OCR
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Tuesday 4 June 2024 – Afternoon

A Level Mathematics A

H240/01 Pure Mathematics

Time allowed: 2 hours

133550574 3

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

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Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

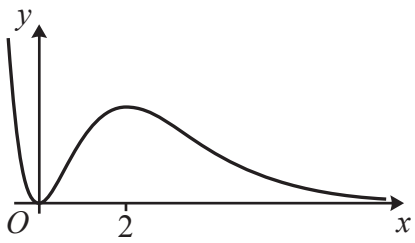
p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u+v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

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1



The diagram shows part of the curve $y = x^2e^{-x}$.

(a) Use the trapezium rule with 4 intervals of equal width to find an estimate for $\int_0^2 x^2e^{-x}dx$. Give your answer correct to **3** significant figures. **[4]**

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for $\int_0^2 x^2e^{-x}dx$. **[1]**

(c) Explain why it is not clear from the diagram whether the value from part **(a)** is an under-estimate or an over-estimate for $\int_0^2 x^2e^{-x}dx$. **[2]**

2 You are given that y is inversely proportional to x^6 and z is directly proportional to the cube root of y.

(a) **(i)** Find an equation for z in terms of x and k, where k is a constant of proportionality. **[2]**

(ii) State which of the diagrams below could represent the graph of z against x. **[1]**

Fig. 1.1

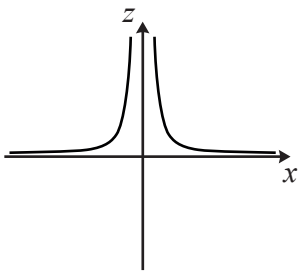


Fig. 1.2

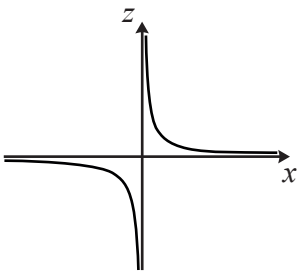


Fig. 1.3

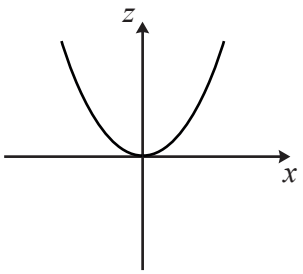
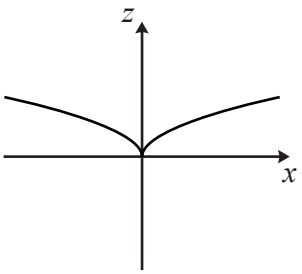


Fig. 1.4



(b) Given that $z = 3$ when $x = 4$, determine the values of x when $z = 12$. **[3]**

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3 **(a)** Find a counterexample to disprove the statement that the product of two prime numbers is always odd. **[1]**

(b) In each of the following cases write one of the symbols $\Rightarrow$, $\Leftrightarrow$, $\Leftarrow$ in the box in the Printed Answer Booklet to make each statement correct.

(i) $x^2 = 3x$ ☐ $x = 3$ **[1]**

(ii) $x > 4$ ☐ $x^3 > 64$ **[1]**

(iii) $x° = 45°$ ☐ $\tan x° = 1$ **[1]**

(c) Prove that the sum of the squares of **any** two odd numbers is always a multiple of 2 but never a multiple of 4. **[4]**

4 A sequence has terms u_1, u_2, u_3, … defined by $u_1 = 2$ and $u_{n+1} = 1 - \frac{1}{u_n}$ for $n \geqslant 1$.

(a) Find the values of u_2, u_3 and u_4. **[2]**

(b) Describe the behaviour of the sequence. **[1]**

(c) Given that $\sum_{n=1}^{k} u_n = 73$, determine the value of k. **[3]**

5 The line $x + 13y = 108$ is the normal to the curve $y = ax^2 + b\sqrt{x}$ at the point (4, 8).

Determine the values of the constants a and b. **[8]**

6 **In this question you must show detailed reasoning.**

The cubic polynomial $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 4x^3 - 25x^2 - 58x + 16$.

(a) Show that $x = \frac{1}{4}$ is a root of the equation $\mathrm{f}(x) = 0$. **[1]**

(b) Hence express $\mathrm{f}(x)$ as the product of a linear factor and a quadratic factor, with all terms in the factors having integer coefficients. **[3]**

(c) Solve the equation $4\mathrm{e}^{3y} - 25\mathrm{e}^{2y} - 58\mathrm{e}^{y} + 16 = 0$, giving each root in the form $y = k\ln 2$ where k is a constant. **[4]**

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7 The point A has coordinates (1, 7), and the point B has coordinates (h, 10).

(a) You are given that the gradient of the line AB is 2.

Find the value of h. **[2]**

(b) You are given that B is the midpoint of AC.

Find the coordinates of the point C. **[2]**

(c) You are given that the straight line through the points A, B and C has two distinct points of intersection with the curve $y = x^2 - 4x + k$.

Determine the set of possible values of k. **[6]**

8 **(a)** State the set of values for which $|x| > x$. **[1]**

(b) You are given that n is an integer such that $|n| \leqslant 9$.

(i) Find the maximum value of $|2n - 1|$. **[1]**

(ii) Find the minimum value of $|2n - 1|$. **[1]**

(c) **(i)** Solve the equation $\left|\frac{1}{2}x - 1\right| = |2x - 3|$. **[3]**

(ii) Explain why the equation $\left|\frac{1}{2}x - 1\right| = 2x - 3$ has only one solution, and state the value of this solution. **[1]**

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9 The depth of the water, d metres, in a tidal river during a given day is modelled by the equation

$$d = 1.9 + 1.1\cos(30t - 60)°$$

where t is the number of hours after midnight.

(A tidal river is one whose level is influenced by tides.)

(a) (i) Find the minimum depth of water given by this model. **[1]**

(ii) Find the value of t when the minimum depth first occurs. **[2]**

(b) A boat can only enter the river when the depth of water is at least 1 metre.

Determine the two periods of time during the day between which this boat will **not** be able to enter the river. Give your answers correct to the **nearest minute**. **[5]**

In reality the depth of the river decreases as this boat travels along the river. An improved model uses the equation

$$d = \mathrm{e}^{-cp}(1.9 + 1.1\cos(30t - 60)°)$$

where c is a positive constant and p is the distance, in kilometres, travelled along the river after entering it.

(c) Explain how this new equation could give an improved model. **[1]**

10 In this question you must show detailed reasoning.

The first three terms of a convergent geometric progression are $2x + 3$, $x + 9$ and $2x - 6$ respectively.

Determine the sum to infinity of this geometric progression. **[8]**

Turn over for questions 11 and 12

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11 A curve has equation $y = 5\ln(1-\cos 2x)$, where x is in radians.

(a) State the values of x for which $5\ln(1-\cos 2x)$ is not defined. **[2]**

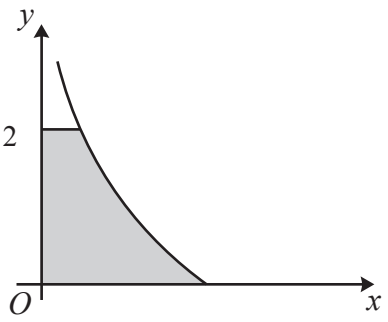
(b) P is the stationary point on the curve that has the smallest positive x-coordinate.

Determine the exact coordinates of P. **[4]**

(c) (i) Show that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 20\mathrm{e}^{-\frac{1}{5}y} = 0$. **[5]**

(ii) State what can be deduced about all of the stationary points on this curve, giving a reason for your answer. **[1]**

12 In this question you must show detailed reasoning.



The diagram shows the curve with parametric equations $x = \dfrac{2}{(2t+1)^4}$, $y = 2t^2 + 3t$ for $t \geqslant 0$.

The shaded region is enclosed by the curve, the x-axis, the y-axis and the line $y = 2$.

(a) Show that the area of the shaded region is given by $\displaystyle\int_a^b \frac{8t+6}{(2t+1)^4}\,\mathrm{d}t$, where a and b are constants to be determined. **[5]**

(b) Determine the exact area of the shaded region. **[6]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a)

$h = (2 - 0) \div 4 = 0.5$

x:	0	0.5	1	1.5	2
y:	0	$0.25e^{-0.5}$	e^{-1}	$2.25e^{-1.5}$	$4e^{-2}$
	= 0	= 0.151633	= 0.367879	= 0.502043	= 0.541341

$$\int_0^2 x^2e^{-x}\,dx \approx \frac{1}{2} \times 0.5 \times \{0 + 0.541341 + 2(0.151633 + 0.367879 + 0.502043)\}$$

$= 0.25 \times \{0.541341 + 2.043109\} = 0.25 \times 2.584450$

$= 0.646112... = 0.646$ (3 s.f.)

1 (b)

Use more trapezia (more intervals), each of a smaller width, over the same interval from $x = 0$ to $x = 2$.

1 (c)

The curve has a point of inflection between $x = 0$ and $x = 2$: it is convex for small x and then concave (gradient starts decreasing). So the tops of the trapezia lie above the curve on part of the interval (over-estimate) but below it on the rest (under-estimate), so it is not clear from the diagram which effect is bigger overall.

2 (a)(i)

$y = \frac{a}{x^6}$, $z = b\sqrt[3]{y}$

$z = b\sqrt[3]{\frac{a}{x^6}} = \frac{b\sqrt[3]{a}}{x^2}$ so $z = \frac{k}{x^2}$ (where $k = b\sqrt[3]{a}$)

2 (a)(ii)

Fig. 1.1

2 (b)

$z = 3$, $x = 4$: $3 = \frac{k}{4^2} = \frac{k}{16} \Rightarrow k = 48$

$z = 12$: $12 = \frac{48}{x^2} \Rightarrow x^2 = 4$

$x = 2$ or $x = -2$

Question number	
3 (a)	2 and 3 are both prime, but $2 \times 3 = 6$, which is even (not odd), so the statement is false.
3 (b)(i)	$x^2 = 3x \quad \Leftarrow \quad x = 3$
3 (b)(ii)	$x > 4 \quad \Leftrightarrow \quad x^3 > 64$
3 (b)(iii)	$x° = 45° \quad \Rightarrow \quad \tan x° = 1$
3 (c)	Let the two odd numbers be $2m + 1$ and $2n + 1$, where m, n are integers. $(2m + 1)^2 + (2n + 1)^2 = 4m^2 + 4m + 1 + 4n^2 + 4n + 1$ $= 4m^2 + 4m + 4n^2 + 4n + 2$ $= 2(2m^2 + 2m + 2n^2 + 2n + 1)$ $2m^2 + 2m + 2n^2 + 2n + 1$ is an integer, so the sum is a multiple of 2. Also sum $= 4(m^2 + m + n^2 + n) + 2$ $4(m^2 + m + n^2 + n)$ is a multiple of 4, but 2 is not a multiple of 4, so the sum always leaves remainder 2 when divided by 4. $\therefore$ it is always a multiple of 2 but never a multiple of 4.
4 (a)	$u_2 = 1 - \frac{1}{2} = \frac{1}{2}$ $u_3 = 1 - \frac{1}{1/2} = 1 - 2 = -1, \qquad u_4 = 1 - \frac{1}{-1} = 1 + 1 = 2$
4 (b)	The sequence is periodic, with period 3 (2, ½, −1, 2, ½, −1, ...).
4 (c)	$u_1 + u_2 + u_3 = 2 + 0.5 - 1 = 1.5$, so each block of 3 terms adds 1.5 $73 \div 1.5 = 48.67 \Rightarrow$ 47 complete blocks: $47 \times 1.5 = 70.5$ $73 = 70.5 + 2.5 = (47 \times 1.5) + 2 + 0.5$ So the sum is 47 blocks of 3 terms plus the next 2 terms (2 and 0.5) $k = 47 \times 3 + 2 = 143$

Question number

5

$(4, 8)$ lies on the curve: $8 = a(4)^2 + b\sqrt{4} \Rightarrow 16a + 2b = 8$

$y = ax^2 + bx^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = 2ax + \frac{b}{2\sqrt{x}}$

At $x = 4$: $\frac{dy}{dx} = 8a + \frac{b}{2 \times 2} = 8a + \frac{1}{4}b$

Normal: $13y = -x + 108 \Rightarrow y = -\frac{1}{13}x + \frac{108}{13}$, gradient $-\frac{1}{13}$

So the gradient of the tangent at $(4, 8)$ is 13.

$8a + \frac{1}{4}b = 13 \Rightarrow 32a + b = 52$

From the first equation: $8a + b = 4$

Subtract: $24a = 48 \Rightarrow a = 2$

$b = 4 - 8(2) = -12$

$a = 2, \quad b = -12$

6 (a)

$f(\frac{1}{4}) = 4(\frac{1}{4})^3 - 25(\frac{1}{4})^2 - 58(\frac{1}{4}) + 16$

$= \frac{1}{16} - \frac{25}{16} - \frac{29}{2} + 16 = -\frac{3}{2} - \frac{29}{2} + 16 = 0$

$f(\frac{1}{4}) = 0$, so $x = \frac{1}{4}$ is a root of $f(x) = 0$.

6 (b)

$x = \frac{1}{4}$ is a root, so $(4x - 1)$ is a factor.

$f(x) = (4x - 1)(x^2 + px - 16)$

Compare x^2 terms: $4p - 1 = -25 \Rightarrow p = -6$

Check x terms: $-64 - p = -64 + 6 = -58$ ✓

$f(x) = (4x - 1)(x^2 - 6x - 16)$

6 (c)

Let $x = e^y$: $4e^{3y} - 25e^{2y} - 58e^y + 16 = f(e^y) = 0$

$x^2 - 6x - 16 = (x - 8)(x + 2)$, so $f(x) = (4x - 1)(x - 8)(x + 2)$

$(4e^y - 1)(e^y - 8)(e^y + 2) = 0$

$e^y = \frac{1}{4}, \quad e^y = 8, \quad e^y = -2$

$e^y = -2$ has no solutions, since $e^y > 0$ for all y.

$y = \ln \frac{1}{4} = \ln 2^{-2} = -2 \ln 2$

$y = \ln 8 = \ln 2^3 = 3 \ln 2$

$y = -2 \ln 2 \quad \text{or} \quad y = 3 \ln 2$

Question number	
7 (a)	Gradient $AB = \frac{10-7}{h-1} = 2 \Rightarrow 3 = 2(h-1) \Rightarrow h - 1 = \frac{3}{2}$
	$h = \frac{5}{2}$
7 (b)	$\frac{1+x}{2} = \frac{5}{2} \Rightarrow x = 4$; $\frac{7+y}{2} = 10 \Rightarrow y = 13$
	C is (4, 13)
7 (c)	Line through A with gradient 2: $y - 7 = 2(x-1) \Rightarrow y = 2x + 5$
	$x^2 - 4x + k = 2x + 5$
	$x^2 - 6x + (k-5) = 0$
	$b^2 - 4ac = (-6)^2 - 4(1)(k-5) = 36 - 4(k-5)$
	Two distinct points of intersection, so $b^2 - 4ac > 0$:
	$36 - 4(k-5) > 0$
	$56 - 4k > 0$
	$k < 14$
8 (a)	$x < 0$
8 (b)(i)	Maximum is 19 (when $n = -9$: $\|2(-9) - 1\| = \|-19\| = 19$)
8 (b)(ii)	Minimum is 1 (when $n = 0$ or $n = 1$: $\|-1\| = \|1\| = 1$)
8 (c)(i)	$\frac{1}{2}x - 1 = 2x - 3 \Rightarrow \frac{3}{2}x = 2 \Rightarrow x = \frac{4}{3}$
	$\frac{1}{2}x - 1 = -(2x-3) = -2x + 3 \Rightarrow \frac{5}{2}x = 4 \Rightarrow x = \frac{8}{5}$
	$x = \frac{4}{3}$ or $x = \frac{8}{5}$
8 (c)(ii)	At $x = \frac{4}{3}$: $2x - 3 = \frac{8}{3} - 3 = -\frac{1}{3} < 0$, but a modulus cannot be
	negative, so $x = \frac{4}{3}$ is not valid. Only solution: $x = \frac{8}{5}$
9 (a)(i)	Minimum when $\cos(30t - 60)° = -1$: $d = 1.9 - 1.1 = 0.8$ m

Question number	
9 (a)(ii)	$\cos(30t - 60)° = -1 \quad \Rightarrow \quad 30t - 60 = 180$
	$30t = 240 \quad \Rightarrow \quad t = 8$
9 (b)	$1.9 + 1.1\cos(30t - 60)° = 1$
	$\cos(30t - 60)° = -\frac{0.9}{1.1} = -0.818181...$
	$30t - 60 = 144.9031...°$ $(\cos^{-1})$
	For $0 \leqslant t < 24$, $-60 \leqslant 30t - 60 < 660$:
	$30t - 60 = 144.903..., \ 215.096..., \ 504.903..., \ 575.096...$
	$30t = 204.903..., \ 275.096..., \ 564.903..., \ 635.096...$
	$t = 6.8301..., \ 9.1698..., \ 18.8301..., \ 21.1698...$
	6.8301 h $= 6$ h 49.8 min $\approx 06{:}50$; 9.1698 h $= 9$ h 10.2 min $\approx 09{:}10$
	18.8301 h $\approx 18{:}50$; 21.1698 h $\approx 21{:}10$
	The depth is less than 1 m (min at $t = 8$ and $t = 20$) between these, so
	the boat cannot enter from 06:50 to 09:10 and from 18:50 to 21:10.
9 (c)	As p increases, e^{-cp} decreases, so the depth gets smaller further along
	the river, and the difference between the maximum and minimum depths
	(the size of the tidal variation) also decreases, as expected upriver.
10	Common ratio: $r = \frac{x + 9}{2x + 3} = \frac{2x - 6}{x + 9}$
	$(x + 9)^2 = (2x + 3)(2x - 6)$
	$x^2 + 18x + 81 = 4x^2 - 12x + 6x - 18 = 4x^2 - 6x - 18$
	$3x^2 - 24x - 99 = 0 \quad \Rightarrow \quad x^2 - 8x - 33 = 0$
	$(x - 11)(x + 3) = 0 \quad \Rightarrow \quad x = 11$ or $x = -3$
	$x = 11$: terms 25, 20, 16, $r = \frac{20}{25} = \frac{4}{5}$
	$x = -3$: terms -3, 6, -12, $r = \frac{6}{-3} = -2$
	S_∞ only exists for $\lvert r\rvert < 1$, and $\lvert -2\rvert = 2 > 1$ (the GP would diverge),
	so $r = -2$ is not valid. Hence $a = 25$, $r = 4/5$.
	$S_\infty = \frac{a}{1 - r} = \frac{25}{1 - 4/5} = \frac{25}{1/5} = 125$

Question number

11 (a)

$\ln$ is only defined for positive values; $1 - \cos 2x \geqslant 0$ always, since $\cos 2x \leqslant 1$.

Not defined when $1 - \cos 2x = 0$, i.e. $\cos 2x = 1$

$2x = 0, \pm 2\pi, \pm 4\pi, \ldots \quad \Rightarrow \quad x = 0, \pm\pi, \pm 2\pi, \ldots$

i.e. $x = k\pi$ for any integer k

11 (b)

$\frac{dy}{dx} = 5 \times \frac{2 \sin 2x}{1 - \cos 2x} = \frac{10 \sin 2x}{1 - \cos 2x}$

$\frac{dy}{dx} = 0 \quad \Rightarrow \quad \sin 2x = 0 \quad \Rightarrow \quad 2x = 0, \pi, 2\pi, \ldots$

$x = 0$ is not in the domain (from (a)), so the smallest positive is $x = \frac{\pi}{2}$

$y = 5 \ln(1 - \cos \pi) = 5 \ln(1 + 1) = 5 \ln 2$ P is $(\frac{\pi}{2}, 5 \ln 2)$

11 (c)(i)

Quotient rule: $\frac{d^2y}{dx^2} = \frac{20 \cos 2x (1 - \cos 2x) - 10 \sin 2x (2 \sin 2x)}{(1 - \cos 2x)^2}$

$= \frac{20 \cos 2x - 20 \cos^2 2x - 20 \sin^2 2x}{(1 - \cos 2x)^2}$

$= \frac{20 \cos 2x - 20(\cos^2 2x + \sin^2 2x)}{(1 - \cos 2x)^2} \qquad (\cos^2 2x + \sin^2 2x = 1)$

$= \frac{20 \cos 2x - 20}{(1 - \cos 2x)^2} = \frac{-20(1 - \cos 2x)}{(1 - \cos 2x)^2} = \frac{-20}{1 - \cos 2x}$

$y = 5 \ln(1 - \cos 2x) \quad \Rightarrow \quad \ln(1 - \cos 2x) = \frac{y}{5} \quad \Rightarrow \quad 1 - \cos 2x = e^{\frac{1}{5}y}$

$\frac{d^2y}{dx^2} = \frac{-20}{e^{\frac{1}{5}y}} = -20e^{-\frac{1}{5}y}$

$\therefore \quad \frac{d^2y}{dx^2} + 20e^{-\frac{1}{5}y} = 0$ as required

11 (c)(ii)

$20e^{-\frac{1}{5}y} > 0$ for all y, so $\frac{d^2y}{dx^2} = -20e^{-\frac{1}{5}y} < 0$ for all x in the domain.

Hence all of the stationary points on the curve are maxima.

12 (a)

Area $= \int x \, dy$ (between $y = 0$ and $y = 2$) $= \int x \, (dy/dt) \, dt$

$y = 2t^2 + 3t \quad \Rightarrow \quad \frac{dy}{dt} = 4t + 3$

Area $= \int \frac{2}{(2t + 1)^4} \times (4t + 3) \, dt = \int \frac{8t + 6}{(2t + 1)^4} \, dt$

Limits: $y = 0$: $2t^2 + 3t = 0 \quad \Rightarrow \quad t(2t + 3) = 0 \quad \Rightarrow \quad t = 0$ (as $t \geqslant 0$)

$y = 2$: $2t^2 + 3t = 2 \quad \Rightarrow \quad 2t^2 + 3t - 2 = 0 \quad \Rightarrow \quad (2t - 1)(t + 2) = 0$

$t = \frac{1}{2}$ or $t = -2$; but $t \geqslant 0$, so $t = \frac{1}{2}$

Area $= \int_0^{\frac{1}{2}} \frac{8t + 6}{(2t + 1)^4} \, dt, \qquad a = 0, \quad b = \frac{1}{2}$

Question number

12 (b)

Let $u = 2t + 1$: $du = 2\,dt$, $8t + 6 = 4(u - 1) + 6 = 4u + 2$

$t = 0 \Rightarrow u = 1$; $t = \frac{1}{2} \Rightarrow u = 2$

$$\int_0^{\frac{1}{2}} \frac{8t + 6}{(2t + 1)^4}\,dt = \int_1^2 \frac{4u + 2}{u^4} \times \frac{1}{2}\,du = \int_1^2 \frac{2u + 1}{u^4}\,du$$

$$= \int_1^2 (2u^{-3} + u^{-4})\,du = \left[-u^{-2} - \frac{1}{3}u^{-3} \right]_1^2$$

$$= \left[-\frac{1}{u^2} - \frac{1}{3u^3} \right]_1^2 = \left(-\frac{1}{4} - \frac{1}{24}\right) - \left(-1 - \frac{1}{3}\right)$$

$$= -\frac{7}{24} + \frac{4}{3} = -\frac{7}{24} + \frac{32}{24}$$

Area $= \frac{25}{24}$