OCR
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Thursday 19 June 2025 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours

188015377

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **16** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim\limits_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

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Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \,|\, A) = \mathrm{P}(B)\mathrm{P}(A \,|\, B)$ **or** $\mathrm{P}(A \,|\, B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

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Section A
Pure Mathematics

1 **(a)** Express $3x^2 - 12x + 17$ in the form $a(x-b)^2 + c$ where a, b and c are constants. **[3]**

(b) State the coordinates of the minimum point of the curve $y = 3x^2 - 12x + 17$. **[1]**

(c) State the equation of the normal to the curve $y = 3x^2 - 12x + 17$ at its minimum point. **[1]**

2 **(a)** The curve $y = \frac{1}{x^3}$ is translated by 4 units in the positive x-direction.

Write down the equation of the curve after it has been translated. **[2]**

(b) The curve $y = 5x^2$ is stretched parallel to the y-axis with scale factor 4.

The point on the curve $y = 5x^2$ with x-coordinate 3 is transformed to the point P.

Write down the coordinates of P. **[2]**

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3 **(a)** Sketch the graph of $y = |2x - 5|$ on the grid provided in the Printed Answer Booklet. **[2]**

(b) Solve the inequality $|2x - 5| < 1$. **[2]**

(c) **In this question you must show detailed reasoning.**

Hence find **all** the possible integer values N that satisfy the inequality

$|2e^{0.1N} - 5| < 1$. **[3]**

4 Functions f and g are defined for all real values of x by

$f(x) = \frac{x-k}{2}$ and $g(x) = x^2 + kx + 5$, where k is a constant.

You are given that the equation $f^{-1}g(x) = 4 - 2kx$ has real distinct roots.

(a) Show that k satisfies the inequality $2k^2 - k - 6 > 0$. **[5]**

(b) **In this question you must show detailed reasoning.**

Hence find the set of values of k. Give your answer in set notation. **[3]**

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5 **(a)** Use the substitution $u = \cos x$ to show that

$$\int \frac{1+\cos x}{\sin x}\,\mathrm{d}x = \ln(1-\cos x) + c.$$ **[4]**

Fig. 1

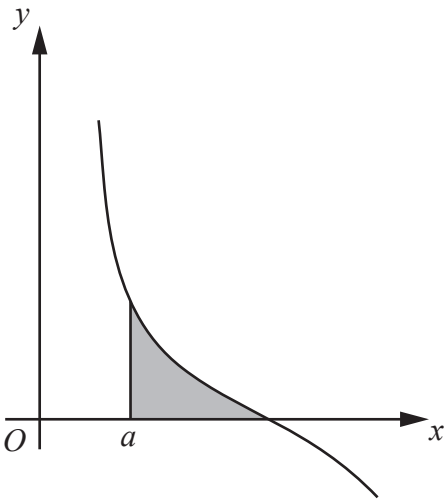


Fig. 1 shows part of the curve $y = \dfrac{1+\cos x}{\sin x}$.

The shaded region is bounded by the curve, the x-axis, and the line $x = a$.

You are given that the area of the shaded region is a square units.

(b) Show that the value of a satisfies the equation $a - \cos^{-1}(1-2\mathrm{e}^{-a}) = 0$. **[4]**

(c) Show by calculation that the value of a lies between 1.1 and 1.2. **[2]**

(d) Use the iterative formula

$$a_{n+1} = \cos^{-1}(1-2\mathrm{e}^{-a_n}),$$

with starting value $a_1 = 1.2$, to find the value of a correct to **2** decimal places. Show the result of each step of the iterative process. **[2]**

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Fig. 2

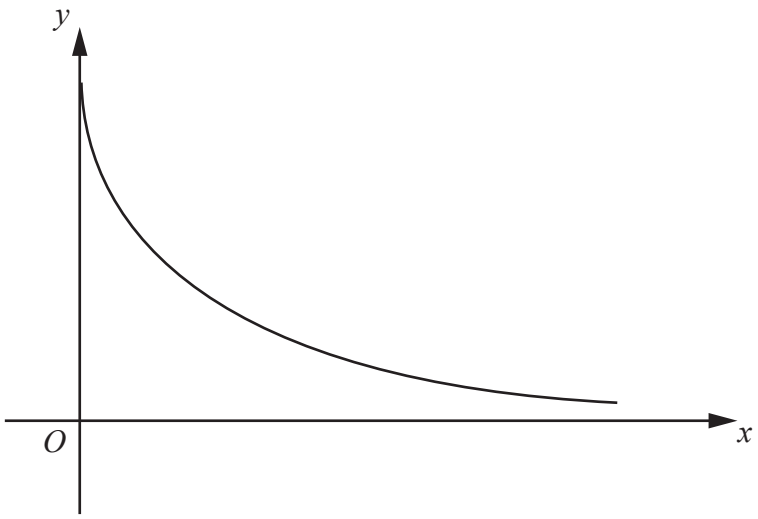


Fig. 2 shows the curve $y = \cos^{-1}(1 - 2e^{-x})$.

(e) Explain why the gradient of the curve $y = \cos^{-1}(1 - 2e^{-x})$ at the point where $x = a$, where a is the value found in part **(d)**, lies in the interval $(-1, 0)$. **[2]**

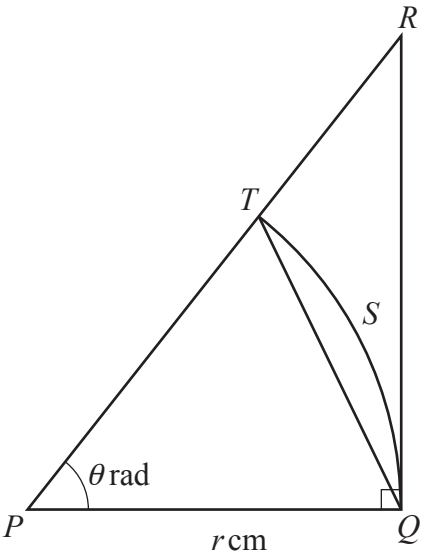
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6 The compound angle formulae for $\sin(A+B)$ and $\sin(A-B)$ are

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ and
$\sin(A-B) = \sin A\cos B - \cos A\sin B$.

(a) By letting $C = A + B$ and $D = A - B$, show that

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right).$$ **[1]**



The diagram shows a right-angled triangle PQR with $PQ = r$ cm. The angle QPR is θ radians. The diagram also shows the sector $PQST$ of a circle with centre P and radius r cm. The line segment QT is a chord of the sector $PQST$.

(b) By considering the areas of triangle PQT, sector $PQST$ and triangle PQR, show that

$$1 < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}.$$ **[4]**

(c) (i) Hence write down a similar inequality interval for the expression $\frac{\sin\theta}{\theta}$. **[1]**

(ii) Hence state $\lim\limits_{\theta\to 0}\frac{\sin\theta}{\theta}$. **[1]**

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(d) Using parts **(a)** and **(c)(ii)**, show from first principles that the derivative of $\sin x$ is $\cos x$, where x is measured in radians. **[4]**

A student attempts to use the result regarding the derivative of $\sin x$ to find the derivative of $\cos x$. The student's attempt is shown below.

Let	$y = \cos x$, where x is measured in radians.
	$y = \sin\left(\frac{\pi}{2} - x\right)$
so	$\frac{\mathrm{d}y}{\mathrm{d}x} = \cos\left(\frac{\pi}{2} - x\right)$
but	$\sin x \equiv \cos\left(\frac{\pi}{2} - x\right)$
therefore	$\frac{\mathrm{d}y}{\mathrm{d}x} = \sin x$.

(e) Identify the error made by the student. **[1]**

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Section B
Mechanics

7 A particle P of mass 3 kg is moving on a smooth horizontal surface under the action of two constant horizontal forces $(5\mathbf{i}+3\mathbf{j})$ N and $(a\mathbf{i}+3b\mathbf{j})$ N. The acceleration of P is $(2\mathbf{i}-3\mathbf{j})\,\text{m s}^{-2}$.

(a) Find the value of a and the value of b. **[3]**

At time $t = 0$ seconds the velocity of P is $\mathbf{u}\,\text{m s}^{-1}$ and at time $t = 5$ seconds the velocity of P is $(7\mathbf{i}-6\mathbf{j})\,\text{m s}^{-1}$.

(b) Find, in terms of **i** and **j**, an expression for **u**. **[2]**

8



Two particles A and B of masses 0.6 kg and 0.8 kg respectively, are attached to the ends of a light inextensible string. Particle A is held at rest at a fixed point and B hangs vertically below A.

Particle A is now released. As the particles fall the air resistance acting on A has a constant magnitude of R N and the air resistance acting on B has a constant magnitude of 0.3 N. The string remains taut throughout the subsequent motion (see diagram).

The downward acceleration of each of the particles is $9.2\,\text{m s}^{-2}$ and the tension in the string is T N.

(a) By applying Newton's second law to B, find the value of T. **[2]**

(b) Hence, or otherwise, find the value of R. **[2]**

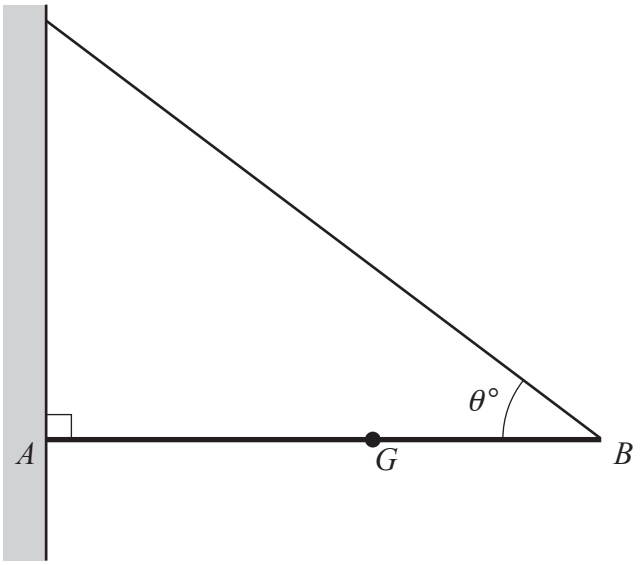
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9 **In this question you must show detailed reasoning.**

A particle moves in a straight line. Its velocity t seconds after leaving a fixed point on the line is $v\,\text{m s}^{-1}$ where $v = 2 + 3t + 0.6t^2$.

Find the distance travelled by the particle from time $t = 0$ until the instant when its acceleration is $15\,\text{m s}^{-2}$. **[6]**

10



The diagram shows a **non-uniform** rod AB of mass 4 kg and length 2 m. The end A of the rod rests against a rough vertical wall. The rod is held in a horizontal position, perpendicular to the wall, by a light inextensible string attached to the rod at B. The other end of the string is attached to the wall at a point vertically above A. The string is inclined at an angle of $\theta°$ to the horizontal, where $\sin\theta = \frac{4}{5}$.

The rod rests in equilibrium in a vertical plane perpendicular to the wall. The rod's weight acts at the point G on AB.

You are given that the magnitude of the moment of the rod's weight about A is 47.04 N m.

(a) Show that the distance AG is 1.2 m. **[1]**

(b) Find the tension in the string. **[2]**

(c) Determine the magnitude of the contact force exerted on the rod by the wall at A. **[5]**

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11 In this question you should take the acceleration due to gravity to be $10\,\text{m s}^{-2}$.

The unit vectors **i** and **j** are in a vertical plane with **i** horizontal and **j** vertically upwards.

A small ball P is projected from a point O on horizontal ground into the air. When P is in the air, it is to be modelled as a particle moving under the influence of gravity only.

At time $t = 2$ seconds, P has velocity $(2\mathbf{i} - 8\mathbf{j})\,\text{m s}^{-1}$.
The position vector of a point on the trajectory of P is $(x\mathbf{i} + y\mathbf{j})$m relative to O.

(a) Show that $y = 6x - 1.25x^2$. **[5]**

(b) Determine the maximum height of P above the ground during its motion. **[3]**

(c) Determine the following as P passes through the point on its trajectory where $x = 2.5$.

- the speed of P
- the direction of motion of P **[5]**

In reality P does not move only under the influence of gravity but is also subject to air resistance.

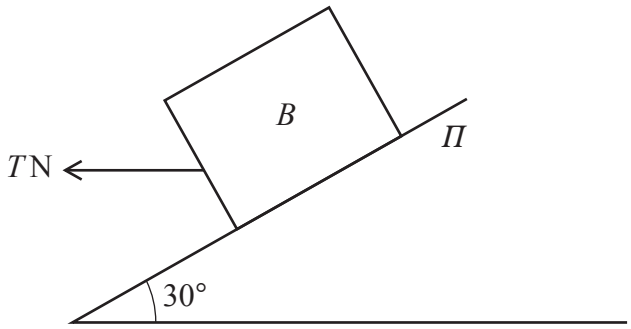
(d) Explain how this would affect your answer to part **(b)**. **[1]**

Turn over for question 12

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12

Fig. 1



A rectangular block B of mass 10 kg lies at rest in limiting equilibrium on a rough plane Π inclined at 30° to the horizontal. A horizontal force of magnitude T N, acting above a line of greatest slope, is applied to B (see **Fig. 1**).

The coefficient of friction between B and the plane is 0.8.

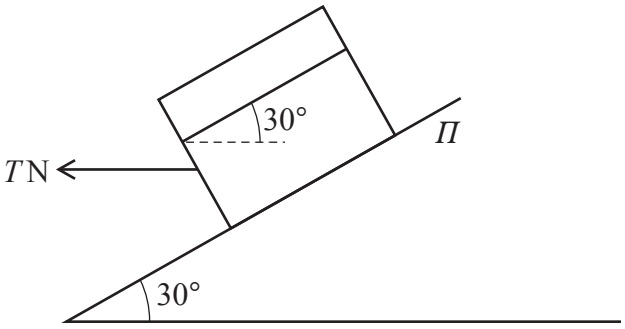
(a) Show that the value of T is 14.9, correct to **3** significant figures. **[6]**

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For the remainder of the question, you should you use this value of *T*.

Fig. 2



Block *B* is now cut at an angle of 30° to the horizontal into two smaller blocks. The upper block has a mass of 4 kg, and the lower block has a mass of 6 kg. The two blocks are held at rest with the lower block on Π. The horizontal force of magnitude T N is now applied to the lower block (see **Fig. 2**).

The two blocks are released from rest and in the subsequent motion the upper block starts to move with acceleration $3.5\,\text{m}\,\text{s}^{-2}$.

(b) Determine the coefficient of friction between the two blocks. **[4]**

(c) Show that the lower block does **not** move. **[3]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a) $3x^2 - 12x + 17 = 3(x^2 - 4x) + 17$

$= 3[(x - 2)^2 - 4] + 17$

$= 3(x - 2)^2 - 12 + 17$

$= 3(x - 2)^2 + 5$ $\quad (a = 3, b = 2, c = 5)$

1 (b) Minimum point $(2, 5)$

1 (c) Tangent at the minimum is horizontal, so the normal is vertical:

$x = 2$

2 (a) $y = \frac{1}{(x - 4)^3}$

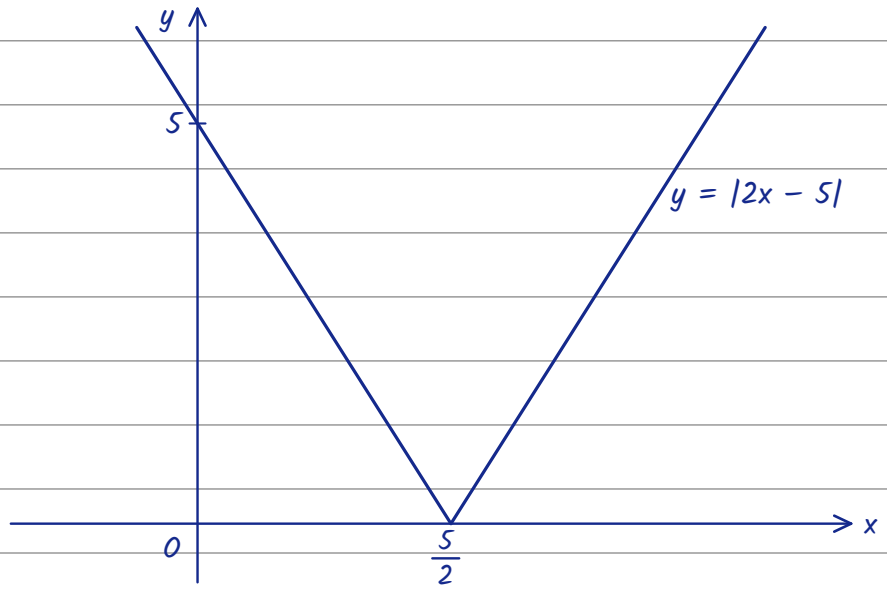
2 (b) On $y = 5x^2$: $x = 3$ gives $y = 5 \times 9 = 45$

Stretch parallel to the y-axis, s.f. 4: y-coordinate $\times 4 = 180$

P is $(3, 180)$

Question number

3 (a)



3 (b) $|2x - 5| < 1 \Rightarrow -1 < 2x - 5 < 1$

$4 < 2x < 6$

$2 < x < 3$

3 (c) Replace x by $e^{0.1N}$: $|2e^{0.1N} - 5| < 1 \Rightarrow 2 < e^{0.1N} < 3$

Take ln: $\ln 2 < 0.1N < \ln 3$

$10 \ln 2 < N < 10 \ln 3$

$6.9314... < N < 10.986...$

So N = 7, 8, 9 and 10

Question number

4 (a)

$y = (x - k)/2 \Rightarrow x = 2y + k$, so $f^{-1}(x) = 2x + k$

$f^{-1}g(x) = 2(x^2 + kx + 5) + k$

$2(x^2 + kx + 5) + k = 4 - 2kx$

$2x^2 + 2kx + 10 + k - 4 + 2kx = 0$

$2x^2 + 4kx + (6 + k) = 0$

Real distinct roots $\Rightarrow b^2 - 4ac > 0$:

$(4k)^2 - 4(2)(6 + k) > 0$

$16k^2 - 8k - 48 > 0$

$\div 8$: $2k^2 - k - 6 > 0$ as required

4 (b)

$2k^2 - k - 6 = 0 \Rightarrow (2k + 3)(k - 2) = 0$

Critical values $k = -3/2$, $k = 2$

Positive quadratic, so > 0 outside the roots: $k < -3/2$ or $k > 2$

$\{ k : k < -\frac{3}{2} \} \cup \{ k : k > 2 \}$

5 (a)

$u = \cos x \Rightarrow \frac{du}{dx} = -\sin x$, so $dx = \frac{du}{-\sin x}$

$\int \frac{1 + \cos x}{\sin x}\, dx = \int \frac{1 + \cos x}{\sin^2 x} \sin x\, dx = -\int \frac{1 + u}{1 - u^2}\, du$

$= -\int \frac{1 + u}{(1 - u)(1 + u)}\, du = -\int \frac{1}{1 - u}\, du$

$= -(-\ln(1 - u)) + c$ $\quad (1 - u > 0$ since $\cos x < 1)$

$= \ln(1 - \cos x) + c$

5 (b)

Curve meets the x-axis when $1 + \cos x = 0$, i.e. $x = \pi$

Area $= \int_a^{\pi} (1 + \cos x)/\sin x\, dx = [\ln(1 - \cos x)]_a^{\pi} = a$

$\ln(1 - \cos \pi) - \ln(1 - \cos a) = a$

Question number

$\ln 2 - \ln(1 - \cos a) = a \quad \Rightarrow \quad \ln \frac{2}{1 - \cos a} = a \quad \Rightarrow \quad \frac{2}{1 - \cos a} = e^a$

$1 - \cos a = 2e^{-a} \quad \Rightarrow \quad \cos a = 1 - 2e^{-a}$

$a = \cos^{-1}(1 - 2e^{-a})$

$\therefore \; a - \cos^{-1}(1 - 2e^{-a}) = 0$

5 (c) Let $f(a) = a - \cos^{-1}(1 - 2e^{-a})$

$f(1.1) = 1.1 - \cos^{-1}(1 - 2e^{-1.1}) = -0.1299... < 0$

$f(1.2) = 1.2 - \cos^{-1}(1 - 2e^{-1.2}) = 0.0381... > 0$

Change of sign (f continuous), so the root a lies between 1.1 and 1.2

5 (d) $a_1 = 1.2$

$a_2 = 1.16188...$ $a_3 = 1.18725...$ $a_4 = 1.17029...$

$a_5 = 1.18159...$ $a_6 = 1.17404...$ $a_7 = 1.17908...$

$a_8 = 1.17571...$ $a_9 = 1.17796...$

$a = 1.18$ (2 d.p.)

5 (e) The graph in Fig. 2 is decreasing for all $x > 0$, so the gradient of $y = \cos^{-1}(1 - 2e^{-x})$ is negative everywhere, so at $x = a$ it is < 0.

The iteration $a_{n+1} = F(a_n)$ with $F(x) = \cos^{-1}(1 - 2e^{-x})$ converged to a, so $|F'(a)| < 1$, i.e. the gradient at $x = a$ is between -1 and 1.

Together: the gradient at $x = a$ lies in the interval $(-1, 0)$.

6 (a) $C = A + B$ and $D = A - B \quad \Rightarrow \quad A = (C + D)/2, \quad B = (C - D)/2$

$\sin C - \sin D = \sin(A + B) - \sin(A - B)$

$= (\sin A \cos B + \cos A \sin B) - (\sin A \cos B - \cos A \sin B)$

$= 2 \cos A \sin B$

$\therefore \; \sin C - \sin D = 2 \cos\left(\frac{C + D}{2}\right) \sin\left(\frac{C - D}{2}\right)$

Question number

6 (b)

$QR = r \tan\theta$

Area of triangle $PQT = \frac{1}{2} r^2 \sin\theta$

Area of sector $PQST = \frac{1}{2} r^2 \theta$

Area of triangle $PQR = \frac{1}{2} \times r \times r \tan\theta = \frac{1}{2} r^2 \tan\theta$

Triangle $PQT \subset$ sector $PQST \subset$ triangle PQR, so

$\frac{1}{2} r^2 \sin\theta < \frac{1}{2} r^2 \theta < \frac{1}{2} r^2 \tan\theta$

$\sin\theta < \theta < \frac{\sin\theta}{\cos\theta}$ $\quad (\div \sin\theta$, which is > 0 for $0 < \theta < \pi/2)$

$\therefore \quad 1 < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}$

6 (c)(i)

Taking reciprocals: $\cos\theta < \frac{\sin\theta}{\theta} < 1$

6 (c)(ii)

As $\theta \to 0$, $\cos\theta \to 1$, so $\lim_{\theta \to 0} (\sin\theta)/\theta = 1$

6 (d)

$f(x) = \sin x$: $\frac{f(x+h) - f(x)}{h} = \frac{\sin(x+h) - \sin x}{h}$

Using (a) with $C = x + h$, $D = x$: $= \frac{2\cos((2x+h)/2)\sin(h/2)}{h}$

$= \cos\left(x + \frac{h}{2}\right) \times \frac{\sin(h/2)}{h/2}$

As $h \to 0$: $\cos(x + h/2) \to \cos x$, and by (c)(ii) with $\theta = h/2$,

$(\sin(h/2))/(h/2) \to 1$

$f'(x) = \lim_{h \to 0} \cos(x + h/2) \times \lim_{h \to 0} (\sin(h/2))/(h/2) = \cos x \times 1$

$\therefore \quad d/dx (\sin x) = \cos x$

Question number	
6 (e)	The line $\frac{dy}{dx} = \cos\left(\frac{\pi}{2} - x\right)$ is wrong: by the chain rule it should be
	$\frac{dy}{dx} = -\cos\left(\frac{\pi}{2} - x\right)$ (the student forgot the factor −1 from $d/dx(\pi/2 - x)$).
7 (a)	$F = ma$: $(5i + 3j) + (ai + 3bj) = 3(2i - 3j)$
	i: $5 + a = 6 \Rightarrow a = 1$
	j: $3 + 3b = -9 \Rightarrow b = -4$
7 (b)	$v = u + at$: $7i - 6j = u + 5(2i - 3j)$
	$u = 7i - 6j - 10i + 15j$
	$u = -3i + 9j$
8 (a)	N2L for B (down): $0.8g - 0.3 - T = 0.8 \times 9.2$
	$7.84 - 0.3 - T = 7.36$
	$T = 0.18$
8 (b)	N2L for A (down): $0.6g + T - R = 0.6 \times 9.2$
	$5.88 + 0.18 - R = 5.52$
	$R = 0.54$
9	$a = dv/dt = 3 + 1.2t$
	$3 + 1.2t = 15 \Rightarrow t = 10$
	$v = 2 + 3t + 0.6t^2 > 0$ for all $t \geq 0$, so distance = displacement
	$s = \int (2 + 3t + 0.6t^2)\, dt = 2t + (3/2)t^2 + 0.2t^3 \ (+ c)$
	Distance $= [2t + 1.5t^2 + 0.2t^3]_0^{10}$
	$= 2(10) + 1.5(10)^2 + 0.2(10)^3 - 0$
	$= 20 + 150 + 200$
	$= 370$ m

Question number	
10 (a)	Moments about A for the weight: $4g \times AG = 47.04$
	$AG = 47.04 \div 39.2 = 1.2$ m as required
10 (b)	Moments about A: $T \sin\theta \times 2 = 47.04$
	$2 \times (4/5)\, T = 47.04 \Rightarrow 1.6T = 47.04$
	$T = 29.4$ N
10 (c)	$\cos\theta = 3/5$. Let X (horizontal) and Y (vertical) be the components
	of the contact force at A.
	Horizontally: $X = T\cos\theta = 29.4 \times 3/5 = 17.64$ N
	Vertically: $Y + T\sin\theta = 4g \Rightarrow Y + 29.4 \times 4/5 = 39.2$
	$Y = 39.2 - 23.52 = 15.68$ N
	$\lvert\text{Contact force}\rvert = \sqrt{(17.64^2 + 15.68^2)} = 23.6015\ldots$
	$= 23.6$ N
11 (a)	Horizontal velocity is constant = 2, and $x = 0$ at $t = 0$: $x = 2t$
	Vertical: $v = u + at \Rightarrow -8 = u_y + (-10)(2) \Rightarrow u_y = 12$
	$y = u_y t + \frac{1}{2}at^2 = 12t - 5t^2$
	$t = x/2$: $y = 12(x/2) - 5(x/2)^2$
	$y = 6x - (5/4)x^2$
	$y = 6x - 1.25x^2$ as required
11 (b)	$dy/dx = 6 - 2.5x$
	Max height when $dy/dx = 0$: $6 - 2.5x = 0 \Rightarrow x = 2.4$
	$y = 6(2.4) - 1.25(2.4)^2 = 14.4 - 7.2$
	Maximum height = 7.2 m

Question number

11 (c)

At $x = 2.5$: $t = 2.5/2 = 1.25$ s

Horizontal velocity = 2, vertical velocity = $12 + (-10)(1.25) = -0.5$

Speed = $\sqrt{(2^2 + (-0.5)^2)} = \sqrt{4.25} = 2.0615\ldots = 2.06$ m s^{-1}

Direction: $dy/dx = 6 - 2.5(2.5) = -0.25$, $\tan\alpha = 0.5/2 = 0.25$

$\alpha = 14.036\ldots°$

P is moving at 14.0° below the horizontal (in the positive x-direction)

11 (d)

With air resistance the maximum height would be less than 7.2 m,

i.e. less than the answer to part (b).

12 (a)

Perpendicular to the plane: $R + T\sin 30 = 10g\cos 30$

Parallel to the plane (friction up the plane, limiting):

$F = T\cos 30 + 10g\sin 30$, and $F = 0.8R$

$T\cos 30 + 10g\sin 30 = 0.8(10g\cos 30 - T\sin 30)$

$T(\cos 30 + 0.8\sin 30) = 8g\cos 30 - 10g\sin 30$

$T = \dfrac{8g\cos 30 - 10g\sin 30}{\cos 30 + 0.8\sin 30} = \dfrac{40\sqrt{3} - 50}{5\sqrt{3} + 4}\ g = 14.9257\ldots$

$T = 14.9$ N (3 s.f.) as required

12 (b)

Upper block (4 kg), interface at 30° to the horizontal.

N2L down the slope: $4g\sin 30 - F_1 = 4 \times 3.5$

$19.6 - F_1 = 14 \Rightarrow F_1 = 5.6$ N

Perpendicular: $R_1 = 4g\cos 30 = 33.948\ldots$ N

Sliding, so $F_1 = \mu R_1$: $5.6 = \mu \times 4g\cos 30$

$\mu = 5.6 \div 33.948\ldots = 0.16495\ldots$

$\mu = 0.165$

Question number

12 (c)

Lower block (6 kg). Normal reaction from the plane:

$R_2 = R_1 + 6g \cos 30 - T \sin 30 = 10g \cos 30 - 14.9 \sin 30 = 77.42... \text{ N}$

$F_{max} = 0.8 \times 77.42... = 61.9 \text{ N}$

Forces down the plane on the lower block (F_1 from the upper block acts down the plane): $5.6 + 6g \sin 30 + 14.9 \cos 30 = 5.6 + 29.4 + 12.90...$ $= 47.9 \text{ N}$

$47.9 < 61.9$, so friction can hold the lower block in equilibrium: the lower block does not move.