



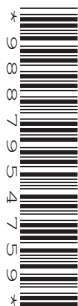
Oxford Cambridge and RSA

Tuesday 6 June 2023 – Afternoon

A Level Mathematics A

H240/01 Pure Mathematics

Time allowed: 2 hours



You must have:

- the Printed Answer Booklet
- a scientific or graphical calculator



INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **12** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae

A Level Mathematics A (H240)

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2, \quad \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Standard deviation

$$\sqrt{\frac{\sum(x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np , variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

- 1 In the triangle ABC , the length $AB = 6$ cm, the length $AC = 15$ cm and the angle $BAC = 30^\circ$.

(a) Calculate the length BC . [2]

D is the point on AC such that the length $BD = 4$ cm.

(b) Calculate the possible values of the angle ADB . [3]

- 2 (a) (i) Show that $\frac{1}{3-2\sqrt{x}} + \frac{1}{3+2\sqrt{x}}$ can be written in the form $\frac{a}{b+cx}$, where a , b and c are constants to be determined. [2]

(ii) Hence solve the equation $\frac{1}{3-2\sqrt{x}} + \frac{1}{3+2\sqrt{x}} = 2$. [2]

(b) In this question you must show detailed reasoning.

Solve the equation $2^{2y} - 7 \times 2^y - 8 = 0$. [4]

- 3 (a) Given that $f(x) = x^2 + 2x$, use differentiation from first principles to show that $f'(x) = 2x + 2$. [4]

(b) The gradient of a curve is given by $\frac{dy}{dx} = 2x + 2$ and the curve passes through the point $(-1, 5)$.

Find the equation of the curve. [3]

- 4 It is given that $ABCD$ is a quadrilateral. The position vector of A is $\mathbf{i} + \mathbf{j}$, and the position vector of B is $3\mathbf{i} + 5\mathbf{j}$.

(a) Find the length AB . [1]

(b) The position vector of C is $p\mathbf{i} + p\mathbf{j}$ where p is a constant greater than 1.

Given that the length AB is equal to the length BC , determine the position vector of C . [3]

(c) The point M is the midpoint of AC .

Given that $\overrightarrow{MD} = 2\overrightarrow{BM}$, determine the position vector of D . [2]

(d) State the name of the quadrilateral $ABCD$, giving a reason for your answer. [2]

- 5 (a) The function $f(x)$ is defined for all values of x as $f(x) = |ax - b|$, where a and b are positive constants.
- (i) The graph of $y = f(x) + c$, where c is a constant, has a vertex at $(3, 1)$ and crosses the y -axis at $(0, 7)$.
- Find the values of a , b and c . [3]
- (ii) Explain why $f^{-1}(x)$ does not exist. [1]
- (b) The function $g(x)$ is defined for $x \geq \frac{q}{p}$ as $g(x) = |px - q|$, where p and q are positive constants.
- (i) Find, in terms of p and q , an expression for $g^{-1}(x)$, stating the domain of $g^{-1}(x)$. [3]
- (ii) State the set of values of p for which the equation $g(x) = g^{-1}(x)$ has no solutions. [1]
- 6 A curve has equation $y = e^{x^2+3x}$.
- (a) Determine the x -coordinates of any stationary points on the curve. [4]
- (b) Show that the curve is convex for all values of x . [5]
- 7 (a) Use the result $\cos(A + B) = \cos A \cos B - \sin A \sin B$ to show that
- $$\cos(A - B) = \cos A \cos B + \sin A \sin B. \quad [2]$$
- The function $f(\theta)$ is defined as $\cos(\theta + 30^\circ)\cos(\theta - 30^\circ)$, where θ is in degrees.
- (b) Show that $f(\theta) = \cos^2\theta - \frac{1}{4}$. [3]
- (c) (i) Determine the following.
- The **maximum** value of $f(\theta)$
 - The smallest **positive** value of θ for which this maximum value occurs [2]
- (ii) Determine the following.
- The **minimum** value of $f(\theta)$
 - The smallest **positive** value of θ for which this minimum value occurs [2]

- 8 (a) Find the first **three** terms in the expansion of $(4 + 3x)^{\frac{3}{2}}$ in ascending powers of x . [4]
- (b) State the range of values of x for which the expansion in part (a) is valid. [1]
- (c) In the expansion of $(4 + 3x)^{\frac{3}{2}}(1 + ax)^2$ the coefficient of x^2 is $\frac{107}{16}$.
Determine the possible values of the constant a . [4]

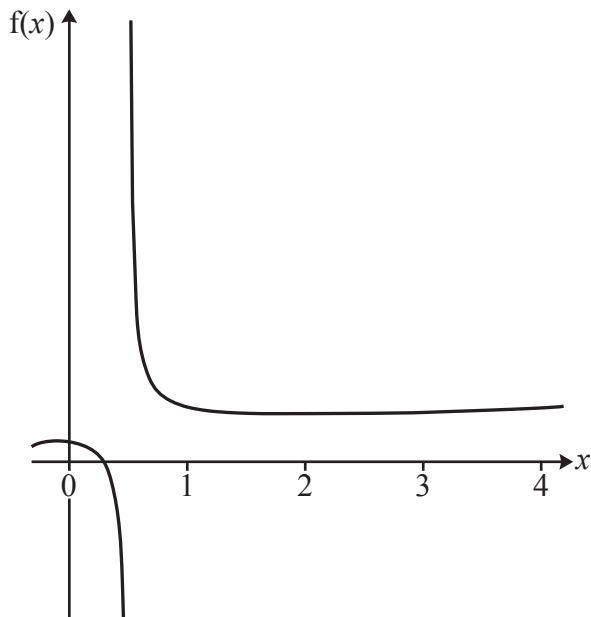
- 9 Conservationists are studying how the number of bees in a wildflower meadow varies according to the number of wildflower plants. The study takes place over a series of weeks in the summer. A model is suggested for the number of bees, B , and the number of wildflower plants, F , at time t weeks after the start of the study.

In the model $B = 20 + 2t + \cos 3t$ and $F = 50e^{0.1t}$.

The model assumes that B and F can be treated as continuous variables.

- (a) State the meaning of $\frac{dB}{dF}$. [1]
- (b) Determine $\frac{dB}{dF}$ when $t = 4$. [4]
- (c) Suggest a reason why this model may not be valid for values of t greater than 12. [1]

10



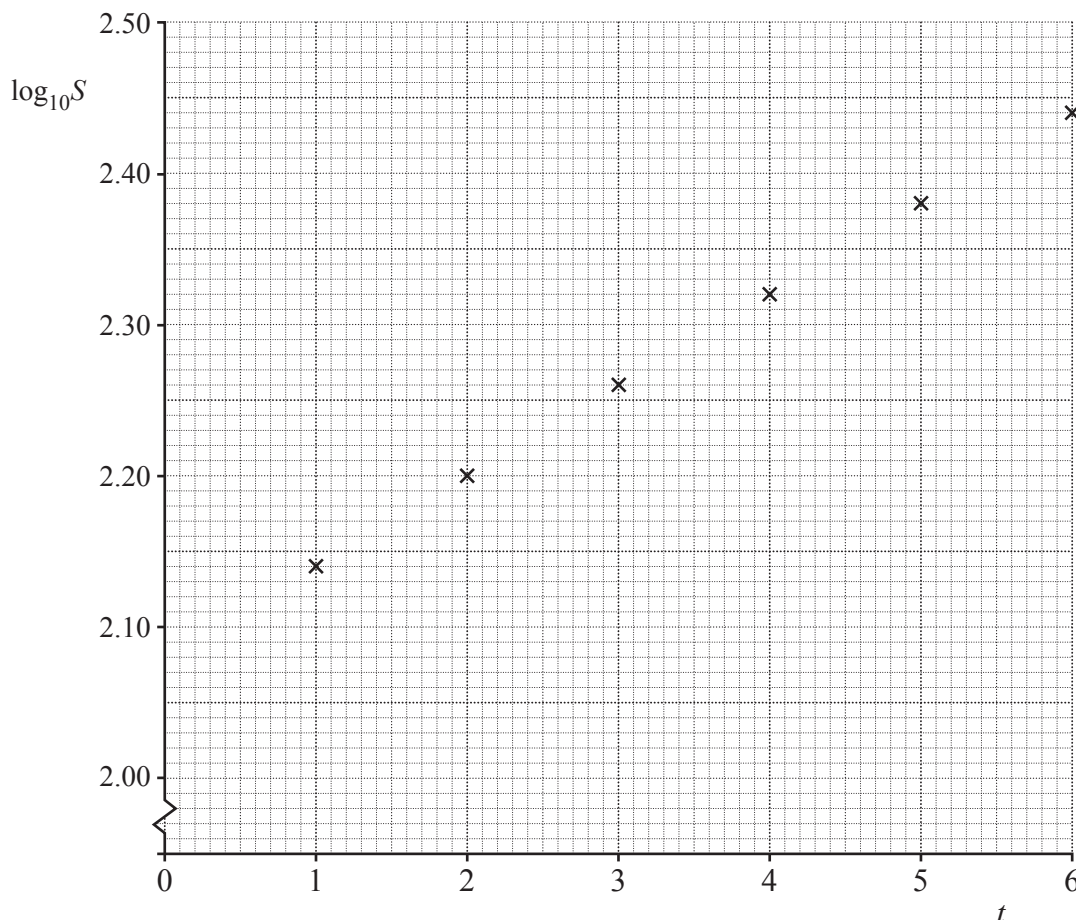
The diagram shows part of the curve $f(x) = \frac{e^x}{4x^2 - 1} + 2$. The equation $f(x) = 0$ has a positive root α close to $x = 0.3$.

- (a) Explain why using the sign change method with $x = 0$ and $x = 1$ will fail to locate α . [1]
- (b) Show that the equation $f(x) = 0$ can be written as $x = \frac{1}{4}\sqrt{(4 - 2e^x)}$. [2]
- (c) Use the iterative formula $x_{n+1} = \frac{1}{4}\sqrt{(4 - 2e^{x_n})}$ with a starting value of $x_1 = 0.3$ to find the value of α correct to 4 significant figures, showing the result of each iteration. [3]
- (d) An alternative iterative formula is $x_{n+1} = F(x_n)$, where $F(x_n) = \ln(2 - 8x_n^2)$.

By considering $F'(0.3)$ explain why this iterative formula will not find α . [3]

- 11 The owners of an online shop believe that their sales can be modelled by $S = ab^t$, where a and b are both positive constants, S is the number of items sold in a month and t is the number of complete months since starting their online shop.

The sales for the first six months are recorded, and the values of $\log_{10} S$ are plotted against t in the graph below. The graph is repeated in the Printed Answer Booklet.



- (a) Explain why the graph suggests that the given model is appropriate. [3]

The owners believe that $a = 120$ and $b = 1.15$ are good estimates for the parameters in the model.

- (b) Show that the graph supports these estimates for the parameters. [2]
- (c) Use the model $S = 120 \times 1.15^t$ to predict the number of items sold in the **seventh** month after opening. [2]
- (d) (i) Use the model $S = 120 \times 1.15^t$ to predict the number of months after opening when the **total** number of items sold after opening will first exceed 70 000. [4]
- (ii) Comment on how reliable this prediction may be. [1]

- 12 (a)** Use the substitution $u = e^x - 2$ to show that

$$\int \frac{7e^x - 8}{(e^x - 2)^2} dx = \int \frac{7u + 6}{u^2(u + 2)} du. \quad [3]$$

- (b)** Hence show that

$$\int_{\ln 4}^{\ln 6} \frac{7e^x - 8}{(e^x - 2)^2} dx = a + \ln b$$

where a and b are rational numbers to be determined. [7]

END OF QUESTION PAPER

**Copyright Information**

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series. If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.