^2 = 10$ |
| 8 (a) | $\int \frac{1}{100 - P}\, dP = \int 1\, dt$ |
| | $-\ln\lvert 100 - P \rvert = t + c$ |
| | $t = 0$, $P = 2000$: $-\ln\lvert 100 - 2000 \rvert = -\ln 1900 = c \Rightarrow c = -\ln 1900$ |
| | $\ln\lvert 100 - P \rvert = -t + \ln 1900$ |
| | $\frac{\lvert 100 - P \rvert}{1900} = e^{-t} \Rightarrow \lvert 100 - P \rvert = 1900e^{-t}$ |
| | Since $P = 2000 > 100$ initially (and P cannot cross 100), $\lvert 100 - P \rvert = P - 100$ |
| | $P - 100 = 1900e^{-t}$ |
| | $P = 1900e^{-t} + 100$ |
| 8 (b) | The population starts at 2000 and decreases (exponentially) over time, approaching 100 (it never falls below 100). |
| 9 (a) | Frequency = frequency density × class width = 0.002 × 40000 = 80 cars |

| Question number | |
|---|---|
| 9 (b) | Number of cars per £ (cars / £) |
| 9 (c) | Split the £50000 to £90000 class into four separate £10000 classes and show the frequency density of each. |
| 9 (d) | $0.002 \times 10000 = 20$ cars (or $\frac{80}{4} = 20$, assuming cars spread evenly) |
| 10 | Let $p$ = probability that a randomly chosen customer of Pierre is satisfied<br>$H_0$: $p = 0.9$ $H_1$: $p < 0.9$ (one-tailed test, 5% level)<br>Under $H_0$, $X \sim B(15, 0.9)$, where $X$ = number of the 15 customers satisfied<br>$P(X \leqslant 11) = 0.05556 = 0.0556$ (3 s.f.)<br>$0.0556 > 0.05$, so the result is not significant: do not reject $H_0$.<br>There is insufficient evidence at the 5% level that Pierre is over-confident,<br>i.e. insufficient evidence that fewer than 90% of his customers are satisfied. |
| 11 (a) | Midpoints: 100, 175, 225, 300; $n = 1000$<br>$\Sigma fx = 162\times100 + 318\times175 + 355\times225 + 165\times300 = 201225$<br>$\Sigma fx^2 = 44180625$<br>mean $= \frac{201225}{1000} = 201.225 = 201$ g (3 s.f.)<br>sd $= \sqrt{\frac{44180625}{1000} - 201.225^2} = 60.738\ldots = 60.7$ g (3 s.f.) |
| 11 (b) | $X \sim N(200, 60^2)$: $P(150 < X < 210) = 0.36385\ldots = 0.364$ (3 s.f.) |
| 11 (c) | $P(X < 160) = 0.25249\ldots$<br>$P(X < x_1) = 0.25249\ldots + 0.6 = 0.85249\ldots$<br>$x_1 = 200 + 60 \times \Phi^{-1}(0.85249\ldots) = 200 + 60 \times 1.04718\ldots = 262.831\ldots$<br>$x_1 = 262.83$ (5 s.f.) |

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|---|---|
| 11 (d) | All masses lie between 112 and 288, i.e. within 200 ± 88 (less than 1.5 sd). But the model gives $P(X < 112) = 0.0712$, so in a sample of 1000 about 71 pebbles would be expected below 112 g (and similarly above 288 g). None were, so the model may not be appropriate (its standard deviation is too large). |
| 11 (e) | Reduce the standard deviation. Take range ≈ 6σ (about 3 sd each side): $288 - 112 = 6\sigma \Rightarrow \sigma = \frac{176}{6} = 29.3\ldots$, so use $\sigma \approx 30$ $(\sigma^2 \approx 900)$ |
| 12 (a) | Let μ = mean journey time (minutes) for all journeys using the new route<br>$H_0$: $\mu = 45.7$ $H_1$: $\mu < 45.7$ |
| 12 (b) | Under $H_0$, $\bar{X} \sim N\left(45.7, \frac{5.6^2}{30}\right)$<br>$P(\bar{X} < a) = 0.025 \Rightarrow a = 45.7 - 1.95996 \times (5.6 \div \sqrt{30})$<br>$a = 43.696\ldots = 43.7$ (3 s.f.)<br>Rejection region: $\bar{X} < 43.7$ minutes |
| 13 (a)(i) | Andy wins with AA (first two points) or BAA<br>$P(\text{Andy wins}) = 0.4^2 + 0.6 \times 0.4^2 = 0.16 + 0.096 = 0.256$ |
| 13 (a)(ii) | Neither player wins only with ABA or BAB<br>$P(\text{neither}) = 0.4 \times 0.6 \times 0.4 + 0.6 \times 0.4 \times 0.6 = 0.096 + 0.144 = 0.24$ |
| 13 (b) | $P(\text{Andy wins match}) = 0.256 + 0.24 \times 0.256 + 0.24^2 \times 0.256 + \ldots$<br>geometric series, $a = 0.256$, $r = 0.24$: $= \frac{0.256}{1 - 0.24} = \frac{0.256}{0.76} = \frac{32}{95} = 0.337$ (3 s.f.) |
| 14 (a)(i) | The existing children are already catered for; the increase shows how many extra school places (and teachers) will be needed. |

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14 (a)(ii) Wigan. It has the greatest increase in the number of children (2120 extra), which is what determines the number of extra teachers needed.

14 (a)(iii) The children stay in the same LA (no great movement of families in or out after 2011), so they go to school in that LA.

14 (b) Manchester and Salford: they have by far the highest increases, both as percentages (47.46% and 29.74%) and in absolute numbers (11720 and 3726).

15 (a) $P(X = 2) = \frac{15}{64} \times \frac{2^2}{2!} = \frac{15}{64} \times \frac{4}{2} = \frac{30}{64} = \frac{15}{32}$ as required

15 (b) $P(X = 3) = \frac{15}{64} \times \frac{2^3}{3!} = \frac{5}{16}$, $P(X = 4) = \frac{15}{64} \times \frac{2^4}{4!} = \frac{5}{32}$

$P(X = 5) = \frac{15}{64} \times \frac{2^5}{5!} = \frac{1}{16}$

Sum 9 from: {2, 2, 5} (3 orders), {2, 3, 4} (6 orders), {3, 3, 3} (1 order)

$P(2,2,5 \text{ any order}) = 3 \times \left(\frac{15}{32}\right)^2 \times \frac{1}{16} = \frac{675}{16384}$

$P(2,3,4 \text{ any order}) = 6 \times \frac{15}{32} \times \frac{5}{16} \times \frac{5}{32} = \frac{2250}{16384}$

$P(3,3,3) = \left(\frac{5}{16}\right)^3 = \frac{125}{4096} = \frac{500}{16384}$

$P(\text{sum} = 9) = \frac{675 + 2250 + 500}{16384} = \frac{3425}{16384}$ (= 0.209045...)

$P(\text{at least one 2 and sum} = 9) = \frac{675 + 2250}{16384} = \frac{2925}{16384}$ (= 0.178528...)

$P(\text{at least one 2} \mid \text{sum} = 9) = \frac{2925}{3425} = \frac{117}{137} = 0.854$ (3 s.f.)

15 (c) Exactly two 2s in the first 9 values, and the 10th value is a 2:

$P = {}^9C_2 \times \left(\frac{15}{32}\right)^2 \times \left(\frac{17}{32}\right)^7 \times \frac{15}{32}$

$= 36 \times 0.2197... \times 0.01194... \times 0.46875 = 0.094466... \times 0.46875$

$= 0.044281... = 0.0443$ (3 s.f.)