OCR
Oxford Cambridge and RSA

Wednesday 4 June 2025 – Afternoon

A Level Mathematics A

H240/01 Pure Mathematics

Time allowed: 2 hours

188115702

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

© OCR 2025 [603/1038/8]
DC (PQ/CB) 356902/3
OCR is an exempt Charity

stemup.co.uk

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim\limits_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

© OCR 2025

stemup.co.uk

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

© OCR 2025

stemup.co.uk

1 Express each of the following in the form px^q, where p and q are constants.

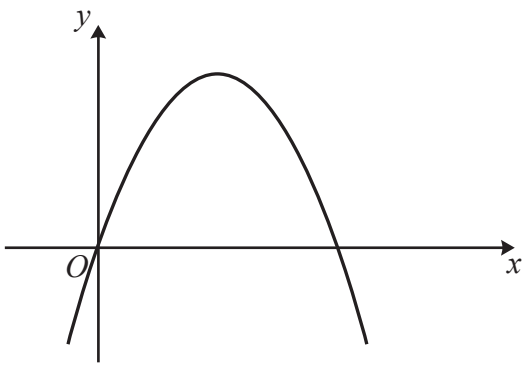
(a) $\dfrac{2}{\sqrt[4]{x}}$ **[1]**

(b) $(5x\sqrt{x})^3$ **[2]**

(c) $\sqrt{2x^3} \times \sqrt{8x^5}$ **[2]**

(d) $x^5(27x^6)^{\frac{1}{3}}$ **[3]**

2



The diagram shows the curve $y = ax(x-b)$, where a and b are constants and $b > 0$.

(a) Given that the curve has a stationary point at $x = 3$, state the value of b. **[1]**

(b) Given also that the stationary point at $x = 3$ is a maximum, state what can be deduced about the value of a. **[1]**

(c) Find the y-coordinate of the stationary point, giving your answer in terms of a. **[1]**

(d) State the range of values of x for which the curve is increasing. **[1]**

3 **(a)** A sequence has terms $u_1, u_2, u_3, \ldots$ defined by $u_1 = 3$ and $u_{n+1} = u_n^2 - 5$ for $n \geqslant 1$.

(i) Find the values of u_2, u_3 and u_4. **[2]**

(ii) Describe the behaviour of the sequence. **[1]**

(b) The second, third and fourth terms of a geometric progression are 12, 8 and $\frac{16}{3}$.

Determine the sum to infinity of this geometric progression. **[3]**

© OCR 2025

stemup.co.uk

4 The position vector of the point A is $0.5\mathbf{i} - 0.5\mathbf{j}$.

(a) Determine whether $0.5\mathbf{i} - 0.5\mathbf{j}$ is a unit vector. **[2]**

(b) Find the direction of $0.5\mathbf{i} - 0.5\mathbf{j}$, giving your answer in degrees. **[2]**

The position vector of the point B is $3.5\mathbf{i} + c\mathbf{j}$, where c is a constant.

(c) Given that $\left|\overrightarrow{AB}\right| = 5$, determine the possible values of c. **[3]**

5 Scientists are comparing h, the average height of a child in cm, with t, the age of the child in years. They suggest the model $h = at + b$ for $t \geqslant 2$, where a and b are constants.

They find that the average height of a 2 year old child is 87 cm and the average height of a 5 year old child is 108 cm.

(a) Find the values of a and b that are consistent with the scientists' findings. **[4]**

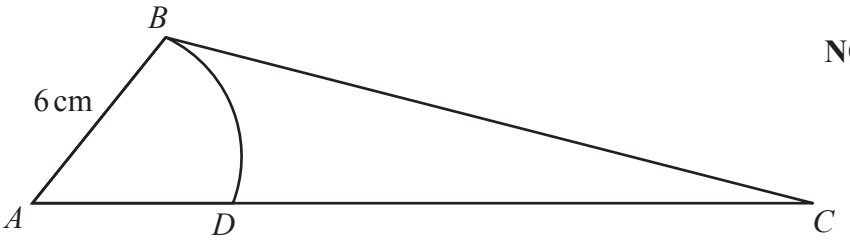
(b) (i) Sam is 4 years old.

Use the model to predict Sam's height. **[2]**

(ii) Comment on the accuracy of this prediction. **[1]**

(c) Suggest **one** possible limitation of this model when predicting the average height of a 12 year old child. **[1]**

6



NOT TO SCALE

The diagram shows a triangle ABC. The arc BD is part of a circle with centre A and radius 6 cm.

The area of the sector ABD is $14.4\,\text{cm}^2$.

(a) Show that angle BAD is 0.8 radians. **[1]**

The area of the triangle ABC is **three** times the area of the sector ABD.

(b) Find the length AC. **[2]**

(c) Find the perimeter of the region BCD. **[5]**

stemup.co.uk

7 **In this question you must show detailed reasoning.**

A curve has parametric equations $x = t^3 + t^2$, $y = t^2 + 2t$ for all real values of t.

The curve passes through the point P with coordinates (2, 3).

(a) Show that the equation of the tangent to the curve at the point P can be written as $5y = 4x + 7$. **[5]**

(b) Determine the coordinates of the point where the tangent to the curve at P meets the curve again. **[4]**

8 **(a)** Find the first **three** terms in the expansion of $(2+x)^{-3}$ in ascending powers of x. **[4]**

(b) Hence find the first **three** terms in the expansion of $\dfrac{\sqrt{1+4x}}{(2+x)^3}$ in ascending powers of x. **[4]**

(c) Determine the range of values of x for which the expansion in part **(b)** is valid, giving a reason for your answer. **[2]**

9 **(a)** Express $3\cos 2x + 4\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. **[3]**

(b) **In this question you must show detailed reasoning.**

Solve the equation $\dfrac{3\cos 2x + 4\sin 2x + 6}{3\cos 2x + 4\sin 2x - 1} = 4$ for $0° < x < 360°$. **[7]**

© OCR 2025

stemup.co.uk

10 The graph of $y = e^x$ can be transformed to the graph of $y = e^{2x-1}$ by a stretch parallel to the x-axis **followed by** a translation.

(a) (i) State the scale factor of the stretch. **[1]**

(ii) Give full details of the translation. **[2]**

Alternatively the graph of $y = e^x$ can be transformed to the graph of $y = e^{2x-1}$ by a stretch parallel to the x-axis and a stretch parallel to the y-axis.

(b) State the scale factor of the stretch parallel to the y-axis. **[1]**

The point P lies on the curve $y = e^{2x-1}$ and has x-coordinate of $\frac{1}{2}$.

(c) Show that the tangent to the curve $y = e^{2x-1}$ at P has equation $y = 2x$. **[4]**

(d) Find the **exact** area enclosed by the curve $y = e^{2x-1}$, the tangent to the curve at P and the y-axis. **[4]**

11 A student is attempting to prove that $\sqrt{5}$ is irrational.
The first three lines of their proof are shown below.

Assume that $\sqrt{5}$ is rational, so it can be written as $\sqrt{5} = \frac{a}{b}$.	**Line 1**
Squaring both sides gives $5 = \frac{a^2}{b^2}$. Hence $a^2 = 5b^2$.	**Line 2**
Hence a must be a multiple of 5, so $a = 5k$, for some integer k.	**Line 3**

(a) State any conditions required on **Line 1**. **[2]**

(b) Explain why **Line 2** means that a must be a multiple of 5. **[2]**

(c) Complete the proof to show that $\sqrt{5}$ is irrational. **[3]**

Turn over for question 12

stemup.co.uk

12 **(a)** Express $\dfrac{2+4x}{x(1+x)(1-x)}$ in partial fractions. **[4]**

The gradient of a curve is given by $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2+4x}{x(1+x)(1-x)\tan y}$ and the curve passes through the point $\left(\frac{1}{2}, \frac{1}{4}\pi\right)$.

(b) Show that the equation of the curve can be written in the form $\cos y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is fully simplified. **[7]**

END OF QUESTION PAPER

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.

© OCR 2025

stemup.co.uk

Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a) $\frac{2}{x^{1/4}} = 2x^{-1/4}$

1 (b) $(5x\sqrt{x})^3 = (5x^{3/2})^3 = 5^3x^{9/2} = 125x^{9/2}$

1 (c) $\sqrt{(2x^3)} \times \sqrt{(8x^5)} = \sqrt{(16x^8)} = 4x^4$

1 (d) $(27x^6)^{1/3} = 27^{1/3}x^2 = 3x^2$

$x^5 \times 3x^2 = 3x^7$

2 (a) Roots at $x = 0$ and $x = b$, so by symmetry the stationary point is at $x = \frac{1}{2}b$

$\frac{1}{2}b = 3 \quad \Rightarrow \quad b = 6$

2 (b) The stationary point is a maximum, so the curve is n-shaped: $a < 0$

2 (c) $y = a \times 3 \times (3 - 6) = -9a$

2 (d) The curve is increasing for $x < 3$

3 (a)(i) $u_2 = 3^2 - 5 = 4$

$u_3 = 4^2 - 5 = 11$

$u_4 = 11^2 - 5 = 116$

3 (a)(ii) The sequence is increasing (it diverges).

Question number	
3 (b)	$r = \frac{8}{12} = \frac{2}{3}$
	First term $a = 12 \div \frac{2}{3} = 18$
	$S_\infty = \frac{a}{1-r} = \frac{18}{1-2/3} = \frac{18}{1/3} = 54$
4 (a)	$\lvert 0.5i - 0.5j \rvert = \sqrt{(0.5^2 + 0.5^2)} = \sqrt{0.5} = 0.7071\ldots$
	$\sqrt{0.5} \neq 1$, so $0.5i - 0.5j$ is not a unit vector.
4 (b)	$\tan^{-1}(0.5 \div 0.5) = 45°$, and the vector is 45° below the positive x-axis
	Direction (anticlockwise from i) $= 360° - 45° = 315°$
4 (c)	$AB = (3.5i + cj) - (0.5i - 0.5j) = 3i + (c + 0.5)j$
	$3^2 + (c + 0.5)^2 = 5^2$
	$c^2 + c + 9.25 = 25 \Rightarrow c^2 + c - 15.75 = 0$
	$(c + 0.5)^2 = 16 \Rightarrow c + 0.5 = 4$ or $c + 0.5 = -4$
	$c = 3.5$ or $c = -4.5$
5 (a)	$t = 2, h = 87$: $2a + b = 87$
	$t = 5, h = 108$: $5a + b = 108$
	Subtract: $3a = 21 \Rightarrow a = 7$
	$14 + b = 87 \Rightarrow b = 73$
5 (b)(i)	$h = 7 \times 4 + 73 = 101$ cm
5 (b)(ii)	The model only gives the average height of a 4 year old. Heights of children the same age vary a lot, so Sam may not be exactly 101 cm.

Question number	
5 (c)	Growth is unlikely to continue at a constant (linear) rate, e.g. there is a growth spurt at puberty, so extrapolating to age 12 is unreliable.
6 (a)	$\frac{1}{2} \times 6^2 \times \theta = 14.4$ $18\theta = 14.4$ $\theta = 0.8$, so angle BAD = 0.8 radians as required
6 (b)	$\frac{1}{2} \times 6 \times AC \times \sin 0.8 = 3 \times 14.4 = 43.2$ $AC = 43.2 \div (3 \sin 0.8) = 20.0737... = 20.1$ cm
6 (c)	Arc BD $= r\theta = 6 \times 0.8 = 4.8$ cm $BC^2 = 6^2 + 20.0737...^2 - 2 \times 6 \times 20.0737... \times \cos 0.8$ $= 271.128...$ BC = 16.4659... cm DC = AC − AD = 20.0737... − 6 = 14.0737... cm Perimeter = 4.8 + 14.0737... + 16.4659... = 35.3396... Perimeter of BCD = 35.3 cm

Question number

7 (a)

$\frac{dx}{dt} = 3t^2 + 2t, \qquad \frac{dy}{dt} = 2t + 2$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t + 2}{3t^2 + 2t}$

At P: $t^2 + 2t = 3 \Rightarrow (t + 3)(t - 1) = 0 \Rightarrow t = 1$ or $t = -3$

$t = -3$ gives $x = -27 + 9 = -18 \neq 2$; $t = 1$ gives $x = 1 + 1 = 2$, so $t = 1$

At $t = 1$: $\frac{dy}{dx} = \frac{2 + 2}{3 + 2} = \frac{4}{5}$

Tangent: $y - 3 = \frac{4}{5}(x - 2)$

$5y - 15 = 4x - 8$

$5y = 4x + 7$ as required

7 (b)

$5(t^2 + 2t) = 4(t^3 + t^2) + 7$

$5t^2 + 10t = 4t^3 + 4t^2 + 7$

$4t^3 - t^2 - 10t + 7 = 0$

$t = 1$ is a repeated root (tangent at $t = 1$):

$(t - 1)(4t^2 + 3t - 7) = 0$

$(t - 1)(t - 1)(4t + 7) = 0$

Other point: $t = -\frac{7}{4}$

$x = (-\frac{7}{4})^3 + (-\frac{7}{4})^2 = -\frac{343}{64} + \frac{196}{64} = -\frac{147}{64}$

$y = (-\frac{7}{4})^2 + 2(-\frac{7}{4}) = \frac{49}{16} - \frac{56}{16} = -\frac{7}{16}$

The tangent meets the curve again at $(-\frac{147}{64}, -\frac{7}{16})$

Question number

8 (a) $(2+x)^{-3} = 2^{-3}(1+\frac{x}{2})^{-3} = \frac{1}{8}(1+\frac{x}{2})^{-3}$

$(1+\frac{x}{2})^{-3} = 1 + (-3)(\frac{x}{2}) + \frac{(-3)(-4)}{2!}(\frac{x}{2})^2 + \ldots$

$= 1 - \frac{3}{2}x + \frac{3}{2}x^2 + \ldots$

$(2+x)^{-3} = \frac{1}{8} - \frac{3}{16}x + \frac{3}{16}x^2 + \ldots$

8 (b) $(1+4x)^{\frac{1}{2}} = 1 + \frac{1}{2}(4x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(4x)^2 + \ldots = 1 + 2x - 2x^2 + \ldots$

$\frac{\sqrt{(1+4x)}}{(2+x)^3} = (1+2x-2x^2)(\frac{1}{8} - \frac{3}{16}x + \frac{3}{16}x^2 + \ldots)$

$= \frac{1}{8} - \frac{3}{16}x + \frac{3}{16}x^2 + \frac{1}{4}x - \frac{3}{8}x^2 - \frac{1}{4}x^2 + \ldots$

$= \frac{1}{8} + \frac{1}{16}x - \frac{7}{16}x^2 + \ldots$

8 (c) $(1+\frac{x}{2})^{-3}$ is valid for $|\frac{x}{2}| < 1$, i.e. $|x| < 2$

$(1+4x)^{\frac{1}{2}}$ is valid for $|4x| < 1$, i.e. $|x| < \frac{1}{4}$

Both expansions must be valid and $|x| < \frac{1}{4}$ lies inside $|x| < 2$,

so the expansion is valid for $|x| < \frac{1}{4}$ $(-\frac{1}{4} < x < \frac{1}{4})$

9 (a) $R = \sqrt{(3^2 + 4^2)} = \sqrt{25} = 5$

$R\cos\alpha = 3$, $R\sin\alpha = 4$ $\Rightarrow$ $\tan\alpha = 4/3$

$\alpha = 53.130\ldots^\circ = 53.1^\circ$

$3\cos 2x + 4\sin 2x = 5\cos(2x - 53.1^\circ)$

Question number

9 (b) $\frac{5\cos(2x - 53.1°) + 6}{5\cos(2x - 53.1°) - 1} = 4$

$5\cos(2x - 53.1°) + 6 = 20\cos(2x - 53.1°) - 4$

$15\cos(2x - 53.1°) = 10$

$\cos(2x - 53.1°) = 2/3$ (denominator $= 10/3 - 1 \neq 0$)

$0° < x < 360° \Rightarrow -53.1° < 2x - 53.1° < 666.9°$

$\cos^{-1}(2/3) = 48.189...°$

$2x - 53.13° = -48.19°,\ 48.19°,\ 311.81°,\ 408.19°$

$2x = 4.940°,\ 101.32°,\ 364.94°,\ 461.32°$

$x = 2.47°,\ 50.7°,\ 182.5°,\ 230.7°$

10 (a)(i) Scale factor ½

10 (a)(ii) After the stretch: $y = e^{2x}$; $e^{2(x - ½)} = e^{2x - 1}$

Translation of ½ unit parallel to the x-axis, in the positive x-direction,

i.e. by the vector (½, 0)

10 (b) $e^{2x - 1} = e^{-1} \times e^{2x}$

Scale factor of the stretch parallel to the y-axis is e^{-1}

10 (c) $\frac{dy}{dx} = 2e^{2x - 1}$

At $x = ½$: gradient $= 2e^0 = 2$, $y = e^0 = 1$

Tangent: $y - 1 = 2(x - ½)$

$y = 2x - 1 + 1 \Rightarrow y = 2x$ as required

Question number

10 (d) Region lies between $x = 0$ and $x = \frac{1}{2}$, with the curve above the tangent.

$\int_0^{\frac{1}{2}} e^{2x-1}\,dx = [\frac{1}{2}e^{2x-1}]_0^{\frac{1}{2}} = \frac{1}{2}(e^0 - e^{-1}) = \frac{1}{2} - \frac{1}{2}e^{-1}$

Area of triangle under $y = 2x$ from 0 to $\frac{1}{2}$: $\frac{1}{2} \times \frac{1}{2} \times 1 = \frac{1}{4}$

$$\text{Area} = \frac{1}{2} - \frac{1}{2e} - \frac{1}{4} = \frac{1}{4} - \frac{1}{2e}$$

11 (a) a and b are positive integers (so $b \neq 0$)

a and b have no common factors other than 1 (a/b is in its simplest form)

11 (b) b is an integer, so $a^2 = 5b^2$ means a^2 is a multiple of 5.

5 is prime, so if 5 is a factor of $a^2 = a \times a$ it must be a factor of a

(any prime factor of a^2 is a prime factor of a), so a is a multiple of 5.

11 (c) $(5k)^2 = 5b^2 \Rightarrow 25k^2 = 5b^2 \Rightarrow b^2 = 5k^2$

So b^2 is a multiple of 5 and, by the same reasoning, b is a multiple of 5.

Then a and b share a common factor of 5, which contradicts the

assumption that a and b have no common factors.

Hence $\sqrt{5}$ cannot be written as a/b, so $\sqrt{5}$ is irrational.

12 (a) $$\frac{2+4x}{x(1+x)(1-x)} \equiv \frac{A}{x} + \frac{B}{1+x} + \frac{C}{1-x}$$

$2 + 4x \equiv A(1+x)(1-x) + Bx(1-x) + Cx(1+x)$

$x = 0$: $2 = A \Rightarrow A = 2$

$x = -1$: $-2 = B(-1)(2) \Rightarrow B = 1$

$x = 1$: $6 = C(1)(2) \Rightarrow C = 3$

$$\frac{2+4x}{x(1+x)(1-x)} = \frac{2}{x} + \frac{1}{1+x} + \frac{3}{1-x}$$

Question number

12 (b)

$$\int \tan y \, dy = \int \frac{2 + 4x}{x(1 + x)(1 - x)} \, dx = \int \left(\frac{2}{x} + \frac{1}{1 + x} + \frac{3}{1 - x} \right) dx$$

$$\ln|\sec y| = 2 \ln|x| + \ln|1 + x| - 3 \ln|1 - x| + c$$

$$\ln|\sec y| = \ln \frac{kx^2(1 + x)}{(1 - x)^3} \qquad \text{(where } c = \ln k\text{)}$$

$$\sec y = \frac{kx^2(1 + x)}{(1 - x)^3} \quad \Rightarrow \quad \cos y = \frac{(1 - x)^3}{kx^2(1 + x)}$$

$$\text{At } (\tfrac{1}{2}, \tfrac{1}{4}\pi): \quad \cos \tfrac{1}{4}\pi = \frac{1}{\sqrt{2}} = \frac{(\tfrac{1}{2})^3}{k \times \tfrac{1}{4} \times 3/2} = \frac{1/8}{3k/8} = \frac{1}{3k}$$

$$3k = \sqrt{2} \quad \Rightarrow \quad k = \frac{\sqrt{2}}{3}$$

$$\cos y = \frac{3(1 - x)^3}{\sqrt{2}\, x^2(1 + x)} = \frac{3\sqrt{2}(1 - x)^3}{2x^2(1 + x)}$$