OCR
Oxford Cambridge and RSA

Tuesday 20 June 2023 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours

6888578800

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient rule $y = \dfrac{u}{v},\ \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta,\ \cos\theta \approx 1 - \frac{1}{2}\theta^2,\ \tan\theta \approx \theta$ where θ is measured in radians

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Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

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Section A
Pure Mathematics

1 Using logarithms, solve the equation

$4^{2x+1} = 5^x$,

giving your answer correct to **3** significant figures. **[3]**

2 **(a)** Express $3\sin x - 4\cos x$ in the form $R\sin(x-\alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of α correct to **4** significant figures. **[3]**

(b) Hence solve the equation $3\sin x - 4\cos x = 2$ for $0° < x < 90°$, giving your answer correct to **3** significant figures. **[2]**

3 The cubic polynomial $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = x^3 + px + q$, where p and q are constants.

(a) **(i)** Given that $\mathrm{f}'(2) = 13$, find the value of p. **[2]**

(ii) Given also that $(x-2)$ is a factor of $\mathrm{f}(x)$, find the value of q. **[2]**

The curve $y = \mathrm{f}(x)$ is translated by the vector $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

(b) Using the values from part **(a)**, determine the equation of the curve after it has been translated. Give your answer in the form $y = x^3 + ax^2 + bx + c$, where a, b and c are integers to be found. **[4]**

4 A circle C has equation $x^2 + y^2 - 6x + 10y + k = 0$.

(a) Find the set of possible values of k. **[2]**

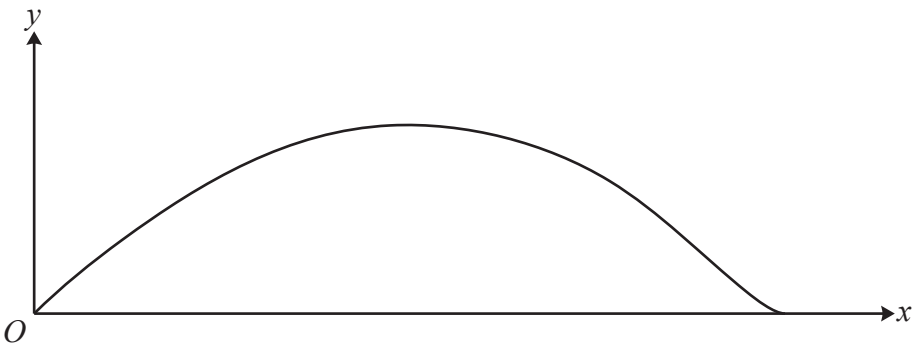
(b) It is given that $k = -46$.

Determine the coordinates of the **two** points on C at which the gradient of the tangent is $\frac{1}{2}$. **[5]**

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5 A mathematics department is designing a new emblem to place on the walls outside its classrooms. The design for the emblem is shown in the diagram below.



The emblem is modelled by the region between the x-axis and the curve with parametric equations

$$x = 1 + 0.2t - \cos t, \qquad y = k\sin^2 t,$$

where k is a positive constant and $0 \leqslant t \leqslant \pi$.

Lengths are in metres and the area of the emblem must be $1\,\text{m}^2$.

(a) Show that $k\int_0^{\pi}(0.2 + \sin t - 0.2\cos^2 t - \sin t\cos^2 t)\,\mathrm{d}t = 1$. **[3]**

(b) Determine the exact value of k. **[6]**

6 The first, third and fourth terms of an arithmetic progression are u_1, u_3 and u_4 respectively, where

$$u_1 = 2\sin\theta, \qquad u_3 = -\sqrt{3}\cos\theta, \qquad u_4 = \tfrac{7}{2}\sin\theta,$$

and $\frac{1}{2}\pi < \theta < \pi$.

(a) Determine the exact value of θ. **[3]**

(b) Hence determine the value of $\sum_{r=1}^{100} u_r$. **[3]**

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7 A car C is moving horizontally in a straight line with velocity $v\,\text{m s}^{-1}$ at time t seconds, where $v > 0$ and $t \geqslant 0$. The acceleration, $a\,\text{m s}^{-2}$, of C is modelled by the equation

$$a = v\left(\frac{8t}{7+4t^2} - \frac{1}{2}\right).$$

(a) In this question you must show detailed reasoning.

Find the times when the acceleration of C is zero. **[3]**

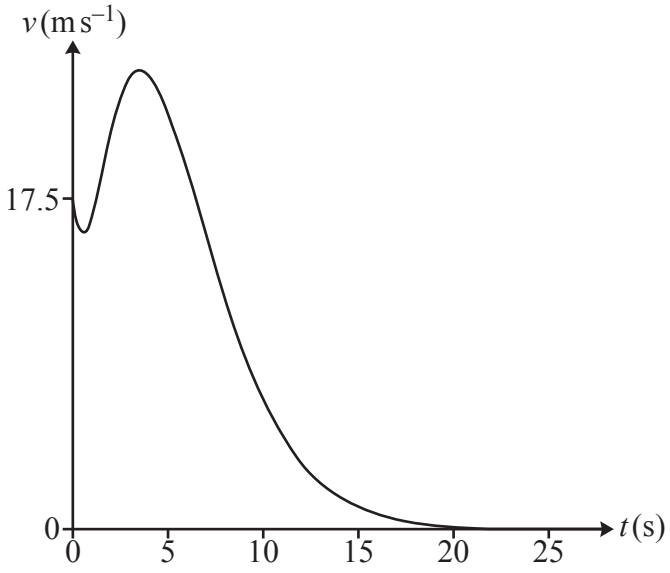
At $t = 0$ the velocity of C is $17.5\,\text{m s}^{-1}$ and at $t = T$ the velocity of C is $5\,\text{m s}^{-1}$.

(b) By setting up and solving a differential equation, show that T satisfies the equation

$$T = 2\ln\left(\frac{7+4T^2}{2}\right).$$ **[6]**

(c) Use an iterative formula, based on the equation in part **(b)**, to find the value of T, giving your answer correct to **4** significant figures. Use an initial value of 11.25 and show the result of each step of the iteration process. **[2]**

(d) The diagram below shows the velocity-time graph for the motion of C.



Find the time taken for C to decelerate from travelling at its maximum speed until it is travelling at $5\,\text{m s}^{-1}$. **[1]**

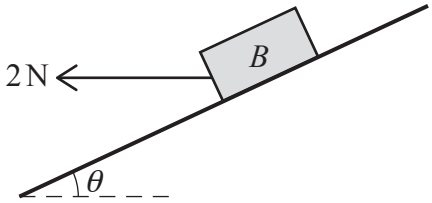
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Section B
Mechanics

8 A particle P moves with constant acceleration $(3\mathbf{i}-2\mathbf{j})\,\mathrm{m\,s^{-2}}$. At time $t = 4$ seconds, P has velocity $6\mathbf{i}\,\mathrm{m\,s^{-1}}$.

Determine the speed of P at time $t = 0$ seconds. **[4]**

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A block B of weight 10 N lies at rest in equilibrium on a rough plane inclined at θ to the horizontal. A horizontal force of magnitude 2 N, acting above a line of greatest slope, is applied to B (see diagram).

(a) Complete the diagram in the Printed Answer Booklet to show all the forces acting on B. **[1]**

It is given that B remains at rest and the coefficient of friction between B and the plane is 0.8.

(b) Determine the greatest possible value of $\tan\theta$. **[5]**

10 A particle P of mass m kg is moving on a smooth horizontal surface under the action of two constant horizontal forces $(-4\mathbf{i}+2\mathbf{j})$ N and $(a\mathbf{i}+b\mathbf{j})$ N. The resultant of these two forces is $\mathbf{R}$ N. It is given that $\mathbf{R}$ acts in a direction which is parallel to the vector $-\mathbf{i}+3\mathbf{j}$.

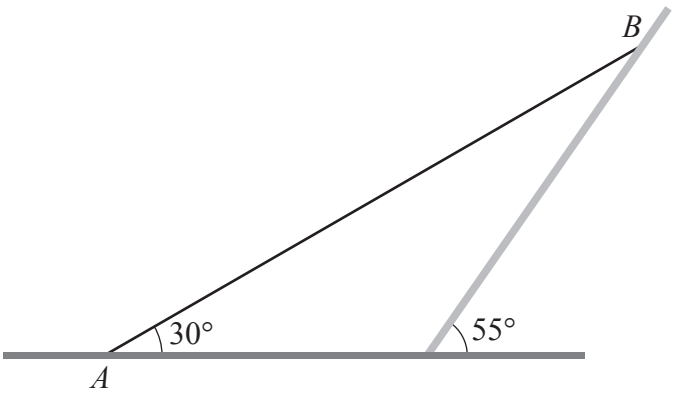
(a) Show that $3a+b=10$. **[3]**

It is given that $a = 6$ and that P moves with an acceleration of magnitude $5\sqrt{10}\,\mathrm{m\,s^{-2}}$.

(b) Determine the value of m. **[4]**

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A uniform rod AB, of weight 20 N and length 2.8 m, rests in equilibrium with the end A in contact with rough horizontal ground and the end B resting against a smooth wall inclined at 55° to the horizontal. The rod, which rests in a vertical plane that is perpendicular to the wall, is inclined at 30° to the horizontal (see diagram).

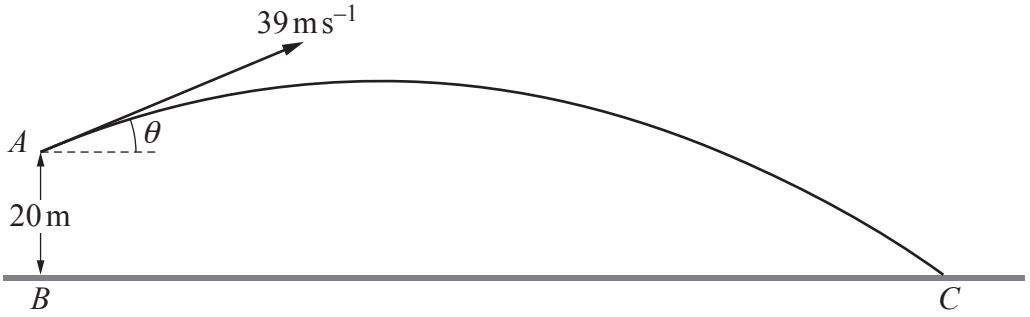
(a) Show that the magnitude of the force acting on the rod at B is 9.56 N, correct to **3** significant figures. **[3]**

(b) Determine the magnitude of the contact force between the rod and the ground. Give your answer correct to **3** significant figures. **[5]**

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12 In this question you should take the acceleration due to gravity to be $10\,\text{m s}^{-2}$.



A small ball P is projected from a point A with speed $39\,\text{m s}^{-1}$ at an angle of elevation θ, where $\sin\theta = \frac{5}{13}$ and $\cos\theta = \frac{12}{13}$. Point A is 20 m vertically above a point B on horizontal ground. The ball first lands at a point C on the horizontal ground (see diagram).

The ball P is modelled as a particle moving freely under gravity.

(a) Find the maximum height of P above the ground during its motion. **[3]**

The time taken for P to travel from A to C is T seconds.

(b) Determine the value of T. **[3]**

(c) State **one** limitation of the model, other than air resistance or the wind, that could affect the answer to part **(b)**. **[1]**

At the instant that P is projected, a second small ball Q is released from rest at B and moves towards C along the horizontal ground.

At time t seconds, where $t \geqslant 0$, the velocity $v\,\text{m s}^{-1}$ of Q is given by

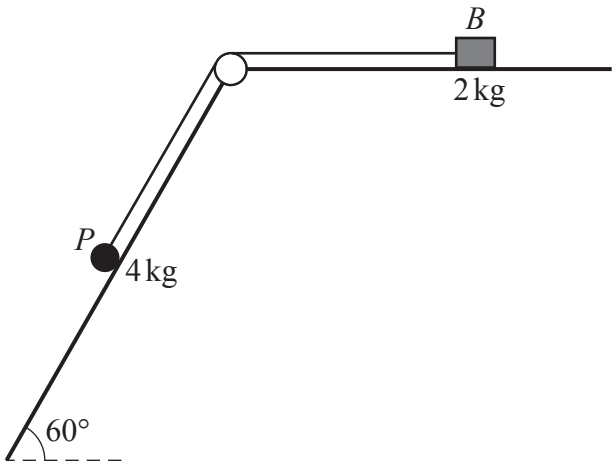
$$v = kt^3 + 6t^2 + \tfrac{3}{2}t,$$

where k is a positive constant.

(d) Given that P and Q collide at C, determine the acceleration of Q immediately before this collision. **[6]**

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The diagram shows a small block B, of mass 2 kg, and a particle P, of mass 4 kg, which are attached to the ends of a light inextensible string. The string is taut and passes over a small smooth pulley fixed at the intersection of a horizontal surface and an inclined plane. The particle can move on the inclined plane, which is rough, and which makes an angle of 60° with the horizontal. The block can move on the horizontal surface, which is also rough.

The system is released from rest, and in the subsequent motion P moves down the plane and B does not reach the pulley.

It is given that the coefficient of friction between P and the inclined plane is twice the coefficient of friction between B and the horizontal surface.

(a) Determine, in terms of g, the tension in the string. **[7]**

When P is moving at $2\,\text{m}\,\text{s}^{-1}$ the string breaks. In the 0.5 seconds after the string breaks P moves 1.9 m down the plane.

(b) Determine the deceleration of B after the string breaks. Give your answer correct to **3** significant figures. **[5]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 $4^{2x+1} = 5^x$

Taking logs: $(2x + 1) \log 4 = x \log 5$

$2x \log 4 + \log 4 = x \log 5$

$x(2 \log 4 - \log 5) = -\log 4$

$x = -\log 4 / (2 \log 4 - \log 5) = \log 4 / (\log 5 - 2 \log 4)$

$x = -1.191844... = -1.19$ (3 s.f.)

2 (a) $R = \sqrt{(3^2 + 4^2)} = 5$

$R \sin(x - \alpha) = R \sin x \cos \alpha - R \cos x \sin \alpha$

$R \cos \alpha = 3, \quad R \sin \alpha = 4 \quad \Rightarrow \quad \tan \alpha = 4/3$

$\alpha = 53.130102...° = 53.13°$

$3 \sin x - 4 \cos x = 5 \sin(x - 53.13°)$

2 (b) $5 \sin(x - 53.13°) = 2 \quad \Rightarrow \quad \sin(x - 53.13°) = 2/5$

$x - 53.13° = \sin^{-1}(0.4) = 23.578...°$

(the other solution $180° - 23.578...°$ gives $x = 209.6°$, outside the range)

$x = 53.130...° + 23.578...° = 76.708...° = 76.7°$

3 (a)(i) $f(x) = x^3 + px + q \quad \Rightarrow \quad f'(x) = 3x^2 + p$

$f'(2) = 3(2)^2 + p = 12 + p = 13 \quad \Rightarrow \quad p = 1$

3 (a)(ii) $(x - 2)$ is a factor so $f(2) = 0$:

$2^3 + 2p + q = 0 \quad \Rightarrow \quad 8 + 2 + q = 0 \quad \Rightarrow \quad q = -10$

Question number	
3 (b)	$f(x) = x^3 + x - 10$. Translation by $(2, -3)$: $y = f(x - 2) - 3$
	$y = (x - 2)^3 + (x - 2) - 10 - 3$
	$(x - 2)^3 = x^3 + 3x^2(-2) + 3x(-2)^2 + (-2)^3 = x^3 - 6x^2 + 12x - 8$
	$y = x^3 - 6x^2 + 12x - 8 + x - 2 - 13$
	$y = x^3 - 6x^2 + 13x - 23$ $\quad (a = -6,\ b = 13,\ c = -23)$
4 (a)	$(x - 3)^2 - 9 + (y + 5)^2 - 25 + k = 0$
	$(x - 3)^2 + (y + 5)^2 = 34 - k$
	For a circle the radius^2 must be positive: $34 - k > 0$
	$k < 34$
4 (b)	$x^2 + y^2 - 6x + 10y - 46 = 0$
	Differentiating implicitly: $2x + 2y\,dy/dx - 6 + 10\,dy/dx = 0$
	$dy/dx = (6 - 2x)/(2y + 10) = (3 - x)/(y + 5)$
	Gradient ½: $(3 - x)/(y + 5) = ½ \Rightarrow y + 5 = 6 - 2x \Rightarrow y = 1 - 2x$
	Substitute: $x^2 + (1 - 2x)^2 - 6x + 10(1 - 2x) - 46 = 0$
	$x^2 + 1 - 4x + 4x^2 - 6x + 10 - 20x - 46 = 0$
	$5x^2 - 30x - 35 = 0 \Rightarrow x^2 - 6x - 7 = 0$
	$(x - 7)(x + 1) = 0 \Rightarrow x = 7$ or $x = -1$
	$x = 7$: $y = 1 - 14 = -13$; $\quad x = -1$: $y = 1 + 2 = 3$
	Points: $(7, -13)$ and $(-1, 3)$
5 (a)	$x = 1 + 0.2t - \cos t \Rightarrow dx/dt = 0.2 + \sin t$
	Area $= \int y\,dx = \int_0^\pi y\,(dx/dt)\,dt = \int_0^\pi k \sin^2 t\,(0.2 + \sin t)\,dt$
	($t = 0$ gives $x = 0$ at O and $t = \pi$ gives the right-hand end of the emblem)
	$\sin^2 t = 1 - \cos^2 t$: $\quad$ Area $= k\int_0^\pi (1 - \cos^2 t)(0.2 + \sin t)\,dt$
	$= k\int_0^\pi (0.2 + \sin t - 0.2\cos^2 t - \sin t \cos^2 t)\,dt$
	Area $= 1\ m^2$, so $k\int_0^\pi (0.2 + \sin t - 0.2\cos^2 t - \sin t \cos^2 t)\,dt = 1$ as required

Question number	
5 (b)	Using $2\cos^2 t = 1 + \cos 2t$: $0.2\cos^2 t = 0.1(1 + \cos 2t) = 0.1 + 0.1\cos 2t$
	$\int (0.2 + \sin t - 0.1 - 0.1\cos 2t - \sin t\cos^2 t)\,dt$
	$= \int (0.1 + \sin t - 0.1\cos 2t - \sin t\cos^2 t)\,dt$
	$= 0.1t - \cos t - 0.05\sin 2t + \frac{1}{3}\cos^3 t$
	(since $d/dt(\cos^3 t) = -3\cos^2 t \sin t$)
	$[\,0.1t - \cos t - 0.05\sin 2t + \frac{1}{3}\cos^3 t\,]_0^{\pi}$
	$= (0.1\pi - (-1) - 0 + \frac{1}{3}(-1)^3) - (0 - 1 - 0 + \frac{1}{3})$
	$= (0.1\pi + 1 - \frac{1}{3}) - (-\frac{2}{3}) = 0.1\pi + 4/3$
	$k(0.1\pi + 4/3) = 1 \Rightarrow k = 1/(\pi/10 + 4/3)$
	$k = 30/(3\pi + 40)$
6 (a)	$u_1 = a = 2\sin\theta$, $u_3 = a + 2d = -\sqrt{3}\cos\theta$, $u_4 = a + 3d = 7/2\sin\theta$
	$d = u_4 - u_3 = 7/2\sin\theta + \sqrt{3}\cos\theta$
	$u_3 = u_1 + 2d$: $-\sqrt{3}\cos\theta = 2\sin\theta + 2(7/2\sin\theta + \sqrt{3}\cos\theta)$
	$-\sqrt{3}\cos\theta = 9\sin\theta + 2\sqrt{3}\cos\theta$
	$-3\sqrt{3}\cos\theta = 9\sin\theta \Rightarrow \tan\theta = -3\sqrt{3}/9 = -\sqrt{3}/3$
	$\frac{1}{2}\pi < \theta < \pi$: $\theta = \pi - \pi/6 = 5\pi/6$
6 (b)	$\sin(5\pi/6) = \frac{1}{2}$, $\cos(5\pi/6) = -\sqrt{3}/2$
	$a = 2\sin\theta = 1$, $d = 7/2\sin\theta + \sqrt{3}\cos\theta = 7/4 - 3/2 = \frac{1}{4}$
	$S_{100} = (100/2)[2a + (100 - 1)d] = 50[2(1) + 99(\frac{1}{4})]$
	$= 50(2 + 24.75) = 50 \times 26.75$
	$\sum_{r=1}^{100} u_r = 1337.5$
7 (a)	$a = 0$ and $v > 0$, so $8t/(7 + 4t^2) - \frac{1}{2} = 0$
	$8t/(7 + 4t^2) = \frac{1}{2} \Rightarrow 16t = 7 + 4t^2$
	$4t^2 - 16t + 7 = 0$
	$(2t - 1)(2t - 7) = 0$
	$t = 0.5$ s or $t = 3.5$ s

Question number

7 (b)

$dv/dt = v(8t/(7 + 4t^2) - \frac{1}{2})$

Separating variables: $\int 1/v \, dv = \int (8t/(7 + 4t^2) - \frac{1}{2}) \, dt$

$\ln v = \ln(7 + 4t^2) - \frac{1}{2}t + c \qquad (v > 0, 7 + 4t^2 > 0)$

$t = 0, v = 17.5$: $\ln 17.5 = \ln 7 + c \Rightarrow c = \ln 17.5 - \ln 7 = \ln(5/2)$

$\ln v = \ln(7 + 4t^2) - \frac{1}{2}t + \ln(5/2)$

$t = T, v = 5$: $\ln 5 = \ln(7 + 4T^2) - \frac{1}{2}T + \ln(5/2)$

$\frac{1}{2}T = \ln(7 + 4T^2) + \ln(5/2) - \ln 5 = \ln[5(7 + 4T^2) / (2 \times 5)]$

$\frac{1}{2}T = \ln((7 + 4T^2)/2)$

$T = 2 \ln((7 + 4T^2)/2)$ as required

7 (c)

$T_{n+1} = 2 \ln((7 + 4T_n^2)/2), \qquad T_0 = 11.25$

$T_1 = 11.09523175... \qquad T_2 = 11.04058716...$

$T_3 = 11.02111643... \qquad T_4 = 11.01415608...$

$T_5 = 11.01166500... \qquad T_6 = 11.01077309...$

$T = 11.01$ (4 s.f.)

7 (d)

Maximum speed when $a = 0$ with v at a maximum: $t = 3.5$ s (from (a) and the graph)

Time taken $= 11.01 - 3.5 = 7.51$ s

8

$v = u + at$: $6i = u + 4(3i - 2j)$

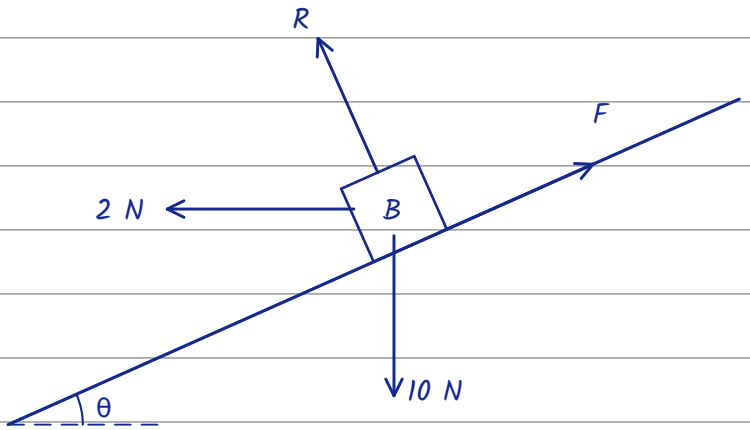
$u = 6i - 12i + 8j = -6i + 8j$

Speed $= |u| = \sqrt{((-6)^2 + 8^2)} = \sqrt{100}$

Speed at $t = 0$ is 10 m s^{-1}

Question number

9 (a)



Forces on B: weight 10 N, normal contact force R, friction F (up the plane), 2 N

9 (b)

Greatest tan θ: B is on the point of slipping down the plane, $F = 0.8R$ up the plane.

Perpendicular to the plane: $R + 2\sin\theta = 10\cos\theta \Rightarrow R = 10\cos\theta - 2\sin\theta$

Parallel to the plane: $F = 2\cos\theta + 10\sin\theta$

At rest: $F \leq \mu R$, so $2\cos\theta + 10\sin\theta \leq 0.8(10\cos\theta - 2\sin\theta)$

$2\cos\theta + 10\sin\theta \leq 8\cos\theta - 1.6\sin\theta$

$11.6\sin\theta \leq 6\cos\theta \Rightarrow \tan\theta \leq 6/11.6 = 30/58 = 15/29$

Greatest possible value of $\tan\theta = 15/29$ $(= 0.517\ldots)$

10 (a)

$R = (-4i + 2j) + (ai + bj) = (a - 4)i + (b + 2)j$

R parallel to $-i + 3j$: $(a - 4)i + (b + 2)j = k(-i + 3j)$

$a - 4 = -k \Rightarrow k = 4 - a$

$b + 2 = 3k = 3(4 - a) = 12 - 3a$

$3a + b = 10$ as required

10 (b)

$a = 6 \Rightarrow b = 10 - 18 = -8$, so $R = 2i - 6j$

$|R| = \sqrt{(2^2 + (-6)^2)} = \sqrt{40} = 2\sqrt{10}$ N

N2L: $|R| = m|a| \Rightarrow \sqrt{40} = m(5\sqrt{10})$

$m = \sqrt{40} / (5\sqrt{10}) = 2\sqrt{10} / (5\sqrt{10}) = 0.4$

Question number	
11 (a)	Force at B, R_B, is perpendicular to the smooth wall, so it acts at 35° to the horizontal; the angle between R_B and the rod is 180° − 35° − 30° = 115°, i.e. 25° to the normal to the rod. Moments about A: $20 \times 1.4 \cos 30° = R_B \times 2.8 \cos 25°$ $R_B = 28 \cos 30° / (2.8 \cos 25°) = 24.2487... / 2.5377... = 9.555533...$ $R_B = 9.56$ N (3 s.f.) as required
11 (b)	Let R_A = normal reaction and F_A = friction at A (friction acts towards the wall). Resolving vertically: $R_A + 9.5555... \sin 35° = 20$ $R_A = 20 - 5.4808... = 14.5191...$ N Resolving horizontally: $F_A = 9.5555... \cos 35° = 7.8274...$ N Contact force $= \sqrt{(F_A^2 + R_A^2)} = \sqrt{(7.8274...^2 + 14.5191...^2)}$ $= \sqrt{272.07...} = 16.4946... = 16.5$ N (3 s.f.)
12 (a)	Vertical component of velocity: $39 \sin \theta = 39 \times 5/13 = 15$ m s^{-1} At greatest height $v_y = 0$: $v^2 = u^2 + 2as$: $0 = 15^2 + 2(-10)h$ $h = 225/20 = 11.25$ m above A Maximum height above ground $= 20 + 11.25 = 31.25$ m
12 (b)	Vertically from A to C (up positive): $s = ut + \frac{1}{2}at^2$ $-20 = (39 \times 5/13)T + \frac{1}{2}(-10)T^2 = 15T - 5T^2$ $5T^2 - 15T - 20 = 0 \Rightarrow T^2 - 3T - 4 = 0$ $(T - 4)(T + 1) = 0$, $T > 0 \Rightarrow T = 4$
12 (c)	The ball is not a particle: in reality it has dimensions (size), which the model ignores.
12 (d)	Horizontal velocity of P $= 39 \cos \theta = 39 \times 12/13 = 36$ m s^{-1} BC $= 36 \times T = 36 \times 4 = 144$ m For Q: $s = \int v \, dt = \int (kt^3 + 6t^2 + 3/2\, t) \, dt = \frac{1}{4}kt^4 + 2t^3 + \frac{3}{4}t^2$ ($s = 0$ at $t = 0$)

Question number

Q reaches C at $t = 4$: $\frac{1}{4}k(4)^4 + 2(4)^3 + \frac{3}{4}(4)^2 = 144$

$64k + 128 + 12 = 144 \Rightarrow 64k = 4 \Rightarrow k = 1/16$

$a = dv/dt = 3kt^2 + 12t + 3/2$

At $t = 4$: $a = 3(1/16)(16) + 12(4) + 1.5 = 3 + 48 + 1.5$

Acceleration of Q = 52.5 m s^{-2}

13 (a)

Let μ = coefficient of friction for B, so 2μ for P. Acceleration a for both.

P, perpendicular to the plane: $R_P = 4g \cos 60° = 2g$

P, N2L down the plane: $4g \sin 60° - F_P - T = 4a$, $F_P = 2\mu R_P$

$4g \sin 60° - 2\mu(4g \cos 60°) - T = 4a \Rightarrow 2\sqrt{3}g - 4\mu g - T = 4a$ (1)

B, vertically: $R_B = 2g$, N2L: $T - \mu(2g) = 2a$

$\Rightarrow 2T - 4\mu g = 4a$ (2)

(1) − (2): $2\sqrt{3}g - T - 2T = 0 \Rightarrow 3T = 2\sqrt{3}g$

$T = (2\sqrt{3}/3)g$ N ($\approx 1.15g$ N)

13 (b)

P after the string breaks: $s = ut + \frac{1}{2}at^2$: $1.9 = 2(0.5) + \frac{1}{2}a_P(0.5)^2$

$1.9 = 1 + 0.125a_P \Rightarrow a_P = 7.2$ m s^{-2}

$T = 0$: $4g \sin 60° - \mu_P(4g \cos 60°) = 4a_P$

$2\sqrt{3}(9.8) - \mu_P(2 \times 9.8) = 28.8 \Rightarrow \mu_P = (33.948... - 28.8)/19.6 = 0.26266...$

$\mu_B = \frac{1}{2}\mu_P = 0.13133...$

B after the string breaks: $-\mu_B(2g) = 2a_B \Rightarrow a_B = -\mu_B g = -1.28704...$

Deceleration of B = 1.29 m s^{-2} (3 s.f.)