



Oxford Cambridge and RSA

Wednesday 4 June 2025 – Afternoon

A Level Mathematics A

H240/01 Pure Mathematics

Time allowed: 2 hours

You must have:

- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ ms}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae

A Level Mathematics A (H240)

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2, \quad \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Standard deviation

$$\sqrt{\frac{\sum(x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np , variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

1 Express each of the following in the form px^q , where p and q are constants.

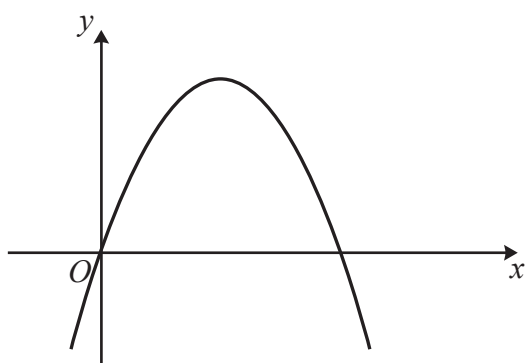
(a) $\frac{2}{\sqrt[4]{x}}$ [1]

(b) $(5x\sqrt{x})^3$ [2]

(c) $\sqrt{2x^3} \times \sqrt{8x^5}$ [2]

(d) $x^5(27x^6)^{\frac{1}{3}}$ [3]

2



The diagram shows the curve $y = ax(x - b)$, where a and b are constants and $b > 0$.

(a) Given that the curve has a stationary point at $x = 3$, state the value of b . [1]

(b) Given also that the stationary point at $x = 3$ is a maximum, state what can be deduced about the value of a . [1]

(c) Find the y -coordinate of the stationary point, giving your answer in terms of a . [1]

(d) State the range of values of x for which the curve is increasing. [1]

3 (a) A sequence has terms $u_1, u_2, u_3, \dots$ defined by $u_1 = 3$ and $u_{n+1} = u_n^2 - 5$ for $n \geq 1$.

(i) Find the values of u_2, u_3 and u_4 . [2]

(ii) Describe the behaviour of the sequence. [1]

(b) The second, third and fourth terms of a geometric progression are 12, 8 and $\frac{16}{3}$.

Determine the sum to infinity of this geometric progression. [3]

- 4 The position vector of the point A is $0.5\mathbf{i} - 0.5\mathbf{j}$.
- (a) Determine whether $0.5\mathbf{i} - 0.5\mathbf{j}$ is a unit vector. [2]
- (b) Find the direction of $0.5\mathbf{i} - 0.5\mathbf{j}$, giving your answer in degrees. [2]

The position vector of the point B is $3.5\mathbf{i} + c\mathbf{j}$, where c is a constant.

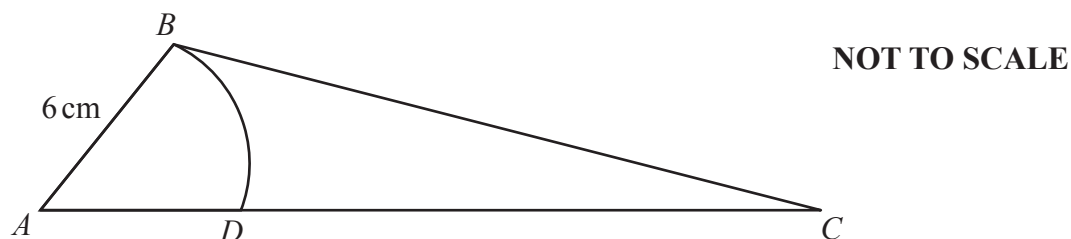
- (c) Given that $|\overrightarrow{AB}| = 5$, determine the possible values of c . [3]

- 5 Scientists are comparing h , the average height of a child in cm, with t , the age of the child in years. They suggest the model $h = at + b$ for $t \geq 2$, where a and b are constants.

They find that the average height of a 2 year old child is 87 cm and the average height of a 5 year old child is 108 cm.

- (a) Find the values of a and b that are consistent with the scientists' findings. [4]
- (b) (i) Sam is 4 years old.
- Use the model to predict Sam's height. [2]
- (ii) Comment on the accuracy of this prediction. [1]
- (c) Suggest **one** possible limitation of this model when predicting the average height of a 12 year old child. [1]

6



The diagram shows a triangle ABC . The arc BD is part of a circle with centre A and radius 6 cm .

The area of the sector ABD is 14.4 cm^2 .

- (a) Show that angle BAD is 0.8 radians. [1]

The area of the triangle ABC is **three** times the area of the sector ABD .

- (b) Find the length AC . [2]
- (c) Find the perimeter of the region BCD . [5]

7 In this question you must show detailed reasoning.

A curve has parametric equations $x = t^3 + t^2$, $y = t^2 + 2t$ for all real values of t .

The curve passes through the point P with coordinates $(2, 3)$.

(a) Show that the equation of the tangent to the curve at the point P can be written as $5y = 4x + 7$. [5]

(b) Determine the coordinates of the point where the tangent to the curve at P meets the curve again. [4]

8 (a) Find the first **three** terms in the expansion of $(2+x)^{-3}$ in ascending powers of x . [4]

(b) Hence find the first **three** terms in the expansion of $\frac{\sqrt{1+4x}}{(2+x)^3}$ in ascending powers of x . [4]

(c) Determine the range of values of x for which the expansion in part **(b)** is valid, giving a reason for your answer. [2]

9 (a) Express $3 \cos 2x + 4 \sin 2x$ in the form $R \cos(2x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

(b) In this question you must show detailed reasoning.

Solve the equation $\frac{3 \cos 2x + 4 \sin 2x + 6}{3 \cos 2x + 4 \sin 2x - 1} = 4$ for $0^\circ < x < 360^\circ$. [7]

- 10 The graph of $y = e^x$ can be transformed to the graph of $y = e^{2x-1}$ by a stretch parallel to the x -axis **followed by** a translation.

- (a) (i) State the scale factor of the stretch. [1]
 (ii) Give full details of the translation. [2]

Alternatively the graph of $y = e^x$ can be transformed to the graph of $y = e^{2x-1}$ by a stretch parallel to the x -axis and a stretch parallel to the y -axis.

- (b) State the scale factor of the stretch parallel to the y -axis. [1]

The point P lies on the curve $y = e^{2x-1}$ and has x -coordinate of $\frac{1}{2}$.

- (c) Show that the tangent to the curve $y = e^{2x-1}$ at P has equation $y = 2x$. [4]
 (d) Find the **exact** area enclosed by the curve $y = e^{2x-1}$, the tangent to the curve at P and the y -axis. [4]

- 11 A student is attempting to prove that $\sqrt{5}$ is irrational.
 The first three lines of their proof are shown below.

Assume that $\sqrt{5}$ is rational, so it can be written as $\sqrt{5} = \frac{a}{b}$.	Line 1
Squaring both sides gives $5 = \frac{a^2}{b^2}$. Hence $a^2 = 5b^2$.	Line 2
Hence a must be a multiple of 5, so $a = 5k$, for some integer k .	Line 3

- (a) State any conditions required on **Line 1**. [2]
 (b) Explain why **Line 2** means that a must be a multiple of 5. [2]
 (c) Complete the proof to show that $\sqrt{5}$ is irrational. [3]

Turn over for question 12

- 12 (a) Express $\frac{2+4x}{x(1+x)(1-x)}$ in partial fractions. [4]

The gradient of a curve is given by $\frac{dy}{dx} = \frac{2+4x}{x(1+x)(1-x)\tan y}$ and the curve passes through the point $(\frac{1}{2}, \frac{1}{4}\pi)$.

- (b) Show that the equation of the curve can be written in the form $\cos y = f(x)$, where $f(x)$ is fully simplified. [7]

END OF QUESTION PAPER

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