

GCE

Mathematics A

H240/01: Pure Mathematics

A Level

Mark Scheme for June 2024

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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MARKING INSTRUCTIONS**PREPARATION FOR MARKING
RM ASSESSOR**

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.
2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal <http://www.rm.com/support/ca>
3. Log-in to RM Assessor and mark the **required number** of practice responses (“scripts”) and the **number of required** standardisation responses.

MARKING

1. Mark strictly to the mark scheme.
2. Marks awarded must relate directly to the marking criteria.
3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. Annotations

Annotation	Meaning
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

8. Subject Specific Marking Instructions

- a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

- b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

- c. The following types of marks are available.

M

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
 - When a value is **not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
- NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

- g. Rules for replaced work and multiple attempts:
- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
 - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
 - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

- h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
- i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance
1	(a)		$0.5 \times 0.5 \{0 + 4e^{-2} + 2(0.25e^{-0.5} + e^{-1} + 2.25e^{-1.5})\}$	B1	1.1a	State the 4 correct non-zero y-values and no others Exact values (including unsimplified) or decimal equivalents (0, 0.1516, 0.3679, 0.5020, 0.5413), which could be truncated or rounded For the first value, if $0e^0 = 1$ is seen then allow credit for the unsimplified value; if however it is only ever seen as 1 then this is B0 but M1M1 could still be awarded B0 if other ordinates seen, unless clearly not intended to be used
				M1*	1.1a	Attempt to find area between $x = 0$ and $x = 2$, using $k\{y_0 + y_n + 2(y_1 + \dots + y_{n-1})\}$ Big brackets need to be seen or implied Attempts at y-values must be correctly placed (but no need to see $y = 0$ explicitly) If no earlier evidence of y-values seen (eg in a table) then allow M1 for the correct structure with 4 of the 5 values being correct Condone using more than 4 intervals as long as values equally spaced between $x = 0$ and $x = 2$
				M1d*	1.1a	Use $k = 0.5 \times 0.5$ so
			= 0.646	A1	1.1	Obtain 0.646 Dep on previous M1 Or using $k = 0.5h$, with h consistent with their different number of intervals Allow answers $> 3sf$, as long as they round to 0.646 A0 if not using 4 strips, even if 0.646 is obtained No credit if no evidence of using the trapezium rule shown

Question			Answer	Marks	AO	Guidance	
				[4]			Using separate strips (a triangle and then trapezia) is an acceptable method, and marks should be awarded as per the main MS (ie y-values / structure / widths / final answer)
1	(b)		Use more trapezia, of a lesser width, over the same interval	B1 [1]	2.4	Convincing reason	Allow just ‘more trapezia’ or ‘narrower trapezia’ Could refer to strips or intervals
1	(c)		E.g. There is a point of inflection within the given range... ... so the trapezia initially over-estimate but then under-estimate	B1 B1 [2]	2.4 2.2a	Curve is both convex and concave The tops of trapezia are both above and below the curve	Comment about the shape Referring to increasing and decreasing gradients is correct, but increasing and decreasing curve is not Allow BOD if muddles about which part of the curve is convex and which is concave Comment about the estimates If candidates refer to ‘it’ rather than ‘trapezia’ then allow BOD B marks are independent See appendix for further examples

Question			Answer	Marks	AO	Guidance	
2	(a)	(i)	$y = \frac{a}{x^6} \quad z = b\sqrt[3]{y}$ Hence $z = k\sqrt[3]{\frac{1}{x^6}}$ Equation is $z = \frac{k}{x^2}$	M1	3.1a	Attempt at least one equation, involving a constant of proportionality	Allow BOD if the constants of proportionality are the same in two equations Allow $\propto$ to be used
				A1	2.1	Correct simplified equation seen	Equation must be simplified, so A0 for eg $z = k\sqrt[3]{\frac{1}{x^6}}$ Must involve just a single constant of proportionality ie k A0 if the same constant of proportionality was used in both initial equations, or if k was used in either of the initial equations
	(a)	(ii)	Identify Fig. 1.1	B1	3.2a	Not dependent on correct equation in (i)	B0 if more than one Fig. identified
				[1]			
2	(b)		$3 = \frac{k}{16}$ $k = 48$ $\frac{48}{x^2} = 12$ $x^2 = 4$ $x = \pm 2$	M1*	1.1	Use $x = 4$ and $z = 3$ to attempt to find k from their equation of proportionality	Their equation must involve x , z and k As far as attempting k
				M1d*	1.1	Attempt to find x using $z = 12$ and their numerical k Dependent on previous M1	Their equation involving x , z and their k Attempt at least one value of x
				A1	1.1	Both values required	Must have had correct final equation in (a)(i) , but could follow A0 if constants of proportionality were not dealt with correctly
				[3]			

Question			Answer	Marks	AO	Guidance	
3	(a)		$2 \times 3 = 6$ which is even, hence counterexample	B1 [1]	2.1	Any product involving 2 and a prime number, evaluated and contradiction identified	eg $2 \times 3 = 6$, which is not odd Condone $2 \times 2 = 4$, which is not odd
3	(b)	(i)	$x^2 = 3x \Leftarrow x = 3$	B1 [1]	2.2a	Correct symbol used	Condone $\Leftarrow$
3	(b)	(ii)	$x > 4 \Leftrightarrow x^3 > 64$	B1 [1]	2.2a	Correct symbol used	Condone $\leftrightarrow$
3	(b)	(iii))	$x^0 = 45^0 \Rightarrow \tan x^0 = 1$	B1 [1]	2.2a	Correct symbol used	Condone $\rightarrow$

Question			Answer	Marks	AO	Guidance	
3	(c)		$(2m + 1)^2 + (2n + 1)^2$	B1	2.1	Correct form seen for the sum of the squares of any two odd numbers	ie two different variables
		$= 4m^2 + 4m + 1 + 4n^2 + 4n + 1$ $= 4m^2 + 4m + 4n^2 + 4n + 2$	M1	2.1	Attempt to square, add and collect like terms for their two distinct odd numbers	Their odd numbers must both be of the form $2p \pm q$ (where q is odd) May involve a single variable eg $(2n + 1)^2 + (2n + 3)^2$ $= 4n^2 + 4n + 1 + 4n^2 + 12n + 9$ $= 8n^2 + 16n + 10$ Allow sign and/or coefficient errors only	
		$2(2m^2 + 2n^2 + 2m + 2n + 1)$ hence multiple of 2	A1FT	2.4	Show it is a multiple of 2, by taking out a common multiple or arguing that the coefficients in all terms are even	FT on their two odd numbers Factorising by 2 is sufficient for A1 ie no comment required Condone dividing by 2 to show that the quotient would be an integer	
		$4(m^2 + n^2 + m + n) + 2$ $4(m^2 + n^2 + m + n)$ is multiple of 4, but 2 is not multiple of 4, so never multiple of 4	A1	2.4	Not dep on previous A1 , but must follow B1 M1 Take out a common multiple from relevant terms, or argue using coefficients of terms, or take out common factor of 4 and argue that remaining factor is not an integer	Must be from any two odd numbers (ie two different variables) Condone dividing by 4 to show that the quotient is not an integer Comment required – either refer to the remainder of 2 (including ‘2 more than a multiple of 4’), or that the entire quotient is not an integer, depending on method used SC B1 for a complete worded argument about two distinct odd numbers eg odd ² = odd for both odd numbers; odd + odd = even, hence multiple of 2	
				[4]			

Question			Answer	Marks	AO	Guidance	
4	(a)		$u_2 = \frac{1}{2}$ $u_3 = -1, u_4 = 2$	B1 B1FT [2]	1.1 1.1	Or 0.5 FT their u_2	Must be seen as $\frac{1}{2}$ and not just $1 - \frac{1}{2}$ Both as simplified numerical values
4	(b)		Periodic, with period 3	B1 [1]	1.2	Any correct description, such as repeating Condone just 'periodic' without the period being stated	ISW an incorrect period eg 'periodic with period 4' B0 if additional incorrect description eg 'periodic AP' Allow recurring, repetitive, cyclic etc Condone looping, circling etc Do not allow harmonic or alternating, even if with another correct description B0 for divergent or oscillating, unless additional detail eg between 3 values Must have a periodic sequence in (a) (with period of at least 3) to gain credit for description See appendix for further examples
4	(c)		$u_1 + u_2 + u_3 = 2 + 0.5 - 1 = 1.5$ so total goes up by 1.5 each time so $73 = 70.5 + 2 + 0.5$ $= (47 \times 1.5) + 2 + 0.5$ $k = 143$	M1 M1 A1 [3]	3.1a 1.1 1.1	Identify that every block of three terms will increase the total by 1.5 (allow use of $2 + 0.5 - 1$ instead) Attempt to identify the number of terms needed eg 47 blocks plus 2 more terms Obtain $k = 143$	Can still award M1 if using the sum of three of their consecutive terms Must have a periodic sequence in (a) to gain any credit for method (but condone one with a period other than 3) $73 \div 1.5$ is sufficient for M1 M1 Allow M1 if using blocks of 1.5 to try to find a sum of 73 eg $48 \times 1.5 = 72$ Can still get M1 if attempting to use the sum of their three terms

Question	Answer	Marks	AO	Guidance
5	$16a + 2b = 8$ $\frac{dy}{dx} = 2ax + \frac{b}{2\sqrt{x}}$ $\frac{dy}{dx} = 8a + \frac{1}{4}b$ gradient of tangent is 13 OR gradient of normal is $-\frac{1}{8a + \frac{1}{4}b}$ or gradient of normal is $-\frac{1}{2ax + \frac{b}{2\sqrt{x}}}$ OR $(8a + \frac{1}{4}b) \times -\frac{1}{13} = -1$	B1 M1 A1 M1 M1	3.1a 2.1 1.1 1.1 1.1	Substitute (4, 8) into the equation of the curve Attempt differentiation Obtain correct derivative Use $x = 4$ correctly in their derivative Attempt to use the relationship between the gradients of perpendicular lines Seen anywhere in solution Allow for unsimplified equation, even if error then occurs To obtain derivative of the form $px + qx^{0.5}$ Can still be awarded if p and q now incorrect numerical values Must be an attempt at differentiation, but could still follow M0 Their derivative could now be part of an equation or an attempt at a perpendicular gradient Attempt the gradient of the tangent, using attempt at gradient of given normal (condone $-\frac{1}{13}x$ if recovered) OR Attempt gradient of the normal using their derivative (either algebraic or in terms of a and b) Condone slips with fractions within fractions as long as intent is clear NB $(8a + \frac{1}{4}b) \times -\frac{1}{13} = -1$ would imply next M1 as well

Question			Answer	Marks	AO	Guidance	
6	(a)		DR $f\left(\frac{1}{4}\right) = 4\left(\frac{1}{4}\right)^3 - 25\left(\frac{1}{4}\right)^2 - 58\left(\frac{1}{4}\right) + 16$ $= \frac{1}{16} - \frac{25}{16} - \frac{29}{2} + 16 = 0$	B1	2.1	Show $f\left(\frac{1}{4}\right) = 0$, some detail required	B0 for just $f(0.25) = 0$, but condone seeing either just the substitution or just the evaluated terms Could use division by $(4x - 1)$ or $(x - \frac{1}{4})$ but must identify remainder of 0
				[1]			
6	(b)		DR $(4x - 1)(x^2 - 6x - 16)$	B1	2.2a	Identify factor of $(4x - 1)$	Allow factor of $(x - \frac{1}{4})$
				M1	1.1	Obtain complete division by $(4x - 1)$ or $(1 - 4x)$	Must be a complete method ie attempt all 3 terms to obtain x^2 and one other correct term (allow one slip in method) Could be implied by $A = 1$ and one other correct if using coefficient matching Condone division by $(x - \frac{1}{4})$, to obtain $4x^2$ and either $-24x$ or -64
				A1	1.1	Obtain correct product	Integer coefficients now required Must be written as a product , so cannot be implied by eg correct quotient appearing following division by $(4x - 1)$ but the two factors never combined Could be $(1 - 4x)(-x^2 + 6x + 16)$

Question			Answer	Marks	AO	Guidance	
				[3]			If division was used in part (a) then quotient must appear in part (b), but evidence for B1M1 could be in (a) If $(x - 8)(x + 2)$ seen before the quadratic factor then both roots must be justified (eg factor theorem), otherwise M0 (but could still get B1)
6	(c)		DR $(4e^y - 1)(e^y - 8)(e^y + 2)$ $e^y = \frac{1}{4}, e^y = 8, e^y = -2$ $y = \ln \frac{1}{4}, y = \ln 8$ $y = -2\ln 2, y = 3\ln 2$ $e^y = -2$ has no solutions as $e^y > 0$ for all y	M1 A1 A1 B1	3.1a 1.1 1.1 2.3	Attempt to find y from at least one positive root for e^y Obtain at least one correct solution in the required form Obtain both correct solutions in required form Reject $e^y = -2$ with a reason	Attempt to link e^y to the root(s) of the cubic in x , and then solve $e^y = k$ to obtain $y = \ln k$, where k is one of their positive roots $y = -2\ln 2$ comes from the given root, but $y = 3\ln 2$ must come from the correct solution of the correct quadratic Must come from the correct solution of the correct quadratic Allow BOD if $\ln(-2)$ also seen Must have some reason, eg 'e ^y is always positive', 'e ^y cannot be negative', 'cannot take log of a negative number', 'not defined', 'not real', 'no solutions' B0 for 'math error', 'does not work', 'not possible', N/A etc $e^y = -2$ must come from the correct solution of the correct quadratic
				[4]			

Question			Answer	Marks	AO	Guidance	
7	(a)		$\frac{10-7}{h-1} = 2$	M1	1.1	Equate attempt at gradient of line to 2	Must be attempting to use $\frac{y_2 - y_1}{x_2 - x_1}$, with consistent order in numerator and denominator; allow one sign slip Could also use informal methods
			$h = \frac{5}{2}$	A1 [2]	1.1	Obtain $h = \frac{5}{2}$ oe	
7	(b)		(4, 13)	B1FT	1.1	Obtain correct x coordinate for C, following their h ie $2h - 1$	Must be clear that their $2h - 1$ is the x coordinate BOD if brackets omitted Must be clear that 13 is the y coordinate BOD if brackets omitted SC B1 if both values correct but given as a vector not a coordinate
				B1	1.1	Obtain correct y coordinate of 13	
				[2]			

Question			Answer	Marks	AO	Guidance	
7	(c)		$y - 7 = 2(x - 1)$	M1	3.1a	Attempt equation of line through A	Allow one sign slip, but M0 if x and y coordinates transposed Could use their B or C instead, but must still be using gradient of 2
			$x^2 - 4x + k = 2x + 5$	M1	1.1	Equate line and curve	
			$x^2 - 6x + (k - 5) = 0$	A1	1.1	Obtain correct quadratic, with like terms collected	Condone no '= 0'
			$b^2 - 4ac = 36 - 4(k - 5)$	M1*	3.1a	Attempt discriminant of 3 term quadratic, resulting from equating line and curve	Correct discriminant for their quadratic Condone any inequality or equality, or no, sign for this mark M0 if using just $x^2 - 4x + k$ If the discriminant is initially embedded in the quadratic formula, then M1 is only awarded when it is considered in isolation
			Two points of intersection so $b^2 - 4ac > 0$ $36 - 4(k - 5) > 0$	M1d*	1.1	Use $b^2 - 4ac > 0$	Inequality sign could be implied by final answer M0 if incorrect inequality sign, including $b^2 - 4ac \geq 0$
			$56 - 4k > 0$ $k < 14$	A1	1.1	Obtain $k < 14$	
				[6]			
			Alt method for final 3 marks				
			$(x - 3)^2 - 9 + k - 5$	M1*		Attempt completed square form	Correct expression for their quadratic Allow unsimplified
			$k - 14 < 0$	M1d*		Set constant term < 0	M0 if constant term ≤ 0 May see more informal method to determine inequality sign
			$k < 14$	A1		Obtain $k < 14$	

Question			Answer	Marks	AO	Guidance	
			Alt method (using differentiation to find point of intersection)				
			$2x - 4 = 2$ $x = 3$	M1		Differentiate equation of curve, equate to 2 and attempt x	
			$y - 7 = 2(x - 1)$	M1		Attempt equation of line through A	Could use their B or C instead
			$y = 11$ $9 - 12 + k = 11$	M1		Attempt y value from line, and use their (3, 11) in equation of curve to attempt k	
			$k = 14$	A1		Obtain $k = 14$	
			One point of intersection when $k = 14$. It is a positive quadratic so translate in negative y direction for two points of intersection	M1		Clear method to determine inequality sign	Could be algebraic or a sketch Could be implied by final answer
			$k < 14$	A1		Obtain $k < 14$	

Question			Answer	Marks	AO	Guidance	
8	(a)		$x < 0$	B1 [1]	2.2a	Correct inequality	B0 for $x \leq 0$ Could use interval notation ie $(-\infty, 0)$ Condone an incorrect attempt at set notation, as long as intention is clear
8	(b)	(i)	max value is 19 (from $n = -9$)	B1 [1]	1.1	State correct value, and no other	Value of n not required, but B0 if 19 comes from a clearly incorrect n B0 if additional solution B0 for $n \leq 19$
8	(b)	(ii)	min value is 1 (from $n = 0$ and/or 1)	B1 [1]	1.1	State correct value, and no other	Value of n not required, but B0 if 1 comes from a clearly incorrect n B0 if additional solution B0 for $n \geq 1$
8	(c)	(i)	$\frac{1}{2}x - 1 = 2x - 3$ $x = \frac{4}{3}$ $\frac{1}{2}x - 1 = -2x + 3$ $x = \frac{8}{5}$	B1 M1 A1 [3]	1.1 1.1 1.1	Obtain $x = \frac{4}{3}$ oe Attempt to solve equation with all signs reversed on one side of the equation, or square both sides and attempt to solve Obtain $x = \frac{8}{5}$ oe	M0 for eg $\frac{1}{2}x - 1 = -2x - 3$ Maximum of 2 marks if additional solutions

Question			Answer	Marks	AO	Guidance	
				[1]			
Question			Answer	Marks	AO	Guidance	
9	(a)	(i)	0.8 m	B1 [1]	3.4	State 0.8 m, units required	Units may be given as m or metres Could be $\frac{4}{5}$ m Could be 80 cm
9	(a)	(ii)	$\cos(30t - 60) = -1$ $30t - 60 = 180$ $t = 8$ (hours)	M1 A1 [2]	3.4 3.4	Identify that minimum occurs when $\cos(30t - 60)$ is -1 , so need $30t - 60 = 180$ No units needed, as value of t is requested	M1 does not require attempt at solution for t No FT on an incorrect d being used from the previous part eg $d = 2.45$ from using $t = 0$ Condone 0800 or 8am Ignore additional values of t that are greater than 8, but A0 for a smaller positive value of t also given
9	(b)		$1.9 + 1.1\cos(30t - 60) = 1$ $\cos(30t - 60) = -0.8181\dots$ $30t - 60 = 144.903$ $30t = 204.90$ $t = 6.830\dots$ $30t - 60 = 215.097, 504.903, 575.097$ $30t = 275.097, 564.903, 635.097$ $t = 9.169\dots, 18.830\dots, 21.169\dots$	M1 A1 M1	3.3 1.1 3.4	Equate model to 1, rearrange and use $\cos^{-1}$ Obtain correct first value of t Attempt all further values of t within $0 < t < 24$	As far as $30t - 60 = k$, using correct order of operations Allow M1 if working in radians (gives $30t - 60 = 2.529$) Implied by first time of 0650 with no errors seen 3sf or better Ignore inequality signs if used Using a valid method M0 if using radians Values of t could also be found using the symmetry of the curve eg $8 + (8 - 6.83) = 9.17$

Question			Answer	Marks	AO	Guidance	
				A1	1.1	Obtain the further 3 correct values and no others	Allow answers to 3sf Ignore inequality signs if used Correct time periods would imply t values
			River cannot be entered 0650 to 0910 and 1850 to 2110	A1	3.2a	Correct two periods, given as time intervals	Could also be given as 6:50am to 9:10am, and 6:50pm to 9:10pm Must be given as intervals and not just times eg A0 for ‘0650 and 0910’ etc BOD if correct intervals given following any incorrect inequality signs Accept 0649 to 0911 and 1849 to 2111 www (from checking times and realising that rounded answers give depths of less than 1 metre) A0 if giving answers in minutes, or hours and minutes, after midnight and not times (eg ‘410 minutes to 550 minutes’ or ‘6 hours 50 minutes to 9 hours 10 minutes’) Condone attempt at interval notation / inequalities as long as intention is clear, and allow BOD if written as a strict inequality such as $0650 < t < 0910$ Special Case If M1A1M0 awarded, then allow SC B1 for giving a correct time period eg 0650 to 0910
				[5]			

Question			Answer	Marks	AO	Guidance	
9	(c)		As p increases, e^{-cp} decreases so difference between max / min depths will decrease	B1 [1]	3.5c	Any sensible suggestion that suggests that amplitudes of the tides will be reduced due to the exponential term	Must refer to the effect of the exponential term in context in some way Condone reference to the exponential term having a ‘damping’ effect on the tides Cannot just restate the question eg ‘river gets shallower’

Question			Answer	Marks	AO	Guidance	
10			DR $r(2x + 3) = x + 9$; $r(x + 9) = 2x - 6$ $r^2(2x + 3) = 2x - 6$ $\frac{x+9}{2x+3} = \frac{2x-6}{x+9}$ oe $x^2 + 18x + 81 = 4x^2 - 12x + 6x - 18$ $3x^2 - 24x - 99 = 0$ $(x - 11)(x + 3) = 0$ $x = 11, x = -3$ $r = \frac{4}{5}, r = -2$ $S_{\infty} = \frac{a}{1-r} = \frac{25}{1-\frac{4}{5}}$ $S_{\infty} = 125$	B1 M1* A1 A1 A1 M1d* A1	3.1a 3.1a 1.1 2.1 1.1 3.2a 1.1	Obtain any correct equation in terms of r and x Attempt equation in terms of only x Obtain any correct equation not involving fractions or brackets Solve quadratic BC to obtain both correct x values Obtain at least $r = \frac{4}{5}$ Attempt sum to infinity, using correct formula, with their r and their attempt at a Obtain 125 only	Could be implied by later work May use other than r This equation would imply the B1 Or correct equation in terms of only r May still have like terms not yet combined May result in a cubic depending on method (probably $6x^3 - 39x^2 - 270x - 297 = 0$) Or solve cubic, to obtain three correct roots (third is likely to be $x = -1.5$) If second value of r given then it must be correct (if third value given then it must be consistent with their correct cubic roots) Must be using their numerical values for a and r with $ r < 1$ ISW using additional value(s) of r M0 if using their x and not attempt at a A0 if additional solution

Question			Answer	Marks	AO	Guidance	
			S_{∞} only exists for $ r < 1$, so $r = -2$ is not a valid solution	B1	2.5	Clear explanation as to why $r = -2$ is discarded	Must be considering correct r value, so B0 if rejecting $x = -3$ as $ -3 > 1$ Could generate the terms $-3, 6, (-12)$ and hence conclude with ‘divergent sequence’ If additional solutions for x and/or r from cubic then they must also be correct and explicitly rejected
				[8]			NB Eliminating x not r is a valid method, and could gain full credit. When solving their quadratic there is no need to see $r = -2$ (and hence $x = -3$) to award the A marks
Question			Answer	Marks	AO	Guidance	
11	(a)		$\cos 2x = 1$	M1	3.1a	Set $\cos 2x = 1$ soi	$1 - \cos 2x = 0$, then $x = 0$, would imply M1 Allow $\cos 2x \geq 1$, but not $\cos 2x \leq 1$ (unless recovered by final answer) Allow $\cos 2x \neq 1$ if considering the values that x cannot take
			$x = 0, \pm\pi, \dots$ $x = k\pi$ for $k \in \mathbb{Z}$	A1	2.5	Identify all multiples of π , including negatives	Allow any clear notation, but must include negative integers as well eg $x = 0, \pm\pi, \pm 2\pi \dots$ (allow ‘etc’ for ‘...’) Condone working in degrees, as long as final answer is in radians
				[2]			

Question			Answer	Marks	AO	Guidance	
11	(b)		$\frac{dy}{dx} = \frac{10\sin 2x}{1 - \cos 2x}$	M1	1.1	Attempt to differentiate	Obtain $\frac{k \sin 2x}{1 - \cos 2x}$, or unsimplified equiv Other derivatives may be seen if trig identities, or log laws, used before differentiation; allow coefficient errors only Could use implicit differentiation on $1 - \cos 2x = e^{\frac{1}{5}y}$ oe to obtain $a \sin 2x = b e^{\frac{1}{5}y} \frac{dy}{dx}$
			$\frac{10\sin 2x}{1 - \cos 2x} = 0$	M1	1.1	Equate their derivative to 0 and attempt to solve for a non-zero x value	Allow M1 as long as their numerator involves a trig term Allow M1 if working in degrees
			$x = \frac{1}{2}\pi$	A1	1.1	Obtain $x = \frac{1}{2}\pi$ only	Must be exact A0 if $x = 0$ also given in final answer
			$y = 5\ln 2$	A1	1.1	Obtain $y = 5\ln 2$ (or $\ln 32$)	Must be exact, simplified, and from $x = \frac{1}{2}\pi$ Allow A1 if $5\ln 2$ comes from 90°
				[4]			
11	(c)	(i)	$\frac{d^2y}{dx^2} = \frac{(20\cos 2x)(1 - \cos 2x) - (10\sin 2x)(2\sin 2x)}{(1 - \cos 2x)^2}$ OR $(20\cos 2x)e^{-\frac{1}{5}y} + (10\sin 2x)\left(-\frac{1}{5}e^{-\frac{1}{5}y}\right)\frac{dy}{dx}$	M1*	3.1a	Attempt differentiation using an appropriate method on their first derivative	Starting with $\frac{k \sin 2x}{1 - \cos 2x}$, or a multiple of any other correct first derivative, including eg $k \cot x$ Must be correct structure for the differentiation method being attempted, allowing coefficient errors only Could use implicit differentiation on $\frac{dy}{dx} = 10\sin 2x \times e^{-\frac{1}{5}y}$

Question			Answer	Marks	AO	Guidance	
			$= \frac{20\cos 2x - 20\cos^2 2x - 20\sin^2 2x}{(1 - \cos 2x)^2}$ $= \frac{20\cos 2x - 20}{(1 - \cos 2x)^2}$ $\frac{d^2y}{dx^2} = \frac{-20(1 - \cos 2x)}{(1 - \cos 2x)^2} = \frac{-20}{1 - \cos 2x}$ $\frac{-20}{1 - \cos 2x} = \frac{-20}{e^{\frac{1}{5}y}} = -20e^{-\frac{1}{5}y}$ OR $20e^{-\frac{1}{5}y} = \frac{20}{e^{\frac{1}{5}y}} = \frac{20}{1 - \cos 2x}$	A1	2.1	Any correct derivative, including unsimplified	If using implicit differentiation then A1 can be awarded if $\frac{dy}{dx}$ is still present
				M1d*	2.1	Attempt to simplify their second derivative using at least one trigonometric identity correctly	Only award M1 for trig identities used after differentiation
				M1	2.4	Correctly replace $1 - \cos 2x$ with $e^{\frac{1}{5}y}$ or vice versa	Used either in their second derivative or in the given answer If using implicit differentiation then the M1 will be awarded before the differentiation attempt
			$\frac{d^2y}{dx^2} = -20e^{-\frac{1}{5}y}$ $\frac{d^2y}{dx^2} + 20e^{-\frac{1}{5}y} = 0. \quad \mathbf{A.G.}$	A1	2.1	Obtain / confirm given answer www	Penalise any clearly incorrect equations, but allow BOD if denominator disappears (eg when using trig identities) but then reappears when relevant
				[5]			

Question			Answer	Marks	AO	Guidance	
11	(c)	(ii)	$20e^{-\frac{1}{5}y} > 0$ for all y, so $\frac{d^2y}{dx^2} < 0$ for all x, hence stationary points are all maxima	B1 [1]	2.2a	Correct conclusion, with justification	Refer to the exponential term being positive, hence second derivative must be negative, hence maxima Could refer to e^k not $e^{-\frac{1}{5}y}$ Could refer to the correct second derivative of $\frac{-20}{1 - \cos 2x}$ and explain why this is always negative, hence maxima (so no need to refer to exponential term with this approach)

Question		Answer	Marks	AO	Guidance	
12	(a)	DR area = $\int x \frac{dy}{dt} dt$ $\frac{dy}{dt} = 4t + 3$ hence $\int \frac{2}{(2t+1)^4} (4t+3) dt$ $\int \frac{8t+6}{(2t+1)^4} dt$ A.G. $a = 0$, from $t = 0$ oe $2t^2 + 3t = 2$ $(2t-1)(t+2) = 0$ $t = \frac{1}{2}$ $t = -2$ but $t > 0$, so $b = \frac{1}{2}$	M1 A1 B1 M1 A1	1.2 2.1 2.2a 2.1 2.1	Attempt $\int x \frac{dy}{dt} dt$ in terms of t, detail required Obtain correct given integrand Determine correct lower limit from solving equation Equate expression for y to 2 Obtain $b = \frac{1}{2}$ as upper limit www	Clear indication that integrand is given by $\int x \frac{dy}{dt} dt$ (condone just $\int x \frac{dy}{dt}$), along with $\frac{dy}{dt} = 4t + 3$ and full substitution into integrand Condone no dt in integrand in initial statement and/or when substituting May instead see integrand as $\int x dy$ with $dy = (4t + 3)dt$ dt required throughout Evidence for $a = 0$ required eg $2t^2 + 3t = 0$ $a = 0$ doesn't need to be seen explicitly, and could be implied by 0 appearing as the lower limit on an integral sign once sufficient evidence seen Mark independently of any integrand attempted Could be implied by $t = \frac{1}{2}$ seen as a limit No need for $t = -2$ to be explicitly rejected $b = \frac{1}{2}$ doesn't need to be seen, and may be implied by appearing as the upper limit on an integral sign Mark independently of any integrand attempted

Question			Answer	Marks	AO	Guidance	
				[5]			
12	(b)		DR $u = 2t + 1$, $du = 2dt$, $8t + 6 = 4u + 2$ $\int \frac{4t+3}{(2t+1)^4} 2dt = \int \frac{2u+1}{u^4} du$ $\int \frac{2}{u^3} + \frac{1}{u^4} du = -\frac{1}{u^2} - \frac{1}{3u^3}$ $\left[-\frac{1}{u^2} - \frac{1}{3u^3} \right]_1^2 = \left(-\frac{1}{4} - \frac{1}{24} \right) - \left(-1 - \frac{1}{3} \right)$ $= \frac{25}{24}$	M1* A1 M1* A1 M1d* A1	3.1a 1.1 3.1a 1.1 3.1a 1.1	Use $u = 2t + 1$ to attempt to change entire integrand to a function of u Obtain correct integrand Attempt integration to obtain integral of form $au^{-2} + bu^{-3}$ Obtain fully correct integral Attempt use of correct limits: either correct t limits (ie $a = 0$ and $b = \frac{1}{2}$) in a t-integral or commensurate upper and lower limits in an integral involving a substitution (eg with $u = 2t + 1$, then upper limit must be 2 and lower limit must be 1) Obtain correct area, any exact equivalent	Attempt to write numerator, denominator and dt in terms of u Condone no du M0 if additional terms Dependent on M1 M1 Minimum evidence needed is two terms ie $\left(-\frac{7}{24} \right) - \left(-\frac{4}{3} \right)$ or $\left(\frac{3}{4} \right) + \left(\frac{7}{24} \right)$ If these values are not seen then M1 can be awarded for term by term substitution seen (ie 4 terms needed), but allow one error Explicit use of limits must be seen in a correct integral for A1

Question			Answer	Marks	AO	Guidance	
				[6]			Candidates may mix and match methods eg start with substitution and then try to do the actual integration by parts – the MS allows M1A1 for changing the integrand to useable form; M1A1 for doing the integration; M1A1 for use of limits
			Alternative method (integration by parts) $u = 8t + 6, u' = 8$ $v' = (2t + 1)^{-4}, v = -\frac{1}{6}(2t + 1)^{-3}$ $I =$ $-\frac{1}{6}(8t + 6)(2t + 1)^{-3} - \int -\frac{8}{6}(2t + 1)^{-3} dt$ $I =$ $(8t + 6) \times -\frac{1}{6}(2t + 1)^{-3} + \frac{8}{6} \times -\frac{1}{4}(2t + 1)^{-2}$ $\left(-\frac{5}{24} - (-1)\right) + \left(-\frac{1}{12} - \left(-\frac{1}{3}\right)\right)$ $= \left(\frac{19}{24}\right) + \left(\frac{1}{4}\right)$ $= \frac{25}{24}$	M1* A1 M1* A1 M1d* A1		Attempt integration by parts Obtain correct first step Attempt integration to obtain integral of form $a(8t + 6)(2t + 1)^{-3} + b(2t + 1)^{-2}$ Obtain fully correct integral Attempt use of correct limits Obtain correct area, any exact equivalent	Correct parts and correct formula Allow unsimplified Allow unsimplified See guidance in main MS

Question			Answer	Marks	AO	Guidance	
			Alternative method (separate fractions) $\frac{4(2t+1)+2}{(2t+1)^4} = \frac{4}{(2t+1)^3} + \frac{2}{(2t+1)^4}$ $\int \frac{8t+6}{(2t+1)^4} dt = \int \frac{4}{(2t+1)^3} + \frac{2}{(2t+1)^4} dt$ $\int \frac{8t+6}{(2t+1)^4} dt = -\frac{1}{(2t+1)^2} - \frac{1}{3(2t+1)^3}$ $\left(-\frac{1}{4} - \frac{1}{24}\right) - \left(-1 - \frac{1}{3}\right)$ $= \frac{25}{24}$	M1* A1 M1* A1 M1d* A1		Attempt to rewrite integrand as separate fractions with constant numerators Obtain correct integrand Attempt integration to obtain integral of form $\frac{a}{(2t+1)^2} + \frac{b}{(2t+1)^3}$ Obtain fully correct integral Attempt use of correct limits Obtain correct area, any exact equivalent	Could be informal method, or use of partial fractions (extending expected knowledge) As far as $\frac{P}{(2t+1)^3} + \frac{Q}{(2t+1)^4}$, with P and Q as constants, and no other fractions Allow unsimplified See guidance in main MS

Question		Answer	Marks	AO	Guidance	
		Alternative method (integrating between curve and x-axis) $\int y \frac{dx}{dt} dt = \int -\frac{16(2t^2 + 3t)}{(2t + 1)^5} dt$ $\int \frac{(u + 2)(4 - 4u)}{u^5} du$ or $2(2t^2 + 3t)(2t + 1)^{-4}$ $-\int 2(4t + 3)(2t + 1)^{-4} dt$ $\frac{2}{u^2} + \frac{4}{3u^3} - \frac{2}{u^4}$ or $2(2t^2 + 3t)(2t + 1)^{-4} +$ $\frac{1}{3}(4t + 3)(2t + 1)^{-3} + \frac{1}{3}(2t + 1)^{-2}$ $\left(\frac{4}{3}\right) - \left(\frac{13}{24}\right) + \frac{1}{4}$	M1* A1 M1* A1 M1d*		Attempt integration by substitution / integration by parts on correct expression Obtain correct integrand Attempt integration to obtain integral of required form Obtain fully correct integral Attempt use of correct limits, and combine with correct area of rectangle (= $\frac{1}{4}$)	Apply the same MS as for integrating between curve and y-axis Using substitution eg $u = 2t + 1$ Using integration by parts – first stage required for M1 Apply the same MS as for integrating between curve and y-axis Allow unsimplified Limits must be $\int_{\frac{1}{2}}^0$ or commensurate u-limits, and used in the correct order See guidance in main MS, but must also add on the correct area of the rectangle

Question			Answer	Marks	AO	Guidance	
			$= \frac{25}{24}$	A1		Obtain correct area, any exact equivalent	

APPENDIX**Exemplar responses for Q1(c)**

Response	Mark	Comment
The graph is both convex & concave in the range. Therefore, the trapezia do not strictly all lie under or over the graph.	B1 B1	
At the beginning the graph is convex and then concave, therefore some of the trapezia are overestimating and some underestimating.	B1 B1	Condone if the order of convex and concave becomes muddled.
Part of the graph is concave, and part of the graph is convex and so you cannot tell as some of the over/underestimates would cancel out.	B1 B1BOD	Comment about curve is sufficient. BOD for some recognition that this is leading to both over and underestimates within the range.
The concavity of the function changes in the range 0 to 2	B1 B0	Acceptable first comment about the shape. No comment about the estimate.
Because the gradient increases and decreases, so can't tell if under or overestimate.	B1 B0	The first comment is acceptable as it describes the nature of the curve in the range. No reason why it may be both an overestimate and underestimate.
As the trapezia lines go both over the curve and under the curve, there are parts which are overestimating and parts which are underestimating.	B0 B1	No comment about the shape of the curve.
When concave it is an underestimate, when convex it is an overestimate.	B0 B1 BOD	No specific comment about the shape of this curve. Allow BOD for the statement about the nature of the estimate.
The trapezia will both go over and under the curve, given its shape so hard to tell if over or underestimate,	B0 B1	No details about the nature of 'its shape'. Second comment is fine.
At the beginning the curve is curving upwards so it will be an overestimate and later curve is curving downwards so will be an underestimate.	B0 B1	'Curving downwards' is too vague. Second comment is fine.

Because the rectangles go over and under the curve.	B0 B0	No comment made about the shape of the graph. 'Rectangles' not acceptable as it is the Trapezium Rule.
The diagram has an unequal slope so can't tell if over or underestimate.	B0 B0	Comment about shape not sufficient. Comment about estimate is not sufficient.

Exemplar responses for Q4(b)

Response	Mark	Comment
Periodic with order 4	B1 isw	Ignore any attempt to give the period of the sequence as it is just the 'general behaviour' that is required.
Repeating, or repetitive or recurring	B1	These are all acceptable descriptions.
Cyclic, or circling or looping	B1	These are all acceptable descriptions.
Repetitive and infinite	B1	The infinite isn't incorrect, so can be ignored.
Oscillating sequence	B0	MS states B0 for oscillating on its own.
Periodic oscillating	B1 isw	Ignore the comment oscillating if with an acceptable description.
Divergent	B0	Insufficient description on its own.
Repeating and divergent	B1 isw	The divergent isn't incorrect, so can be ignored.
Repeating and convergent	B0	An incorrect description, alongside an acceptable statement cannot be condoned.
Harmonic	B0	Incorrect description.
Periodic harmonic sequence	B0	An incorrect description, alongside an acceptable statement cannot be condoned.

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