



Oxford Cambridge and RSA

Thursday 20 June 2024 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours



You must have:

- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ ms}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **12** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae

A Level Mathematics A (H240)

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2, \quad \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Standard deviation

$$\sqrt{\frac{\sum(x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np , variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

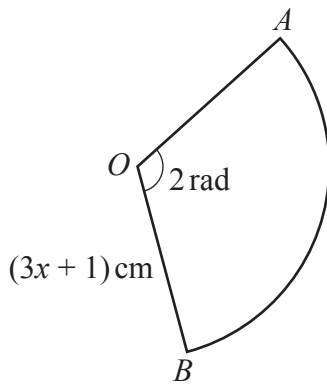
Section A
Pure Mathematics

1 Simplify each of the following.

(a) $(2a^2)^3 \times \frac{3}{4}a^{-1}$ [2]

(b) $\frac{4x^2 - 9}{(2x^2 + 5x - 12)(2x + 3)}$ [2]

2 In this question you must show detailed reasoning.



The diagram shows a sector AOB of a circle with centre O and radius $(3x + 1)\text{cm}$. The angle AOB is 2 radians. The area of sector AOB is less than $(44x - 7)\text{cm}^2$.

Find the set of possible values of x . Give your answer in set notation. [5]

3 (a) Expand $(3 - 2x)^{-2}$ in ascending powers of x up to and including the term in x^2 . [4]

(b) State the set of values of x for which this expansion is valid. [1]

(c) When $\frac{a+x}{(3-2x)^2}$ is expanded in ascending powers of x , the coefficient of x is zero.

Determine the value of the constant a . [2]

- 4 (a) Show that the equation $2 \cot^2 x - 9 \operatorname{cosec} x - 3 = 0$ can be expressed in the form

$$5 \sin^2 x + 9 \sin x - 2 = 0. \quad [3]$$

- (b) (i) In this question you must show detailed reasoning.

Hence solve, for $0 < \theta < \pi$,

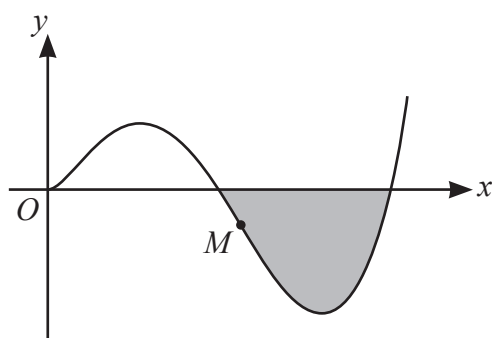
$$2 \cot^2 2\theta - 9 \operatorname{cosec} 2\theta - 3 = 0.$$

Give your answers correct to 3 decimal places. [4]

The small angle approximation for $\sin 2\theta$ is used to find an approximation for the smallest positive solution of the equation $2 \cot^2 2\theta - 9 \operatorname{cosec} 2\theta - 3 = 0$.

- (ii) Show that this approximate solution is accurate to 2 decimal places. [2]

5



The diagram shows the curve with equation $y = (x^3 - 2x^2) \ln x$. The curve has a point of inflection at the point M .

- (a) (i) Show that the x -coordinate of M satisfies the equation

$$x = \frac{6 + (4 - 6x) \ln x}{5}. \quad [5]$$

- (ii) Use an iterative formula, based on the equation in part (a)(i), to determine the x -coordinate of M correct to 2 decimal places. Use an initial value of 1.1 and show the result of each step of the iterative process. [2]

- (b) Determine the exact area of the shaded region, giving your answer in the form $p \ln q - r$, where p and r are positive rational numbers and q is a positive integer. [6]

- 6 The curve C is defined, for $0 \leq t < 2\pi$, by the parametric equations

$$x = 4k + k \sin t, \quad y = 2 + 4 \cos t,$$

where k is a constant.

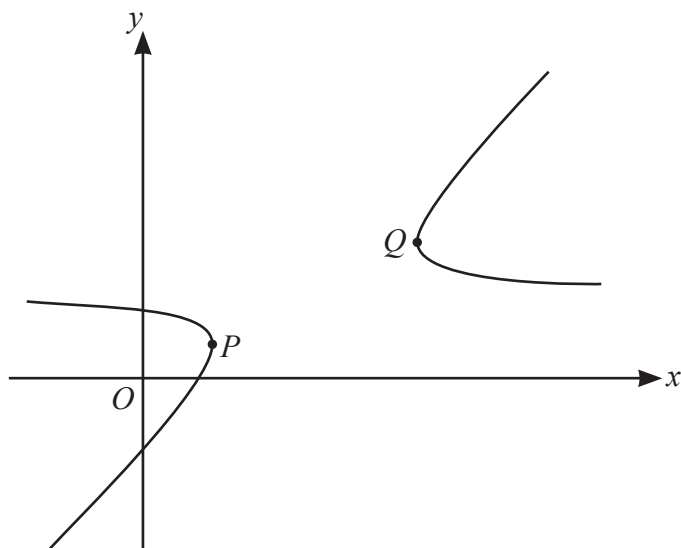
- (a) Find a cartesian equation for C . You do **not** need to simplify your answer. [2]

You are given that C is a circle.

- (b) (i) Determine the radius of C . [2]

- (ii) Find the possible coordinates for the centre of C . [2]

7



The diagram shows the curve $5x - 2xy + 2y^2 - k = 0$, where k is a positive integer.

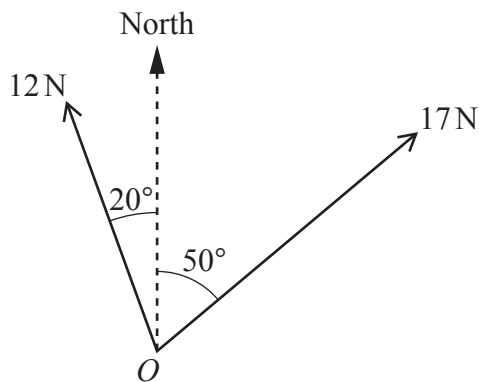
At the points P and Q on the curve, the tangents to the curve are parallel to the y -axis.

Given that the difference in the y -coordinates of P and Q is 3, determine the x -coordinates of P and Q . [7]

Section B
Mechanics

- 8 A particle P is moving with constant acceleration $(-5\mathbf{i} + 2\mathbf{j})\text{ m s}^{-2}$. At time $t = 0$ seconds, P is at the origin and has velocity $(\mathbf{i} + 3\mathbf{j})\text{ m s}^{-1}$.
- (a) Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, the displacement of P at time $t = 2$ seconds. [2]
- (b) Determine the speed of P at time $t = 2$ seconds. [4]

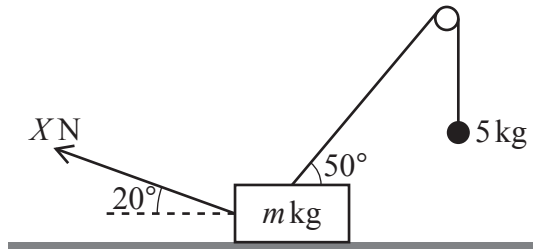
9



Two horizontal forces of magnitudes 17 N and 12 N act at a point O along bearings of 050° and 340° respectively (see diagram).

- (a) Determine the magnitude and bearing of the resultant force. [6]
- A third horizontal force $\mathbf{F}$ is now applied at O . The three forces are in equilibrium.
- (b) State the magnitude of $\mathbf{F}$ and give the bearing along which it acts. [2]

10



A block of mass $m \text{ kg}$ is on smooth horizontal ground with one end of a light inextensible rope attached to its upper surface. The other end of the rope is attached to an object of mass 5 kg . The rope passes over a small smooth pulley, and the object hangs vertically below the pulley. The part of the rope between the block and the pulley makes an angle of 50° with the horizontal. A force of magnitude $X \text{ N}$ acts on the block at an angle of 20° above the horizontal in the vertical plane containing the rope (see diagram).

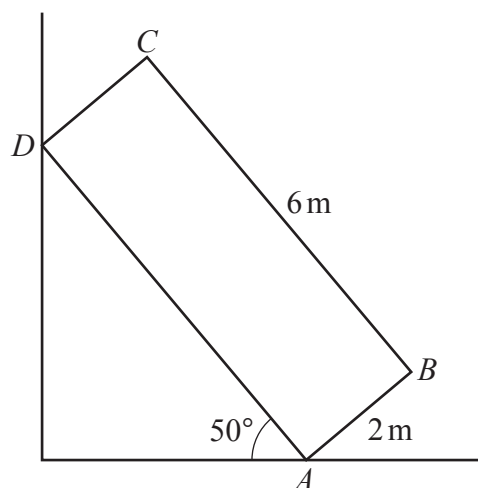
You are given that the block is in equilibrium.

(a) Determine the value of X . [3]

You are also given that the magnitude of the contact force exerted by the ground on the block is 147 N .

(b) Determine the value of m . [3]

11



A uniform rectangular lamina $ABCD$ has a mass of 0.5 kg . The length of AB is 2 m , and the length of BC is 6 m . The lamina is in limiting equilibrium with corner A in contact with rough horizontal ground and corner D in contact with a smooth vertical wall. The lamina rests in a vertical plane that is perpendicular to the wall, with AD inclined at 50° to the horizontal (see diagram).

- (a) By taking moments, show that the magnitude of the normal contact force between the lamina and the wall is 1.24 N , correct to **3** significant figures. [4]
- (b) Determine the coefficient of friction between the lamina and the ground. [3]

12 A particle P moves in a straight line. The velocity $v \text{ ms}^{-1}$ of P at time t seconds is given by

$$v = \frac{1}{12}kt(t-3) \quad \text{for } 0 \leq t \leq 6,$$

$$v = \frac{54k}{t^2} \quad \text{for } 6 \leq t \leq 9,$$

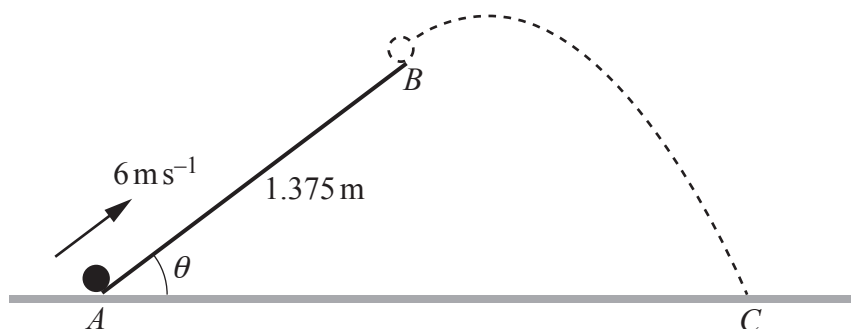
where k is a positive constant.

- (a) Sketch, on the axes in the Printed Answer Booklet, the velocity-time graph for P for values of t from 0 to 9. [3]
- (b) State the value of t in the interval $0 \leq t \leq 9$ when the acceleration of P is zero. [1]
- (c) **In this question you must show detailed reasoning.**

You are given that the total distance travelled by P in the interval $0 \leq t \leq 9$ is 84 m .

Find the value of k . [6]

13



The points A and B are the lower and upper ends, respectively, of a line of greatest slope on a plane inclined at an angle θ to the horizontal, where $\sin \theta = 0.6$ and $AB = 1.375 \text{ m}$ (see diagram).

A particle P is projected up the plane with speed 6 m s^{-1} from A towards B .

The plane at A is fixed to the ground which is horizontal.

The surface of the plane is rough and the coefficient of friction between P and the plane is 0.25.

(a) Show that the speed of P at B is 3.8 m s^{-1} . **[6]**

The particle leaves the slope at B and moves freely under gravity.

The particle first lands at a point C on the horizontal ground. The time taken for P to travel from A to C is T seconds.

(b) Determine the value of T . **[6]**

END OF QUESTION PAPER

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