OCR
Oxford Cambridge and RSA

Tuesday 11 June 2024 – Afternoon

A Level Mathematics A

H240/02 Pure Mathematics and Statistics

Time allowed: 2 hours

135930028

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **16** pages.

ADVICE
- Read each question carefully before you start your answer.

© OCR 2024 [603/1038/8]
DC (ST/CT) 335857/5

OCR is an exempt Charity

stemup.co.uk

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient rule $y = \frac{u}{v}, \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

© OCR 2024

stemup.co.uk

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\overline{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \overline{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\overline{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \overline{x}^2}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0,1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u+v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

© OCR 2024

stemup.co.uk

Section A
Pure Mathematics

1 Differentiate the following with respect to x.

(a) $3x^4 - \frac{2}{x^2}$ **[2]**

(b) $4\sqrt{x} - 9$ **[2]**

2 The vector $\begin{pmatrix} a \\ b \end{pmatrix}$ has magnitude 6 and direction 60° above the positive x-axis.

Determine the exact values of a and b. **[4]**

© OCR 2024

stemup.co.uk

3 The function f is defined by $f(x) = x^3 - x^2 - 5x - 3$.

(a) Show that $(x-3)$ is a factor of $f(x)$. **[1]**

(b) Factorise $f(x)$ completely. **[2]**

Three students attempted to draw the graph of $y = (x-a)(x-1)(x+1)$, each using a different value of the constant a. Not all of their graphs were correct. Their graphs are given in the diagrams below. Copies of the diagrams are provided in the Printed Answer Booklet.

(c) Underneath each diagram in the Printed Answer Booklet,

- **either** give the value of a for which this is the correct graph of $y = f(x)$,
- **or**, if there is no value of a for which this graph is correct, write "No value of a". **[3]**

Fig. 1.1

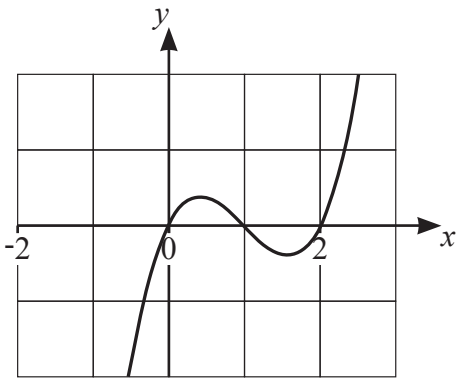


Fig. 1.2

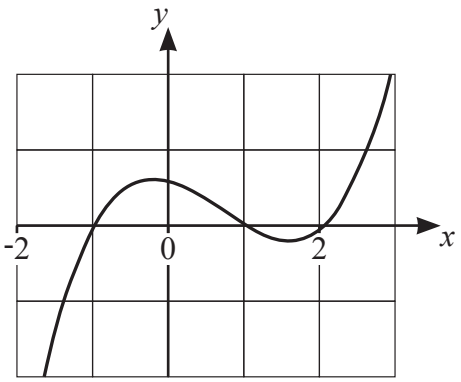
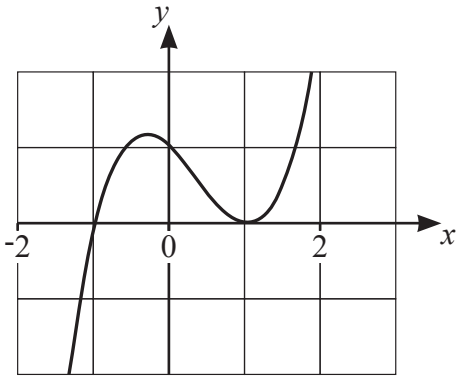
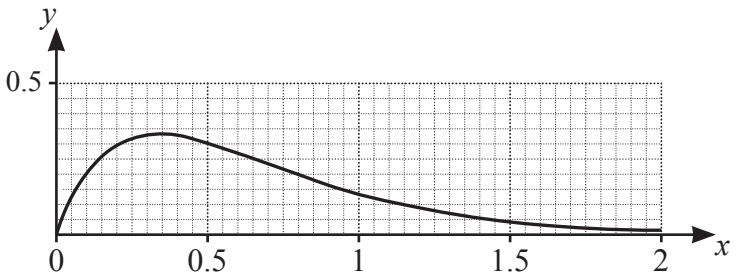


Fig. 1.3



stemup.co.uk

4 The diagram shows part of the graph of $y = xe^{1-3x}$.



(a) Use the sign change method to determine, correct to **2** decimal places, the root of the equation $xe^{1-3x} - 0.2 = 0$, that lies between $x = 0.5$ and $x = 1$. **[3]**

(b) Determine the exact x-coordinate of the maximum point of the curve $y = xe^{1-3x}$. **[3]**

(c) **In this question you must show detailed reasoning.**

Determine the exact area of the region enclosed by the curve $y = xe^{1-3x}$, the x-axis and the line $x = 1$. **[4]**

5 A scientist is monitoring the decline in the population of a certain endangered species of animal in an area where their natural habitat has been damaged.

As a model, the scientist proposes that the rate of decline per year of the population is given by $\frac{1}{80}P^2$, where P is the size of the population t years after the start of the modelling.

(a) Explain how this model gives rise to the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = -\frac{1}{80}P^2.$$ **[1]**

The scientist notes that at the start of the monitoring the population is 120.

(b) Use the model to determine an expression for P in terms of t. **[4]**

(c) Use the model to determine the time it takes for the population to reach 10. **[2]**

The model predicts that the population will never reach zero.

(d) By considering the case when $t \geqslant 160$, or otherwise, comment on the validity of the model for large values of t. **[1]**

© OCR 2024

stemup.co.uk

6 **In this question you must show detailed reasoning.**

(a) **(i)** Use the formula for $\cos(A+B)$, and the double angle formulae, to show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. **[2]**

(ii) Use this result to solve the equation $4\cos^3\theta - 3\cos\theta - \frac{\sqrt{2}}{2} = 0$ for $0° \leqslant \theta \leqslant 180°$. **[3]**

(b) **(i)** Show that $\left(x + \frac{\sqrt{2}}{2}\right)\left(4x^2 - 2\sqrt{2}x - 1\right) = 4x^3 - 3x - \frac{\sqrt{2}}{2}$. **[1]**

(ii) Hence find the exact roots of the equation $4x^3 - 3x - \frac{\sqrt{2}}{2} = 0$. **[2]**

(c) Use the results from parts **(a)(ii)** and **(b)(ii)** to show that $\cos 15° = \frac{\sqrt{2}+\sqrt{6}}{4}$. **[2]**

7 Two arithmetic progressions, A and B, each have 100 terms denoted by a_i and b_i respectively, where $i = 1, 2, 3, \ldots 100$.

The common difference of A is d, where d is a positive integer.

The two progressions have the following properties.

- $a_1 = b_{100} = 4$
- $b_1 = a_{100}$

(a) You are given that there is at least one value of i for which $b_i = 10 + a_i$.

Show that, in this case,

$i = \frac{101}{2} - \frac{5}{d}$. **[6]**

(b) Hence show that it is impossible for the equation $b_i = 10 + a_i$ to hold unless d takes certain values, which should be stated. **[2]**

stemup.co.uk

Section B
Statistics

8 Sweets from a certain manufacturer are sold in packets. Thirty per cent of the sweets are orange, and these are randomly distributed amongst the packets. Each packet contains 15 sweets.

The number of orange sweets in a randomly chosen packet is denoted by X.

(a) Find the following probabilities.

(i) $P(X = 4)$ **[1]**

(ii) $P(X \geqslant 4)$ **[2]**

(b) **(i)** Write down an expression for $P(X = r)$. **[1]**

(ii) Explain the connection between the expression in part **(b)(i)** and the binomial expansion of $(0.7 + 0.3)^n$, for a specific value of n which should be stated. **[2]**

stemup.co.uk

9 **(a)** The masses, M grams, of bags of flour are modelled by the distribution N(1002, 2.25).

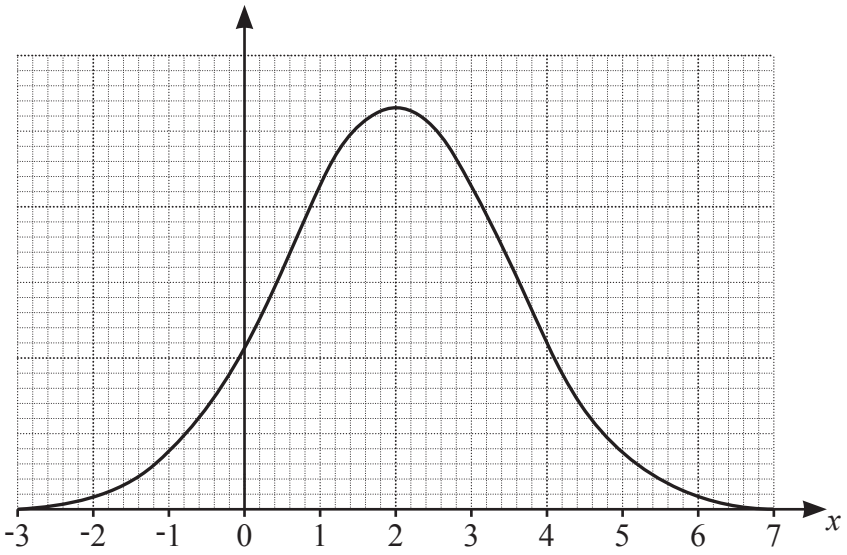
Find P($1000 < M < 1005$). **[1]**

(b) The masses, in grams, of bags of sugar are modelled by the distribution $N(\mu, \sigma^2)$.

You are given that 20% of bags have masses greater than 502 g and 30% of bags have masses less than 499 g.

Determine the values of μ and σ. Give your answers correct to **2** decimal places. **[5]**

(c) The diagram shows the probability distribution of a normal variable, X.



(i) Write down estimates of the x-coordinates of the points of inflection on the graph. **[1]**

(ii) Hence write down an estimate of the standard deviation of X, explaining your method. **[1]**

stemup.co.uk

10 Each month, the manager of a large store records the number, c, of customers who visit the store, and the amount, £h, spent on heating during that month. The manager wants to test whether there is linear correlation between c and h.

For a randomly chosen year the value of Pearson's product-moment correlation coefficient, r, between c and h was -0.798, correct to 3 significant figures.

(a) Using the table below, carry out the test at the 1% significance level. **[5]**

(b) Describe briefly two main features of a scatter diagram that could be drawn to illustrate the values of c and h for this year. There is no need to draw a diagram. **[2]**

(c) The manager makes the following statement.

"The value of r shows that when we spend more on heating, fewer customers visit the store. So we should spend less on heating."

Comment briefly on this statement, making reference to the context. **[2]**

(d) Give a statement about a probability to explain the meaning of the value 0.7155 in the table below. **[2]**

Critical values of Pearson's product-moment correlation coefficient

	1-tail test	5%	2.5%	1%	0.5%
	2-tail test	10%	5%	2%	1%
n	10	0.5494	0.6319	0.7155	0.7646
	11	0.5214	0.6021	0.6851	0.7348
	12	0.4973	0.5760	0.6581	0.7079
	13	0.4762	0.5529	0.6339	0.6835

© OCR 2024

stemup.co.uk

11 The chart below represents the percentage increases (PI) in the numbers of employees using four different methods of travel to work from 2001 and 2011, in five different Local Authorities (LAs) in Wales.

Local Authority	Method of transport			
	Work mainly at or from home	Underground, metro, light rail, tram	Train	Driving a car or van
Caerphilly				
Merthyr Tydfil				
Neath Port Talbot				
Rhondda, Cynon, Taff				
The Vale of Glamorgan				

Key:	$-10\% < PI \leqslant +10\%$	$+10\% < PI \leqslant +30\%$	$+30\% < PI \leqslant +50\%$	$+50\% < PI \leqslant +90\%$	$+90\% < PI$

(a) (i) State, with a reason, which of the four methods of transport probably had the greatest overall percentage growth in these LAs between 2001 and 2011. **[1]**

(ii) Explain why your answer to part **(a)(i)** is **not** definite. **[1]**

(b) A student suggests that the chart can be used to estimate the total percentage change for these methods of transport in each individual LA.

Give **two** reasons why the student is likely to be wrong. **[2]**

(c) A student wants to investigate the trend from 2001 to 2011 in numbers using underground, metro, light rail or tram. The actual numbers of people using these methods in these LAs in 2001 were all less than 50 (and in one case was 4).

Explain why this means that the chart does **not** provide very helpful information for the student. **[1]**

(d) Let D denote the number of people in the Vale of Glamorgan whose usual method of travel to work is "Driving a car or van", and let H denote the number of people in the Vale of Glamorgan who "Work mainly at or from home".

Between 2001 and 2011 the increase in D was approximately 3.5 times the increase in H.

Use this fact and the information in the chart to estimate the ratio $D : H$ in 2001. **[3]**

© OCR 2024

stemup.co.uk

12 Ryan has to choose one student at random from a group of 11 students. Ryan makes the choice using a single throw of two fair, six-sided dice, together with the following table.

Total score on the two dice	2	3	4	5	6	7	8	9	10	11	12
Student chosen	A	B	C	D	E	F	G	H	I	J	K

(a) Show that this sampling method is **not** random. **[2]**

Sasha suggests making the choice using a single throw of two fair, six-sided dice, together with the following table.

Scores on the two dice	1, 1	1, 2	2, 1	1, 3	3, 1	1, 4	4, 1	1, 5	5, 1	1, 6	6, 1
Student chosen	A	B	C	D	E	F	G	H	I	J	K

Ryan says that a further instruction is needed to complete the method.

(b) (i) Write a suitable further instruction. **[1]**

(ii) Using Sasha's method, state the probability of choosing student E. **[1]**

© OCR 2024

stemup.co.uk

13 At an election last year, 20% of voters in Aytown voted for the Now Party. A researcher plans to test whether the proportion of voters in Aytown who support the Now Party this year is greater than 0.2. The hypotheses for the test will be $H_0: p = 0.2$ and $H_1: p > 0.2$.

The researcher surveys a random sample of 200 voters in Aytown and notes the number X who say they support the Now Party.

The random variable Y has a normal distribution which is a good approximation to the distribution of X when the null hypothesis is correct.

The significance level of the test is 5%.

(a) Find the value of y such that $P(Y > y) = 0.05$. **[2]**

(b) Use your answer to part **(a)** to determine the smallest value of X that would result in rejecting the null hypothesis. **[3]**

14 For a certain value of the constant p, the random variable X has the probability distribution given in the table.

x	1	2	3	4
$P(X = x)$	p	$\frac{1}{6}p$	p^2	$\frac{1}{2}$

Two independent values, X_1 and X_2, of X are found.

Determine $P(X_2 = 2X_1 \,|\, X_2 > X_1)$. **[8]**

END OF QUESTION PAPER

stemup.co.uk

BLANK PAGE

© OCR 2024

stemup.co.uk

BLANK PAGE

© OCR 2024

stemup.co.uk

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.

© OCR 2024

stemup.co.uk

Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a) $y = 3x^4 - 2x^{-2}$

$dy/dx = 12x^3 + 4x^{-3} \quad = 12x^3 + \frac{4}{x^3}$

1 (b) $y = 4x^{1/2} - 9$

$dy/dx = 2x^{-1/2} \quad = \frac{2}{\sqrt{x}}$

2 $a = 6\cos 60° = 6 \times \frac{1}{2}$

$a = 3$

$b = 6\sin 60° = 6 \times \frac{\sqrt{3}}{2}$

$b = 3\sqrt{3}$

Check: $a^2 + b^2 = 9 + 27 = 36 = 6^2$ ✓

3 (a) $f(3) = 3^3 - 3^2 - 5 \times 3 - 3 = 27 - 9 - 15 - 3 = 0$

$\therefore$ by the factor theorem, $(x - 3)$ is a factor of $f(x)$

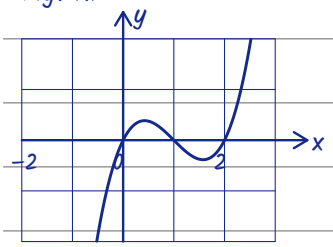
3 (b) $f(x) = (x - 3)(x^2 + 2x + 1)$

$= (x - 3)(x + 1)(x + 1) \quad = (x - 3)(x + 1)^2$

Question number

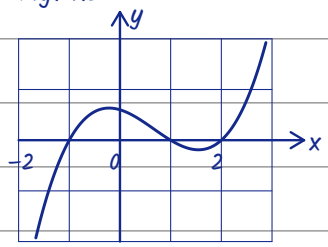
3 (c)

Fig. 1.1



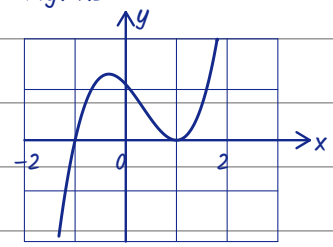
No value of a

Fig. 1.2



$a = 2$

Fig. 1.3



$a = 1$

Roots of $y = (x - a)(x - 1)(x + 1)$ are $x = a, 1, -1$

4 (a) Let $g(x) = xe^{1-3x} - 0.2$

$g(0.785) = 0.785e^{1-2.355} - 0.2 = 0.00249 > 0$

$g(0.795) = 0.795e^{1-2.385} - 0.2 = -0.00099 < 0$

Change of sign and g is continuous, so the root lies between 0.785 and 0.795

$\therefore$ root $x = 0.79$ (2 d.p.)

4 (b) $dy/dx = e^{1-3x} + x(-3e^{1-3x}) = e^{1-3x}(1 - 3x)$

At the maximum $dy/dx = 0$; $e^{1-3x} > 0$ for all x (never 0)

so $1 - 3x = 0 \quad \Rightarrow \quad x = \frac{1}{3}$

Question number

4 (c) By parts: $u = x$, $dv/dx = e^{1-3x} \Rightarrow du/dx = 1$, $v = -\frac{1}{3}e^{1-3x}$

$$\text{Area} = \int_0^1 xe^{1-3x}\,dx = \left[-\tfrac{1}{3}xe^{1-3x}\right]_0^1 + \int_0^1 \tfrac{1}{3}e^{1-3x}\,dx$$

$$= \left[-\tfrac{1}{3}xe^{1-3x}\right]_0^1 - \left[\tfrac{1}{9}e^{1-3x}\right]_0^1$$

$$= \left(-\tfrac{1}{3}e^{-2} - 0\right) - \left(\tfrac{1}{9}e^{-2} - \tfrac{1}{9}e\right)$$

$$= \frac{1}{9}e - \frac{4}{9}e^{-2} \qquad = \frac{e^3 - 4}{9e^2}$$

5 (a) dP/dt is the rate of change of P with respect to time t.

The population is declining at rate $(1/80)P^2$, so P is decreasing and dP/dt is negative: $dP/dt = -(1/80)P^2$

5 (b)

$$\frac{1}{P^2}\,dP = -\frac{1}{80}\,dt$$

$$\int P^{-2}\,dP = \int -\frac{1}{80}\,dt$$

$$-\frac{1}{P} = -\frac{t}{80} + c$$

$t = 0$, $P = 120$: $\quad -\frac{1}{120} = c$

$$\frac{1}{P} = \frac{t}{80} + \frac{1}{120} = \frac{3t + 2}{240}$$

$$P = \frac{240}{3t + 2}$$

Question number

5 (c) $10 = \frac{240}{3t + 2} \Rightarrow 3t + 2 = 24 \Rightarrow 3t = 22$

$t = \frac{22}{3}$ = 7⅓ years (7.33 years)

5 (d) When $t = 160$, $P = \frac{240}{482} = 0.498$, so $P < 1$ for $t \geq 160$

The number of animals must be a whole number, so a population below 1 means the actual population would be 0 (extinct). The model is not valid for large t.

6 (a)(i) $\cos 3\theta = \cos(2\theta + \theta) = \cos 2\theta \cos \theta - \sin 2\theta \sin \theta$

$= (2\cos^2\theta - 1)\cos \theta - 2\sin \theta \cos \theta \sin \theta$

$= 2\cos^3\theta - \cos \theta - 2(1 - \cos^2\theta)\cos \theta$

$= 2\cos^3\theta - \cos \theta - 2\cos \theta + 2\cos^3\theta$

$= 4\cos^3\theta - 3\cos \theta$ as required

6 (a)(ii) $\cos 3\theta = \frac{\sqrt{2}}{2}$ $0° \leq 3\theta \leq 540°$

$3\theta = 45°, 360° - 45° = 315°, 360° + 45° = 405°$

$\theta = 15°, 105°, 135°$

6 (b)(i) $(x + \frac{\sqrt{2}}{2})(4x^2 - 2\sqrt{2}x - 1)$

$= 4x^3 - 2\sqrt{2}x^2 - x + 2\sqrt{2}x^2 - 2x - \frac{\sqrt{2}}{2}$

$= 4x^3 - 3x - \frac{\sqrt{2}}{2}$ as required

Question number

6 (b)(ii) $x + \frac{\sqrt{2}}{2} = 0 \quad \Rightarrow \quad x = -\frac{\sqrt{2}}{2}$

$4x^2 - 2\sqrt{2}x - 1 = 0 \quad \Rightarrow \quad x = \frac{2\sqrt{2} \pm \sqrt{(8+16)}}{8} = \frac{2\sqrt{2} \pm 2\sqrt{6}}{8}$

Roots: $x = -\frac{\sqrt{2}}{2}, \quad x = \frac{\sqrt{2}+\sqrt{6}}{4}, \quad x = \frac{\sqrt{2}-\sqrt{6}}{4}$

6 (c) Put $x = \cos\theta$: the equation in (b)(ii) is $4\cos^3\theta - 3\cos\theta - \frac{\sqrt{2}}{2} = 0$,

so its roots are cos 15°, cos 105°, cos 135° from (a)(ii)

cos 15° > 0, cos 105° < 0, cos 135° < 0

$-\frac{\sqrt{2}}{2} < 0, \quad \frac{\sqrt{2}-\sqrt{6}}{4} < 0, \quad \frac{\sqrt{2}+\sqrt{6}}{4} > 0$

cos 15° is the only positive root, so $\cos 15° = \frac{\sqrt{2}+\sqrt{6}}{4}$

7 (a) $a_i = 4 + (i-1)d$, so $b_1 = a_{100} = 4 + 99d$

$b_{100} = b_1 + 99e = 4 \quad \Rightarrow \quad 4 + 99d + 99e = 4 \quad \Rightarrow \quad e = -d$

so the common difference of B is $-d$

$b_i = 4 + 99d + (i-1)(-d) = 4 + 100d - id$

$b_i = 10 + a_i$: $\quad 4 + 100d - id = 10 + 4 + (i-1)d$

$4 + 100d - id = 14 + id - d$

$2id = 101d - 10$

$i = \frac{101}{2} - \frac{5}{d}$ as required

Question number	
7 (b)	i must be a (positive) integer, but $\frac{101}{2} - \frac{5}{d}$ is not always an integer.
	Need $\frac{5}{d}$ = (an integer) + ½, i.e. $\frac{10}{d}$ is an odd integer
	$d = 2$: $i = 50.5 - 2.5 = 48$ ✓ $d = 10$: $i = 50.5 - 0.5 = 50$ ✓
	Other positive integer d give non-integer i, so impossible
	unless $d = 2$ or $d = 10$
8 (a)(i)	$X \sim B(15, 0.3)$
	$P(X = 4) = {}^{15}C_4 \times 0.3^4 \times 0.7^{11} = 0.21862... = 0.219$
8 (a)(ii)	$P(X \geqslant 4) = 1 - P(X \leqslant 3) = 1 - 0.29687...$
	$= 0.70313... = 0.703$
8 (b)(i)	$P(X = r) = \binom{15}{r} 0.7^{15-r} \times 0.3^r$
8 (b)(ii)	$n = 15$
	$(0.7 + 0.3)^{15} = \Sigma \binom{15}{r} 0.7^{15-r}\, 0.3^r$ $(r = 0$ to $15)$
	$P(X = r)$ is the $(r + 1)$th term of the binomial expansion
	of $(0.7 + 0.3)^{15}$ (and the terms sum to $1^{15} = 1$)
9 (a)	$M \sim N(1002, 1.5^2)$
	$P(1000 < M < 1005) = 0.88604... = 0.886$

Question number	
9 (b)	Let $S \sim N(\mu, \sigma^2)$ be the mass of a bag of sugar
	$P(S > 502) = 0.2 \Rightarrow \Phi^{-1}(0.8) = 0.84162$
	$P(S < 499) = 0.3 \Rightarrow \Phi^{-1}(0.3) = -0.52440$
	$\frac{502 - \mu}{\sigma} = 0.84162$ $\quad$ $\frac{499 - \mu}{\sigma} = -0.52440$
	$502 - \mu = 0.84162\sigma$ and $499 - \mu = -0.52440\sigma$
	Subtract: $3 = (0.84162 + 0.52440)\sigma = 1.36602\sigma$
	$\sigma = 2.19616... \Rightarrow \mu = 502 - 0.84162 \times 2.19616... = 500.1517...$
	$\mu = 500.15$, $\sigma = 2.20$ (2 d.p.)
9 (c)(i)	$x = 0.5$ and $x = 3.5$
9 (c)(ii)	The points of inflection are one standard deviation either
	side of the mean ($\mu \approx 2$): $\sigma \approx 3.5 - 2 = 1.5$
10 (a)	H_0: $\rho = 0$, H_1: $\rho \neq 0$ (two-tailed test)
	where ρ is the population correlation coefficient between
	the number of customers c and the amount spent on heating h
	$n = 12$, 1% two-tail critical value = 0.7079
	$\lvert r \rvert = 0.798 > 0.7079$, so reject H_0
	There is sufficient evidence at the 1% level of a linear
	correlation between the number of customers (c) and the
	amount spent on heating (h)
10 (b)	The points lie fairly close to a straight line
	with a negative gradient (sloping downwards)

Question number	
10 (c)	Correlation does not imply causation. A third factor, e.g. the time of year / outside temperature, may affect both: in cold winter months heating costs are high and fewer customers visit, so spending less on heating would not necessarily bring in more customers.
10 (d)	If there is no correlation in the population, then for a random sample of 10 pairs, $P(r \geq 0.7155) = 0.01$ (the probability that r is at least 0.7155 is 1%)
11 (a)(i)	Train, because it has the darkest shading: all five LAs show increases of over 50% (two of them over 90%).
11 (a)(ii)	The chart only shows ranges, not exact values, and we do not know the sizes of the LAs (a small % of a large number may be more than a large % of a small number).
11 (b)	1. We do not know the initial numbers (proportions) using each method of transport in each LA. 2. We do not know the exact % increases: the ranges are wide and there is no upper limit for the +90% < PI class.
11 (c)	With such small numbers, a small actual change gives a large % increase, so the chart gives a misleading impression of the overall trend.

Question number	
11 (d)	Vale of Glamorgan: D in 10% to 30% class, midpoint 20% H in 30% to 50% class, midpoint 40% Increase in D ≈ 3.5 × increase in H: $0.2D \approx 3.5 \times 0.4H = 1.4H$ $D/H \approx \frac{1.4}{0.2} = 7$ D : H ≈ 7 : 1
12 (a)	P(total 2) = $\frac{1}{36}$ (only 1,1) but P(total 3) = $\frac{2}{36}$ (1,2 or 2,1) so A and B do not have the same probability of being chosen. The probabilities are not equal, so the method is not random.
12 (b)(i)	If the pair of scores is not in the table, throw both dice again; repeat until one of the listed pairs is obtained.
12 (b)(ii)	$P(E) = \frac{1}{11}$
13 (a)	Under H_0: $X \sim B(200, 0.2)$, $\mu = 200 \times 0.2 = 40$, $\sigma^2 = 200 \times 0.2 \times 0.8 = 32$, so $Y \sim N(40, 32)$ $P(Y > y) = 0.05 \Rightarrow y = 40 + 1.6449 \times \sqrt{32} = 49.3047...$ $y = 49.3$
13 (b)	Using $X \sim B(200, 0.2)$: $P(X \geqslant 50) = 1 - P(X \leqslant 49) = 0.04935... < 0.05$ $P(X \geqslant 49) = 1 - P(X \leqslant 48) = 0.06903... > 0.05$ Smallest value of X that leads to rejecting H_0 is 50

Question number

14

$p + \frac{1}{6}p + p^2 + \frac{1}{2} = 1 \quad \Rightarrow \quad 6p^2 + 7p - 3 = 0$

$(3p - 1)(2p + 3) = 0 \quad \Rightarrow \quad p = \frac{1}{3}$ or $p = -\frac{3}{2}$ (invalid, $p > 0$)

$P(X = x)$: $\frac{1}{3}, \frac{1}{18}, \frac{1}{9}, \frac{1}{2}$ for $x = 1, 2, 3, 4$

$P(X_2 = 2X_1 \mid X_2 > X_1) = \frac{P(X_2 = 2X_1 \cap X_2 > X_1)}{P(X_2 > X_1)} = \frac{P(X_2 = 2X_1)}{P(X_2 > X_1)}$

Numerator: (1, 2), (2, 4): $\frac{1}{3} \times \frac{1}{18} + \frac{1}{18} \times \frac{1}{2} = \frac{1}{54} + \frac{1}{36} = \frac{5}{108}$

Denominator: (1,2), (1,3), (1,4), (2,3), (2,4), (3,4):

$\frac{1}{54} + \frac{1}{27} + \frac{1}{6} + \frac{1}{162} + \frac{1}{36} + \frac{1}{18} = \frac{6 + 12 + 54 + 2 + 9 + 18}{324} = \frac{101}{324}$

$P(X_2 = 2X_1 \mid X_2 > X_1) = \frac{5}{108} \div \frac{101}{324} = \frac{15}{101}$ $(= 0.149)$