OCR
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Thursday 12 June 2025 – Afternoon

A Level Mathematics A

H240/02 Pure Mathematics and Statistics

Time allowed: 2 hours

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient rule $y = \dfrac{u}{v}, \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

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Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\overline{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \overline{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\overline{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \overline{x}^2}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u+v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

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Section A
Pure Mathematics

1 **In this question you must show detailed reasoning.**

Solve the inequality $x^2 + 3x \leqslant 10$. **[3]**

2 **In this question you must show detailed reasoning.**

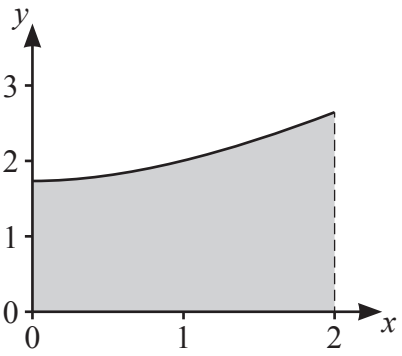
Solve the following equations.

(a) $(x^2 - 5)^{\frac{3}{2}} = 8$ **[3]**

(b) $e^{3y} = 2$ **[2]**

(c) $x^4 - 3x^2 - 4 = 0$ **[4]**

3 The diagram shows part of the curve with equation $y = \sqrt{x^2 + 3}$, between $x = 0$ and $x = 2$.



(a) **(i)** Use the trapezium rule, with intervals of 0.5, to determine an approximation to the area of the shaded region. **[3]**

(ii) Use rectangles with the same intervals as in part **(i)** to determine lower and upper bounds for the exact area of the shaded region. **[2]**

(b) Write an expression for the exact area of the shaded region in terms of a limit, involving x and intervals of width δx. **[2]**

4 **(a)** Prove that $(\cos\theta + \sin\theta)^2 \equiv 1 + \sin 2\theta$. **[2]**

(b) Hence or otherwise prove that $\sec 2\theta + \tan 2\theta \equiv \dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$. **[4]**

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5 **In this question you must show detailed reasoning.**

A circle has diameter PQ where P is $(-5, 1)$ and Q is $(5, 1)$. The line $x + 2y = 12$ meets the circle at A and B.

Find the exact length AB. **[8]**

6 A curve has equation $y = 3x^4 - 4x^3 - 6x^2 + 12x$.

(a) **(i)** Find $\frac{dy}{dx}$. **[2]**

(ii) Use your answer to part **(a)(i)** to find the x-coordinates of the stationary points on the curve. **[2]**

(b) Show that there is exactly **one** point on the curve which is a point of inflection but not a stationary point. **[3]**

7 An arithmetic progression has first term a and common difference d, where a and d are non-zero. The first, third and fourth terms of the arithmetic progression are consecutive terms of a geometric progression with common ratio r.

(a) **(i)** Show that $r = \frac{a + 2d}{a}$. **[1]**

(ii) Find d in terms of a. **[2]**

(b) Find the common ratio of the geometric progression. **[2]**

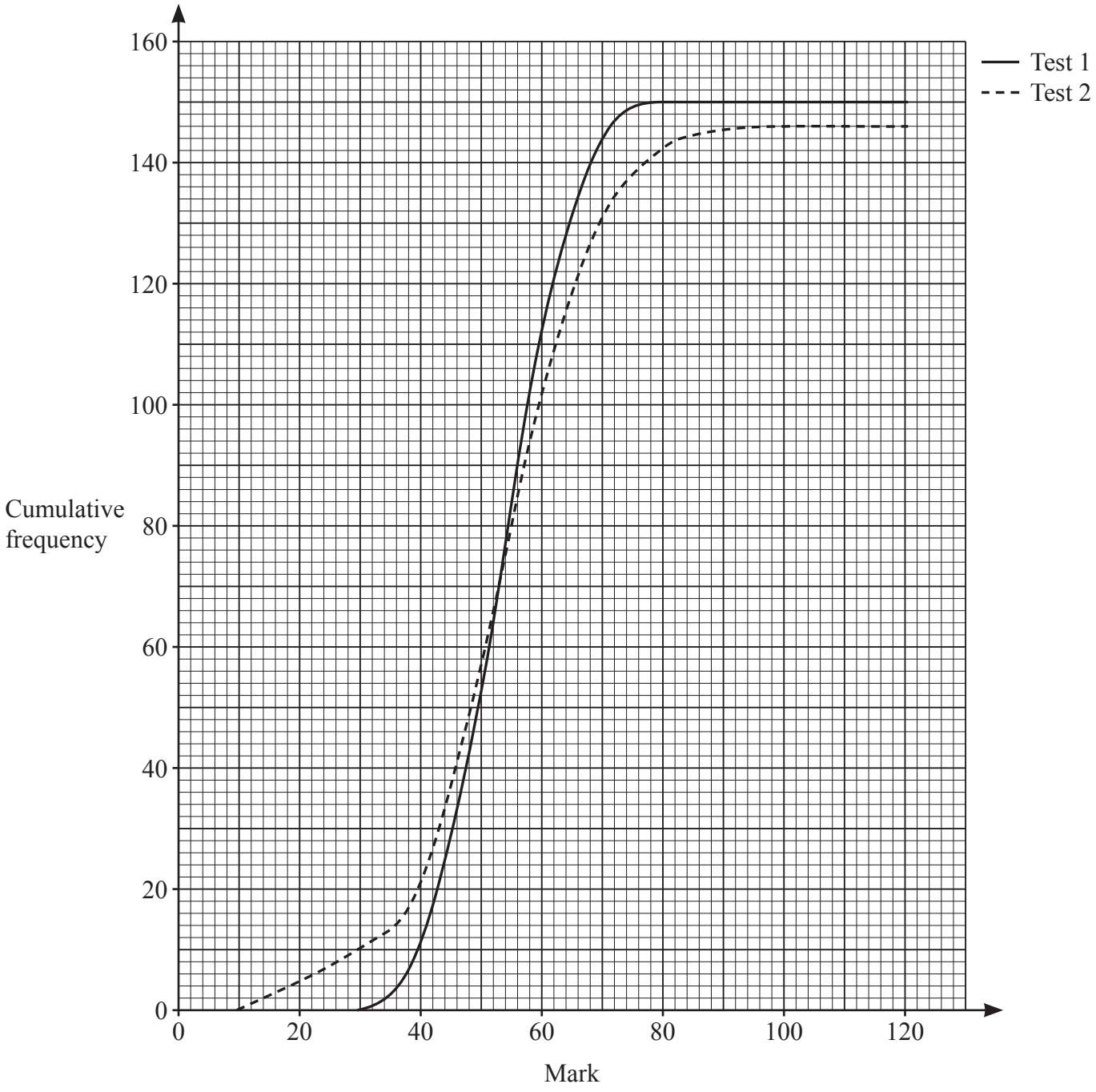
8 **In this question you must show detailed reasoning.**

Use integration by parts to find the exact value of $\int_0^{\frac{1}{2}} \ln(3 - 2x)\,dx$. **[5]**

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Section B
Statistics

9 Some students in a year-group took two tests, Test 1 and Test 2. The maximum mark in each test was 120.
The students' marks are illustrated in the cumulative frequency diagram. The diagram is reproduced in the Printed Answer Booklet.



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(a) All the students in the year-group took Test 1, but some students missed Test 2.

State the number of students who missed Test 2. **[1]**

(b) Find an estimate of the number of students whose marks for Test 1 were in the range 45 to 65 inclusive. **[2]**

(c) The final parts of the graphs are horizontal.

For Test 1, explain what this means about the results. **[1]**

(d) It is required that the number of students obtaining grade A* should be the same on both tests. The minimum mark for grade A* on Test 2 was 76.

Find the minimum mark for grade A* on Test 1. **[1]**

A teacher commented that the median marks on both tests were very similar. This suggests that, on average, the levels of difficulty on both tests were about the same.

(e) Use the cumulative frequency diagram to make another comparison between Test 1 and Test 2 about the easier questions on both papers. **[1]**

The marks for Test 1 are summarised in the table.

Test 1 mark	$\leqslant 29$	30–39	40–49	50–59	60–69	70–79	$\geqslant 80$
Frequency	0	10	40	60	33	7	0

(f) **(i)** Find estimates of the mean and standard deviation of the Test 1 marks. **[3]**

(ii) Use your answers to part **(f)(i)** to comment on whether there may be any outliers amongst the Test 1 marks. **[2]**

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10 **(a)** The random variable W has the distribution B(12, $\frac{1}{3}$).

Find P($W \geqslant 7$). **[2]**

(b) The random variable X has the distribution N(45, 4^2).

Find P($X > 50$). **[1]**

(c) The random variable Y has the distribution N(25, σ^2).

Given that P($Y < 30$) = 0.75, find σ. **[2]**

11 A train company claims that a particular daily service arrives on time on 95% of days. Riley suspects that the true percentage is less than 95%. Riley tests the company's claim by choosing a random sample of 50 days during the period November 2023 to March 2024. Riley finds that the service arrived on time on 44 of these days.

(a) Use a binomial distribution to test at the 5% significance level whether Riley's suspicion is justified. **[7]**

(b) By referring to one of the necessary assumptions for using a binomial distribution, suggest a reason why the conclusion may **not** be reliable. **[1]**

12 Sam has 9 cards, each with a different non-zero digit printed on it.

1	2	3	4	5	6	7	8	9

Sam chooses 5 cards at random and places them in a random order in a straight line, to form a 5-digit number.

(a) **(i)** Find the probability that the 5-digit number is greater than 50 000. **[1]**

(ii) Show that the probability that the 5-digit number is greater than 58 600 is $\frac{13}{28}$. **[4]**

(b) Given that the 5-digit number is greater than 50 000, determine the probability that it is greater than 58 600. **[2]**

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13 The table shows excerpts from the census data for Age Structure in 2001 and 2011 for four Local Authorities (LAs) in South West England.

		Age								
LA	**Year**	**0 to 4**	**5 to 7**	**8 to 9**	**10 to 14**	**15**	**16 to 17**	**18 to 19**	**20 to 24**	**25 to 29**
Bournemouth	2001	8 171	5 103	3 579	8 752	1 681	3 255	4 289	12 901	11 785
	2011	10 275	4 862	2 999	8 399	1 732	3 517	6 141	17 130	14 935
City of Bristol	2001	23 453	12 887	8 794	22 871	4 791	8 690	11 812	34 798	32 001
	2011	29 633	14 371	8 466	21 703	4 408	8 922	13 711	44 371	40 752
Plymouth	2001	13 213	8 535	6 091	16 078	3 106	6 103	7 454	17 245	14 885
	2011	15 336	7 956	4 991	13 645	2 954	6 011	8 839	24 343	18 888
Swindon	2001	11 392	7 194	4 862	12 121	2 178	4 337	3 798	10 212	13 816
	2011	14 083	7 551	4 722	12 433	2 593	5 141	4 690	12 859	15 075

Researchers want to investigate whether there is evidence that those people who were residents in these LAs in 2001 were still resident in the same LAs in 2011.

(a) One researcher suggests that, for each of the LAs, they should compare the sum of the data for 2001 for Age 15 to 19 with the data for 2011 for Age 25 to 29.

Explain why this comparison is relevant. **[1]**

(b) The census data also includes data for the following age ranges:

Age						
30 to 44	**45 to 59**	**60 to 64**	**65 to 74**	**75 to 84**	**85 to 89**	**90 +**

Explain why **none** of these data ranges are helpful in this context. **[1]**

(c) The data for one of these four LAs show that some of the 0 to 4 year olds living in that LA in 2001 are definitely no longer living there in 2011.

Explain which LA this is. **[1]**

(d) One of the researchers says that there has been little movement in or out of Bournemouth between 2001 and 2011 for those who were aged 5 to 7 in 2001.

(i) Use values from the table to show that this statement is **not** contradicted by the data. **[2]**

(ii) Explain why the statement is **not** necessarily correct. **[1]**

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14 The masses of stones on a certain beach are normally distributed with mean 0.36 kg and standard deviation 0.12 kg.

(a) Find the probability that two stones chosen at random from this beach both have masses between 0.3 kg and 0.4 kg. **[2]**

(b) Determine the probability that the mean mass of a random sample of 40 stones from this beach is less than 0.32 kg. **[2]**

Some geologists suspect that the mean mass, μ kg, of stones on a second beach is different from the mean for the first beach. They carry out a hypothesis test, at the 5% significance level, of the null hypothesis $\mu = 0.36$ against the alternative hypothesis $\mu \neq 0.36$.

They weigh each of a random sample of 40 stones on the second beach and find the sample mean, $\overline{X}$ kg, of their masses. You may assume that the standard deviation of the masses of stones on the second beach is 0.12 kg.

(c) Determine the range of values of $\overline{X}$ for which the null hypothesis would **not** be rejected. **[5]**

15 The probability distribution of the random variable X is modelled as follows.

- $P(X = 1) = p$ where p is a constant.
- $P(X = x) = 2P(X = x - 1)$ for $x = 2, 3, 4$.
- $P(X = x) = 0$ for all other values of x.

Three values, X_1, X_2 and X_3, of X are chosen at random.

Determine $P(X_3 > X_1 + X_2)$. **[4]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1

$x^2 + 3x - 10 \leq 0$

$(x - 2)(x + 5) \leq 0$

Critical values: $x = 2$ or $x = -5$

Positive x^2 coefficient, so the quadratic is ≤ 0 between the roots

$-5 \leq x \leq 2$

2 (a)

$(x^2 - 5)^{3/2} = 8$

$x^2 - 5 = 8^{2/3} = 4$

$x^2 = 9$

$x = \pm 3$

2 (b)

ln both sides: $3y = \ln 2$

$y = \frac{1}{3} \ln 2 \quad (= 0.231 \text{ to } 3 \text{ s.f.})$

2 (c)

Quadratic in x^2: let $u = x^2$

$u^2 - 3u - 4 = 0$

$(u - 4)(u + 1) = 0 \Rightarrow u = 4$ or $u = -1$

$x^2 = 4 \Rightarrow x = \pm 2$

$x^2 = -1$ has no real roots

$x = 2$ or $x = -2$

Question number

3 (a)(i) $h = 0.5$, $y = \sqrt{(x^2 + 3)}$

x:	0	0.5	1	1.5	2
y:	$\sqrt{3}$	$\sqrt{3.25}$	2	$\sqrt{5.25}$	$\sqrt{7}$
	1.7321	1.8028	2	2.2913	2.6458

Area $\approx \frac{1}{2} \times 0.5 \times \{\sqrt{3} + \sqrt{7} + 2(\sqrt{3.25} + 2 + \sqrt{5.25})\}$

$= 0.25 \times 16.5659... = 4.1414...$

Area ≈ 4.14 (3 s.f.)

3 (a)(ii) The curve is increasing for $0 \leq x \leq 2$, so:

Lower $= 0.5 \times (\sqrt{3} + \sqrt{3.25} + 2 + \sqrt{5.25}) = 0.5 \times 7.8261...$

$= 3.91$ (3 s.f.)

Upper $= 0.5 \times (\sqrt{3.25} + 2 + \sqrt{5.25} + \sqrt{7}) = 0.5 \times 8.7398...$

$= 4.37$ (3 s.f.)

$3.91 <$ exact area < 4.37

3 (b) $\text{Area} = \lim_{\delta x \to 0} \sum_{x=0}^{2} \sqrt{(x^2 + 3)}\, \delta x$

4 (a) $\text{LHS} \equiv (\cos\theta + \sin\theta)^2$

$\equiv \cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta$

$\equiv (\cos^2\theta + \sin^2\theta) + 2\sin\theta\cos\theta$

$\equiv 1 + \sin 2\theta \equiv \text{RHS}$

$\therefore (\cos\theta + \sin\theta)^2 \equiv 1 + \sin 2\theta$

Question number

4 (b)

$$\text{LHS} \equiv \frac{1}{\cos 2\theta} + \frac{\sin 2\theta}{\cos 2\theta} \equiv \frac{1 + \sin 2\theta}{\cos 2\theta}$$

Using (a), and $\cos 2\theta \equiv \cos^2\theta - \sin^2\theta$:

$$\equiv \frac{(\cos\theta + \sin\theta)^2}{\cos^2\theta - \sin^2\theta}$$

$$\equiv \frac{(\cos\theta + \sin\theta)^2}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}$$

$$\equiv \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} \equiv \text{RHS}$$

5

Centre = midpoint of PQ = (0, 1)

Radius = ½ × PQ = ½ × 10 = 5

Circle: $x^2 + (y - 1)^2 = 25$

Line: $x = 12 - 2y$

$(12 - 2y)^2 + (y - 1)^2 = 25$

$144 - 48y + 4y^2 + y^2 - 2y + 1 = 25$

$5y^2 - 50y + 120 = 0$

$y^2 - 10y + 24 = 0$

$(y - 4)(y - 6) = 0 \Rightarrow y = 4$ or $y = 6$

$y = 4$: $x = 12 - 8 = 4$; $y = 6$: $x = 12 - 12 = 0$

A(4, 4) and B(0, 6)

$AB^2 = (4 - 0)^2 + (4 - 6)^2 = 16 + 4 = 20$

$AB = \sqrt{20} = 2\sqrt{5}$

6 (a)(i)

$$\frac{dy}{dx} = 12x^3 - 12x^2 - 12x + 12$$

Question number

6 (a)(ii) Stationary points: $12x^3 - 12x^2 - 12x + 12 = 0$

$x^3 - x^2 - x + 1 = 0$

$x^2(x - 1) - (x - 1) = 0$

$(x - 1)(x^2 - 1) = (x - 1)^2(x + 1) = 0$

$x = 1$ and $x = -1$

6 (b) $\frac{d^2y}{dx^2} = 36x^2 - 24x - 12$

Points of inflection: $36x^2 - 24x - 12 = 0$

$12(3x^2 - 2x - 1) = 0 \Rightarrow 12(3x + 1)(x - 1) = 0$

$x = 1$ or $x = -\frac{1}{3}$ (distinct roots, so d^2y/dx^2 changes sign at each)

$x = 1$ is a stationary point (from (a)(ii)), so it is not counted.

At $x = -\frac{1}{3}$: $\frac{dy}{dx} = 12(x - 1)^2(x + 1) = \frac{128}{9} \neq 0$

So only $x = -\frac{1}{3}$ is a point of inflection that is not stationary:

exactly one such point.

7 (a)(i) AP terms: a, $a + d$, $a + 2d$, $a + 3d$

3rd term of AP is $a + 2d$, the 2nd term of the GP: $ar = a + 2d$

$\therefore\ r = \frac{a + 2d}{a}$

Question number

7 (a)(ii) 4th term: $ar^2 = a + 3d \Rightarrow \frac{a + 3d}{a + 2d} = \frac{a + 2d}{a}$

$a(a + 3d) = (a + 2d)^2$

$a^2 + 3ad = a^2 + 4ad + 4d^2$

$0 = 4d^2 + ad = d(4d + a)$

$d \neq 0$, so $d = -\frac{a}{4}$

7 (b) $r = \frac{a + 2d}{a} = \frac{a - \frac{1}{2}a}{a} = \frac{1}{2}$

8 $u = \ln(3 - 2x)$, $\frac{dv}{dx} = 1 \Rightarrow \frac{du}{dx} = \frac{-2}{3 - 2x}$, $v = x$

$\int \ln(3 - 2x)\, dx = x \ln(3 - 2x) - \int \frac{-2x}{3 - 2x}\, dx$

$= x \ln(3 - 2x) + \int \left(-1 + \frac{3}{3 - 2x}\right) dx$

$= x \ln(3 - 2x) - x - \frac{3}{2}\ln(3 - 2x)$

$\int_0^{1/2} \ln(3 - 2x)\, dx = \left[x \ln(3 - 2x) - x - \frac{3}{2}\ln(3 - 2x)\right]_0^{1/2}$

$= \left(\frac{1}{2}\ln 2 - \frac{1}{2} - \frac{3}{2}\ln 2\right) - \left(0 - 0 - \frac{3}{2}\ln 3\right)$

$= \frac{3}{2}\ln 3 - \ln 2 - \frac{1}{2}$

9 (a) 150 took Test 1, 146 took Test 2

$150 - 146 = 4$

Question number	
9 (b)	Test 1: cf at 45 ≈ 28, cf at 65 ≈ 130 130 − 28 = 102 students
9 (c)	No student scored more than about 78 marks on Test 1 (the cumulative frequency reaches 150 at a mark of about 78).
9 (d)	Test 2: cf at 76 ≈ 141, so 146 − 141 = 5 students got A* Test 1: 5 students needed, so cf = 150 − 5 = 145 cf of 145 on Test 1 at a mark of about 69 Minimum mark for A* on Test 1 ≈ 69
9 (e)	The easier questions on Test 1 were easier than the easier questions on Test 2: the lowest marks on Test 1 (about 30) are much higher than on Test 2 (about 10), so weaker students scored better on Test 1.
9 (f)(i)	Midpoints: 34.5, 44.5, 54.5, 64.5, 74.5; $\Sigma f = 150$ $\Sigma fx = 34.5\times10 + 44.5\times40 + 54.5\times60 + 64.5\times33 + 74.5\times7$ $= 8045$ $\bar{x} = \frac{8045}{150} = 53.633\ldots = 53.6$ (3 s.f.) $\Sigma fx^2 = 445467.5$ $s = \sqrt{\left(\frac{445467.5}{150} - 53.633\ldots^2\right)} = \sqrt{93.249\ldots} = 9.6565\ldots$ $s = 9.66$ (3 s.f.)
9 (f)(ii)	$\bar{x} - 2s = 53.6 - 2 \times 9.66 = 34.3$ $\bar{x} + 2s = 53.6 + 2 \times 9.66 = 72.9$ Marks in the 30–39 class may be below 34.3 and marks in the 70–79 class may be above 72.9, so there may be outliers at both ends.

Question number

10 (a) $W \sim B(12, \frac{1}{3})$

$P(W \geqslant 7) = 1 - P(W \leqslant 6) = 1 - 0.93355...$

$= 0.066447...$ $= 0.0664$ (3 s.f.)

10 (b) $P(X > 50) = 0.10564...$ $= 0.106$ (3 s.f.)

10 (c) $\frac{30 - 25}{\sigma} = \Phi^{-1}(0.75) = 0.67449...$

$\sigma = \frac{5}{0.67449...} = 7.4130...$ $= 7.41$ (3 s.f.)

11 (a) H_0: $p = 0.95$ H_1: $p < 0.95$

where p is the proportion of days on which the service arrives on time

Under H_0, $X \sim B(50, 0.95)$, X = number of days on time

Observed $X = 44$

$P(X \leqslant 44) = 0.03777...$ $= 0.0378$

$0.0378 < 0.05$, so reject H_0

There is sufficient evidence at the 5% level to suggest that the proportion of days the service arrives on time is less than 0.95.

11 (b) The probability of arriving on time may not be constant: all the days are in November to March, when bad weather may cause more delays, so p may not be the same on every day (or across the year).

12 (a)(i) First card must be 5, 6, 7, 8 or 9

$P(> 50000) = \frac{5}{9}$

Question number

12 (a)(ii) Number > 58600 if:

first digit 6, 7, 8 or 9: $\frac{4}{9}$

first digits 59: $\frac{1}{9} \times \frac{1}{8} = \frac{1}{72}$

first digits 58, then 6, 7 or 9: $\frac{1}{9} \times \frac{1}{8} \times \frac{3}{7} = \frac{3}{504} = \frac{1}{168}$

$P(> 58600) = \frac{4}{9} + \frac{1}{72} + \frac{1}{168} = \frac{224 + 7 + 3}{504} = \frac{234}{504} = \frac{13}{28}$

12 (b) Every number > 58600 is also > 50000, so

$P(> 58600 \mid > 50000) = \frac{13/28}{5/9} = \frac{13}{28} \times \frac{9}{5} = \frac{117}{140}$ $(= 0.836)$

13 (a) People aged 15 to 19 in 2001 would be aged 25 to 29 in 2011 (ten years later), so this compares the same group of people (the same cohort).

13 (b) The only two ranges exactly 10 years apart are 65 to 74 and 75 to 84, and numbers in these will fall because of deaths (ageing population), so they cannot show whether people stayed; in the 15-year ranges (30 to 44, 45 to 59) only some people move to the next range after 10 years.

13 (c) City of Bristol: those aged 0 to 4 in 2001 are aged 10 to 14 in 2011, but 21703 (2011, 10 to 14) < 23453 (2001, 0 to 4), so the number has decreased and some of them must have left.

Question number

13 (d)(i) Aged 5 to 7 in 2001 → aged 15 to 17 in 2011

2001: 5103 2011: 1732 + 3517 = 5249

5103 → 5249, only a small increase: little change in overall numbers,

so the statement is not contradicted by the data.

13 (d)(ii) The totals could stay similar even if many individuals moved out and

were replaced by others moving in, so they may not be the same people.

14 (a) $M \sim N(0.36, 0.12^2)$

$P(0.3 < M < 0.4) = 0.32202...$

$P(\text{both}) = 0.32202...^2 = 0.10369... = 0.104$ (3 s.f.)

14 (b) $\bar{X} \sim N\left(0.36, \frac{0.12^2}{40}\right)$

$P(\bar{X} < 0.32) = 0.017507... = 0.0175$ (3 s.f.)

14 (c) Under H_0: $\bar{X} \sim N\left(0.36, \frac{0.12^2}{40}\right)$, 2.5% in each tail

Lower: $a = 0.36 - 1.96 \times \frac{0.12}{\sqrt{40}} = 0.32281...$

Upper: $b = 0.36 + 1.96 \times \frac{0.12}{\sqrt{40}} = 0.39718...$

H_0 not rejected for $0.323 < \bar{X} < 0.397$

Question number

15

$p + 2p + 4p + 8p = 1 \Rightarrow 15p = 1 \Rightarrow p = \frac{1}{15}$

$P(X = 1), P(X = 2), P(X = 3), P(X = 4) = \frac{1}{15}, \frac{2}{15}, \frac{4}{15}, \frac{8}{15}$

$X_3 > X_1 + X_2$ needs $(X_1, X_2, X_3) = (1, 1, 3), (1, 1, 4), (1, 2, 4), (2, 1, 4)$

$P = (\frac{1}{15})^2 \times \frac{4}{15} + (\frac{1}{15})^2 \times \frac{8}{15} + 2 \times \frac{1}{15} \times \frac{2}{15} \times \frac{8}{15}$

$= \frac{4 + 8 + 32}{3375} = \frac{44}{3375}$