OCR
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Thursday 20 June 2024 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours

1336236120

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient rule $y = \dfrac{u}{v},\ \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta,\ \cos\theta \approx 1 - \frac{1}{2}\theta^2,\ \tan\theta \approx \theta$ where θ is measured in radians

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Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

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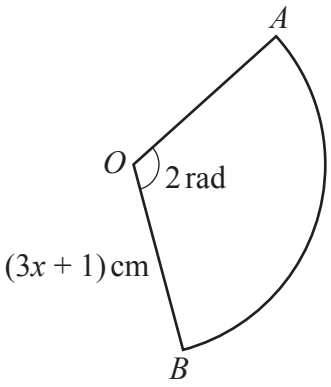
Section A
Pure Mathematics

1 Simplify each of the following.

(a) $(2a^2)^3 \times \frac{3}{4}a^{-1}$ **[2]**

(b) $\dfrac{4x^2-9}{(2x^2+5x-12)(2x+3)}$ **[2]**

2 **In this question you must show detailed reasoning.**



The diagram shows a sector AOB of a circle with centre O and radius $(3x+1)$ cm. The angle AOB is 2 radians. The area of sector AOB is less than $(44x-7)\text{cm}^2$.

Find the set of possible values of x. Give your answer in set notation. **[5]**

3 **(a)** Expand $(3-2x)^{-2}$ in ascending powers of x up to and including the term in x^2. **[4]**

(b) State the set of values of x for which this expansion is valid. **[1]**

(c) When $\dfrac{a+x}{(3-2x)^2}$ is expanded in ascending powers of x, the coefficient of x is zero.

Determine the value of the constant a. **[2]**

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4 **(a)** Show that the equation $2\cot^2 x - 9\operatorname{cosec} x - 3 = 0$ can be expressed in the form

$5\sin^2 x + 9\sin x - 2 = 0$. **[3]**

(b) **(i)** **In this question you must show detailed reasoning.**

Hence solve, for $0 < \theta < \pi$,

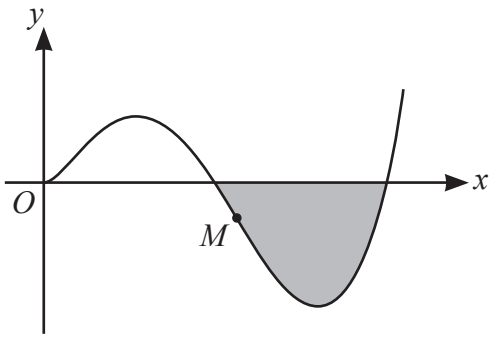
$2\cot^2 2\theta - 9\operatorname{cosec} 2\theta - 3 = 0$.

Give your answers correct to **3** decimal places. **[4]**

The small angle approximation for $\sin 2\theta$ is used to find an approximation for the smallest positive solution of the equation $2\cot^2 2\theta - 9\operatorname{cosec} 2\theta - 3 = 0$.

(ii) Show that this approximate solution is accurate to **2** decimal places. **[2]**

5



The diagram shows the curve with equation $y = (x^3 - 2x^2)\ln x$. The curve has a point of inflection at the point M.

(a) **(i)** Show that the x-coordinate of M satisfies the equation

$$x = \frac{6 + (4 - 6x)\ln x}{5}.$$ **[5]**

(ii) Use an iterative formula, based on the equation in part **(a)(i)**, to determine the x-coordinate of M correct to **2** decimal places. Use an initial value of 1.1 and show the result of each step of the iterative process. **[2]**

(b) Determine the exact area of the shaded region, giving your answer in the form $p\ln q - r$, where p and r are positive rational numbers and q is a positive integer. **[6]**

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6 The curve C is defined, for $0 \leqslant t < 2\pi$, by the parametric equations

$x = 4k + k\sin t, \quad y = 2 + 4\cos t,$

where k is a constant.

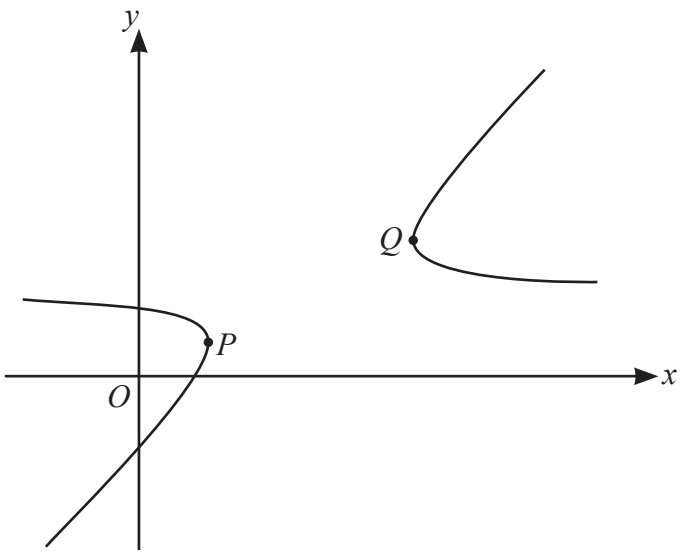
(a) Find a cartesian equation for C. You do **not** need to simplify your answer. **[2]**

You are given that C is a circle.

(b) (i) Determine the radius of C. **[2]**

(ii) Find the possible coordinates for the centre of C. **[2]**

7



The diagram shows the curve $5x - 2xy + 2y^2 - k = 0$, where k is a positive integer.

At the points P and Q on the curve, the tangents to the curve are parallel to the y-axis.

Given that the difference in the y-coordinates of P and Q is 3, determine the x-coordinates of P and Q. **[7]**

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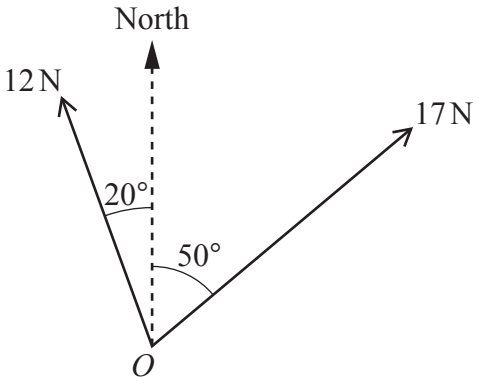
Section B
Mechanics

8 A particle P is moving with constant acceleration $(-5\mathbf{i}+2\mathbf{j})\,\mathrm{m\,s^{-2}}$. At time $t = 0$ seconds, P is at the origin and has velocity $(\mathbf{i}+3\mathbf{j})\,\mathrm{m\,s^{-1}}$.

(a) Find, in terms of **i** and **j**, the displacement of P at time $t = 2$ seconds. **[2]**

(b) Determine the speed of P at time $t = 2$ seconds. **[4]**

9



Two horizontal forces of magnitudes 17 N and 12 N act at a point O along bearings of 050° and 340° respectively (see diagram).

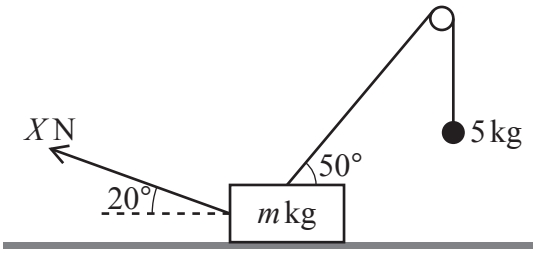
(a) Determine the magnitude and bearing of the resultant force. **[6]**

A third horizontal force **F** is now applied at O. The three forces are in equilibrium.

(b) State the magnitude of **F** and give the bearing along which it acts. **[2]**

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A block of mass m kg is on smooth horizontal ground with one end of a light inextensible rope attached to its upper surface. The other end of the rope is attached to an object of mass 5 kg. The rope passes over a small smooth pulley, and the object hangs vertically below the pulley. The part of the rope between the block and the pulley makes an angle of 50° with the horizontal. A force of magnitude X N acts on the block at an angle of 20° above the horizontal in the vertical plane containing the rope (see diagram).

You are given that the block is in equilibrium.

(a) Determine the value of X. **[3]**

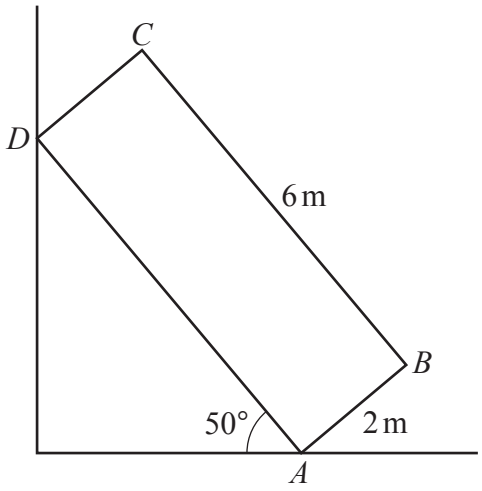
You are also given that the magnitude of the contact force exerted by the ground on the block is 147 N.

(b) Determine the value of m. **[3]**

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A uniform rectangular lamina $ABCD$ has a mass of 0.5 kg. The length of AB is 2 m, and the length of BC is 6 m. The lamina is in limiting equilibrium with corner A in contact with rough horizontal ground and corner D in contact with a smooth vertical wall. The lamina rests in a vertical plane that is perpendicular to the wall, with AD inclined at 50° to the horizontal (see diagram).

(a) By taking moments, show that the magnitude of the normal contact force between the lamina and the wall is 1.24 N, correct to **3** significant figures. **[4]**

(b) Determine the coefficient of friction between the lamina and the ground. **[3]**

12 A particle P moves in a straight line. The velocity $v\,\text{m}\,\text{s}^{-1}$ of P at time t seconds is given by

$v = \frac{1}{12}kt(t-3)$ for $0 \leqslant t \leqslant 6$,

$v = \dfrac{54k}{t^2}$ for $6 \leqslant t \leqslant 9$,

where k is a positive constant.

(a) Sketch, on the axes in the Printed Answer Booklet, the velocity-time graph for P for values of t from 0 to 9. **[3]**

(b) State the value of t in the interval $0 \leqslant t \leqslant 9$ when the acceleration of P is zero. **[1]**

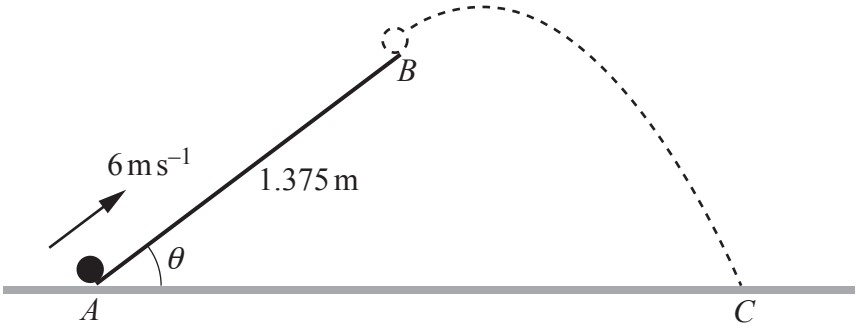
(c) **In this question you must show detailed reasoning.**

You are given that the total distance travelled by P in the interval $0 \leqslant t \leqslant 9$ is 84 m.

Find the value of k. **[6]**

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The points A and B are the lower and upper ends, respectively, of a line of greatest slope on a plane inclined at an angle θ to the horizontal, where $\sin\theta = 0.6$ and $AB = 1.375\,\text{m}$ (see diagram).

A particle P is projected up the plane with speed $6\,\text{m s}^{-1}$ from A towards B.

The plane at A is fixed to the ground which is horizontal.

The surface of the plane is rough and the coefficient of friction between P and the plane is 0.25.

(a) Show that the speed of P at B is $3.8\,\text{m s}^{-1}$. **[6]**

The particle leaves the slope at B and moves freely under gravity.

The particle first lands at a point C on the horizontal ground. The time taken for P to travel from A to C is T seconds.

(b) Determine the value of T. **[6]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a)

$(2a^2)^3 = 8a^6$

$8a^6 \times \frac{3}{4}a^{-1} = 6a^{6-1}$

$= 6a^5$

1 (b)

$4x^2 - 9 = (2x - 3)(2x + 3)$

$2x^2 + 5x - 12 = (2x - 3)(x + 4)$

$(2x - 3)(2x + 3) \,/\, [(2x - 3)(x + 4)(2x + 3)]$

$= 1/(x + 4)$

2

Area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2}(3x + 1)^2(2) = (3x + 1)^2$

$(3x + 1)^2 < 44x - 7$

$9x^2 + 6x + 1 < 44x - 7$

$9x^2 - 38x + 8 < 0$

$(9x - 2)(x - 4) < 0$

Critical values: $x = 2/9$ and $x = 4$

Positive quadratic, so it is negative between the roots: $2/9 < x < 4$

(radius $3x + 1 > 0$ for all of these x)

$\{ x : 2/9 < x < 4 \}$

3 (a)

$(3 - 2x)^{-2} = 3^{-2}(1 - \frac{2}{3}x)^{-2} = (1/9)(1 - \frac{2}{3}x)^{-2}$

$(1 - \frac{2}{3}x)^{-2} = 1 + (-2)(-\frac{2}{3}x) + [(-2)(-3)/2!](-\frac{2}{3}x)^2 + \ldots$

$= 1 + (4/3)x + 3 \times (4/9)x^2 + \ldots$

$= 1 + (4/3)x + (4/3)x^2 + \ldots$

$(3 - 2x)^{-2} = (1/9)(1 + (4/3)x + (4/3)x^2 + \ldots)$

$= 1/9 + (4/27)x + (4/27)x^2 + \ldots$

Question number	
3 (b)	Valid for $\lvert\frac{2}{3}x\rvert < 1$, i.e. $\lvert x\rvert < 3/2$ $(-3/2 < x < 3/2)$
3 (c)	$(a + x)/(3 - 2x)^2 = (a + x)(1/9 + (4/27)x + \ldots)$
	Coefficient of x: $(4/27)a + 1/9 = 0$
	$(1/9)((4/3)a + 1) = 0 \Rightarrow (4/3)a = -1$
	$a = -3/4$
4 (a)	$\cot x = \cos x / \sin x$, $\operatorname{cosec} x = 1/\sin x$
	$2(\cos^2 x / \sin^2 x) - 9(1/\sin x) - 3 = 0$
	Multiply through by $\sin^2 x$: $2\cos^2 x - 9 \sin x - 3 \sin^2 x = 0$
	$2(1 - \sin^2 x) - 9 \sin x - 3 \sin^2 x = 0$
	$2 - 2\sin^2 x - 9 \sin x - 3 \sin^2 x = 0$
	$2 - 9 \sin x - 5 \sin^2 x = 0$
	$\Rightarrow$ $5 \sin^2 x + 9 \sin x - 2 = 0$ as required
4 (b)(i)	Using (a) with $x = 2\theta$: $5 \sin^2 2\theta + 9 \sin 2\theta - 2 = 0$
	$(5 \sin 2\theta - 1)(\sin 2\theta + 2) = 0$
	$\sin 2\theta = 0.2$ or $\sin 2\theta = -2$
	$\sin 2\theta = -2$ is impossible since $-1 \leq \sin 2\theta \leq 1$, so $\sin 2\theta = 0.2$ only
	$0 < \theta < \pi \Rightarrow 0 < 2\theta < 2\pi$
	$2\theta = \sin^{-1}(0.2) = 0.201357\ldots$ or $2\theta = \pi - 0.201357\ldots = 2.940235\ldots$
	$\theta = 0.1006789\ldots$ or $\theta = 1.4701173\ldots$
	$\theta = 0.101$, $\theta = 1.470$ (3 d.p.)

Question number	
4 (b)(ii)	Small angle: $\sin 2\theta \approx 2\theta$ in $5\sin^2 2\theta + 9\sin 2\theta - 2 = 0$
	$5(2\theta)^2 + 9(2\theta) - 2 = 0 \Rightarrow 20\theta^2 + 18\theta - 2 = 0$
	$10\theta^2 + 9\theta - 1 = 0 \Rightarrow (10\theta - 1)(\theta + 1) = 0$
	Smallest positive solution: $\theta = 0.1 = 0.10$
	Exact smallest solution from (b)(i) is 0.1006789... = 0.10 (2 d.p.),
	so the approximation 0.10 is accurate to 2 decimal places.
5 (a)(i)	$y = (x^3 - 2x^2)\ln x$. Product rule:
	$dy/dx = (x^3 - 2x^2)(1/x) + (3x^2 - 4x)\ln x = x^2 - 2x + (3x^2 - 4x)\ln x$
	$d^2y/dx^2 = 2x - 2 + (3x^2 - 4x)(1/x) + (6x - 4)\ln x$
	$= 2x - 2 + 3x - 4 + (6x - 4)\ln x = 5x - 6 + (6x - 4)\ln x$
	At the point of inflection M, $d^2y/dx^2 = 0$:
	$5x - 6 + (6x - 4)\ln x = 0$
	$5x = 6 - (6x - 4)\ln x = 6 + (4 - 6x)\ln x$
	$x = [6 + (4 - 6x)\ln x] / 5$ as required
5 (a)(ii)	$x_{n+1} = [6 + (4 - 6x_n)\ln x_n] / 5$, $x_1 = 1.1$
	$x_2 = 1.150438...$ $x_3 = 1.118643...$
	$x_4 = 1.139191...$ $x_5 = 1.126105...$
	$x_6 = 1.134521...$ $x_7 = 1.129141...$
	$x_8 = 1.132594...$ $x_9 = 1.130384...$
	x-coordinate of M is 1.13 (2 d.p.)
5 (b)	Curve meets x-axis where $(x^3 - 2x^2)\ln x = 0$: $x = 1$ ($\ln x = 0$), $x = 2$
	Integration by parts: $u = \ln x$, $dv/dx = x^3 - 2x^2$, $v = \frac{1}{4}x^4 - \frac{2}{3}x^3$
	$\int (x^3 - 2x^2)\ln x\, dx = (\frac{1}{4}x^4 - \frac{2}{3}x^3)\ln x - \int (\frac{1}{4}x^4 - \frac{2}{3}x^3)(1/x)\, dx$
	$= (\frac{1}{4}x^4 - \frac{2}{3}x^3)\ln x - \int (\frac{1}{4}x^3 - \frac{2}{3}x^2)\, dx$
	$= (\frac{1}{4}x^4 - \frac{2}{3}x^3)\ln x - x^4/16 + 2x^3/9 \ (+ c)$
	$\int_1^2 = [\,(4 - 16/3)\ln 2 - 1 + 16/9\,] - [\,0 - 1/16 + 2/9\,]$

Question number	
	$= -(4/3)\ln 2 + 7/9 - 23/144 = -(4/3)\ln 2 + 89/144$
	Region is below the x-axis, so Area $= (4/3)\ln 2 - 89/144$
	$(p = 4/3,\ q = 2,\ r = 89/144)$
6 (a)	$\sin t = (x - 4k)/k, \quad \cos t = (y - 2)/4$
	$\sin^2 t + \cos^2 t = 1$:
	$(x - 4k)^2/k^2 + (y - 2)^2/16 = 1$
6 (b)(i)	Circle $\Rightarrow$ denominators equal: $k^2 = 16$
	$(x - 4k)^2 + (y - 2)^2 = 16$, so radius $r = \sqrt{16} = 4$
6 (b)(ii)	$k = 4$ or $k = -4$; centre is $(4k, 2)$
	Centre $(16, 2)$ or $(-16, 2)$
7	Differentiate $5x - 2xy + 2y^2 - k = 0$ implicitly with respect to x:
	$5 - 2y - 2x(dy/dx) + 4y(dy/dx) = 0$
	$dy/dx = (2y - 5)/(4y - 2x)$
	Tangent parallel to y-axis $\Rightarrow$ denominator = 0: $4y - 2x = 0,\ x = 2y$
	Substitute into the curve: $5(2y) - 2(2y)y + 2y^2 - k = 0$
	$10y - 4y^2 + 2y^2 - k = 0 \Rightarrow 2y^2 - 10y + k = 0$
	$y = [10 \pm \sqrt{(100 - 8k)}]/4$, difference in roots $= 2\sqrt{(100 - 8k)}/4 = \sqrt{(100 - 8k)}/2$
	$\sqrt{(100 - 8k)}/2 = 3 \Rightarrow 100 - 8k = 36 \Rightarrow k = 8$
	$2y^2 - 10y + 8 = 0 \Rightarrow y^2 - 5y + 4 = 0 \Rightarrow (y - 1)(y - 4) = 0$
	$y_P = 1,\ y_Q = 4$ and $x = 2y$
	x-coordinate of P is 2, x-coordinate of Q is 8
8 (a)	$s = ut + \frac{1}{2}at^2 = 2(i + 3j) + \frac{1}{2} \times 2^2 \times (-5i + 2j)$
	$= 2i + 6j - 10i + 4j$
	$s = (-8i + 10j)$ m

Question number	
8 (b)	$v = u + at = (i + 3j) + 2(-5i + 2j)$
	$v = -9i + 7j$
	$\text{Speed} = \lvert v \rvert = \sqrt{((-9)^2 + 7^2)} = \sqrt{130}$
	$= 11.4017\ldots = 11.4 \text{ m s}^{-1}$
9 (a)	Resolve north (↑): $12 \cos 20° + 17 \cos 50° = 22.2037\ldots$
	Resolve east (→): $17 \sin 50° - 12 \sin 20° = 8.91851\ldots$
	$\text{Magnitude} = \sqrt{((12 \cos 20° + 17 \cos 50°)^2 + (17 \sin 50° - 12 \sin 20°)^2)}$
	$= \sqrt{(22.2037\ldots^2 + 8.91851\ldots^2)} = 23.9278\ldots$
	$\tan \theta = (17 \sin 50° - 12 \sin 20°)/(12 \cos 20° + 17 \cos 50°) = 8.91851\ldots/22.2037\ldots$
	$\theta = 21.8837\ldots°$ east of north
	Resultant force is 23.9 N on a bearing of 022°
9 (b)	F is equal and opposite to the resultant:
	Magnitude of F is 23.9 N, bearing $022° + 180° = 202°$
10 (a)	Object (5 kg) in equilibrium, resolve vertically: $T = 5g = 49$ N
	Block, resolve horizontally: $T \cos 50° = X \cos 20°$
	$X = 5g \cos 50° / \cos 20° = 33.5179\ldots$
	$X = 33.5$
10 (b)	Block, resolve vertically: $X \sin 20° + 147 + T \sin 50° = mg$
	$33.5179\ldots \times \sin 20° + 147 + 5g \sin 50° = 9.8m$
	$11.4639\ldots + 147 + 37.5360\ldots = 9.8m \Rightarrow 196 = 9.8m$
	$m = 20$
11 (a)	Weight $0.5g$ acts at the centre G of the lamina.
	Horizontal distance of G from A = $3 \cos 50° - 1 \sin 50°$
	Height of D above A = $6 \sin 50°$

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Moments about A: $R_D \times 6 \sin 50° = 0.5g \times (3 \cos 50° - 1 \sin 50°)$

$6 \sin 50° R_D = 1.5g \cos 50° - 0.5g \sin 50° = 5.69536...$

$R_D = (1.5g \cos 50° - 0.5g \sin 50°)/(6 \sin 50°) = 1.23912...$

Normal contact force = 1.24 N (3 s.f.) as required

11 (b)

Resolve vertically: $R_A = 0.5g = 4.9$ N

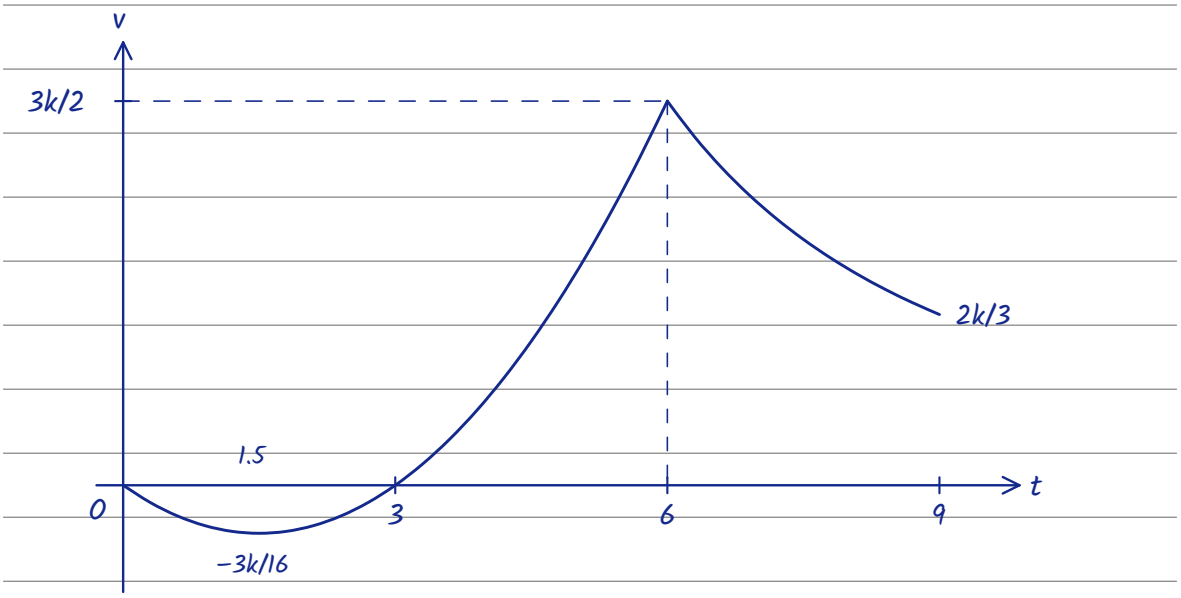
Resolve horizontally: $F_A = R_D = 1.24$ N

Limiting equilibrium: $F = \mu R \Rightarrow 1.24 = 0.5g\,\mu$

$\mu = 1.24/4.9 = 0.25306...$

$\mu = 0.253$

12 (a)



12 (b)

$a = dv/dt = (1/12)k(2t - 3) = 0 \Rightarrow t = 1.5$ s

12 (c)

$v < 0$ for $0 < t < 3$, so distance $= |\int_0^3 v\,dt| + \int_3^6 v\,dt + \int_6^9 v\,dt$

$(k/12)\int (t^2 - 3t)\,dt = (k/12)[\,t^3/3 - (3/2)t^2\,]$

$(k/12)[\,t^3/3 - (3/2)t^2\,]_0^3 = (k/12)(9 - 27/2 - 0) = -(3/8)k$

$(k/12)[\,t^3/3 - (3/2)t^2\,]_3^6 = (k/12)[(72 - 54) - (9 - 27/2)] = (k/12)(45/2) = (15/8)k$

$\int_6^9 54k/t^2\,dt = [\,-54k/t\,]_6^9 = -6k + 9k = 3k$

Total distance: $(3/8)k + (15/8)k + 3k = 8k$

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$(21/4)k = 84$

$k = 16$

13 (a)

$\sin\theta = 0.6 \Rightarrow \cos\theta = 0.8$. Particle mass m kg.

Perpendicular to plane: $R = mg\cos\theta = 0.8mg$

Friction (down the plane, opposing motion): $F = 0.25R = 0.25 \times 0.8mg = 0.2mg$

N2L up the plane: $-0.25 \times 0.8mg - mg \times 0.6 = ma$

$a = -0.8g = -7.84 \text{ m s}^{-2}$

$v^2 = u^2 + 2as = 6^2 + 2 \times (-7.84) \times 1.375$

$v^2 = 36 - 21.56 = 14.44$

$v = \sqrt{14.44} = 3.8 \text{ m s}^{-1}$ as required

13 (b)

A to B: $v = u + at \Rightarrow 3.8 = 6 + (-7.84)t_1$

$t_1 = 2.2/7.84 = 55/196 = 0.280612\ldots$ s

Height of B above ground $= 1.375\sin\theta = 1.375 \times 0.6 = 0.825$ m

B to C (vertical, up positive): $u = 3.8\sin\theta = 3.8 \times 0.6 = 2.28$, $a = -g$, $s = -0.825$

$-0.825 = 2.28t_2 + \frac{1}{2}(-9.8)t_2^2 \Rightarrow 4.9t_2^2 - 2.28t_2 - 0.825 = 0$

$t_2 = [2.28 + \sqrt{(2.28^2 + 4 \times 4.9 \times 0.825)}]/(2 \times 4.9) = 0.704346\ldots$ s $(t_2 > 0)$

$T = t_1 + t_2 = 0.280612\ldots + 0.704346\ldots = 0.984958\ldots$

$T = 0.985$