OCR
Oxford Cambridge and RSA

Tuesday 6 June 2023 – Afternoon

A Level Mathematics A

H240/01 Pure Mathematics

Time allowed: 2 hours

9887954759

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient rule $y = \frac{u}{v}, \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

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Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \,|\, A) = \mathrm{P}(B)\mathrm{P}(A \,|\, B)$ **or** $\mathrm{P}(A \,|\, B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

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1 In the triangle ABC, the length $AB = 6\,\text{cm}$, the length $AC = 15\,\text{cm}$ and the angle $BAC = 30°$.

(a) Calculate the length BC. **[2]**

D is the point on AC such that the length $BD = 4\,\text{cm}$.

(b) Calculate the possible values of the angle ADB. **[3]**

2 **(a)** **(i)** Show that $\frac{1}{3-2\sqrt{x}} + \frac{1}{3+2\sqrt{x}}$ can be written in the form $\frac{a}{b+cx}$, where a, b and c are constants to be determined. **[2]**

(ii) Hence solve the equation $\frac{1}{3-2\sqrt{x}} + \frac{1}{3+2\sqrt{x}} = 2$. **[2]**

(b) **In this question you must show detailed reasoning.**

Solve the equation $2^{2y} - 7 \times 2^{y} - 8 = 0$. **[4]**

3 **(a)** Given that $\text{f}(x) = x^2 + 2x$, use differentiation from first principles to show that $\text{f}'(x) = 2x + 2$. **[4]**

(b) The gradient of a curve is given by $\frac{\text{d}y}{\text{d}x} = 2x + 2$ and the curve passes through the point $(-1, 5)$.

Find the equation of the curve. **[3]**

4 It is given that $ABCD$ is a quadrilateral. The position vector of A is $\mathbf{i} + \mathbf{j}$, and the position vector of B is $3\mathbf{i} + 5\mathbf{j}$.

(a) Find the length AB. **[1]**

(b) The position vector of C is $p\mathbf{i} + p\mathbf{j}$ where p is a constant greater than 1.

Given that the length AB is equal to the length BC, determine the position vector of C. **[3]**

(c) The point M is the midpoint of AC.

Given that $\overrightarrow{MD} = 2\overrightarrow{BM}$, determine the position vector of D. **[2]**

(d) State the name of the quadrilateral $ABCD$, giving a reason for your answer. **[2]**

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5 **(a)** The function $f(x)$ is defined for all values of x as $f(x) = |ax - b|$, where a and b are positive constants.

(i) The graph of $y = f(x) + c$, where c is a constant, has a vertex at (3, 1) and crosses the y-axis at (0, 7).

Find the values of a, b and c. **[3]**

(ii) Explain why $f^{-1}(x)$ does not exist. **[1]**

(b) The function $g(x)$ is defined for $x \geqslant \frac{q}{p}$ as $g(x) = |px - q|$, where p and q are positive constants.

(i) Find, in terms of p and q, an expression for $g^{-1}(x)$, stating the domain of $g^{-1}(x)$. **[3]**

(ii) State the set of values of p for which the equation $g(x) = g^{-1}(x)$ has no solutions. **[1]**

6 A curve has equation $y = e^{x^2+3x}$.

(a) Determine the x-coordinates of any stationary points on the curve. **[4]**

(b) Show that the curve is convex for all values of x. **[5]**

7 **(a)** Use the result $\cos(A+B) = \cos A \cos B - \sin A \sin B$ to show that

$\cos(A-B) = \cos A \cos B + \sin A \sin B$. **[2]**

The function $f(\theta)$ is defined as $\cos(\theta + 30°)\cos(\theta - 30°)$, where θ is in degrees.

(b) Show that $f(\theta) = \cos^2\theta - \frac{1}{4}$. **[3]**

(c) **(i)** Determine the following.

- The **maximum** value of $f(\theta)$
- The smallest **positive** value of θ for which this maximum value occurs **[2]**

(ii) Determine the following.

- The **minimum** value of $f(\theta)$
- The smallest **positive** value of θ for which this minimum value occurs **[2]**

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8 **(a)** Find the first **three** terms in the expansion of $(4+3x)^{\frac{3}{2}}$ in ascending powers of x. **[4]**

(b) State the range of values of x for which the expansion in part **(a)** is valid. **[1]**

(c) In the expansion of $(4+3x)^{\frac{3}{2}}(1+ax)^2$ the coefficient of x^2 is $\frac{107}{16}$.

Determine the possible values of the constant a. **[4]**

9 Conservationists are studying how the number of bees in a wildflower meadow varies according to the number of wildflower plants. The study takes place over a series of weeks in the summer. A model is suggested for the number of bees, B, and the number of wildflower plants, F, at time t weeks after the start of the study.

In the model $B = 20 + 2t + \cos 3t$ and $F = 50e^{0.1t}$.

The model assumes that B and F can be treated as continuous variables.

(a) State the meaning of $\frac{dB}{dF}$. **[1]**

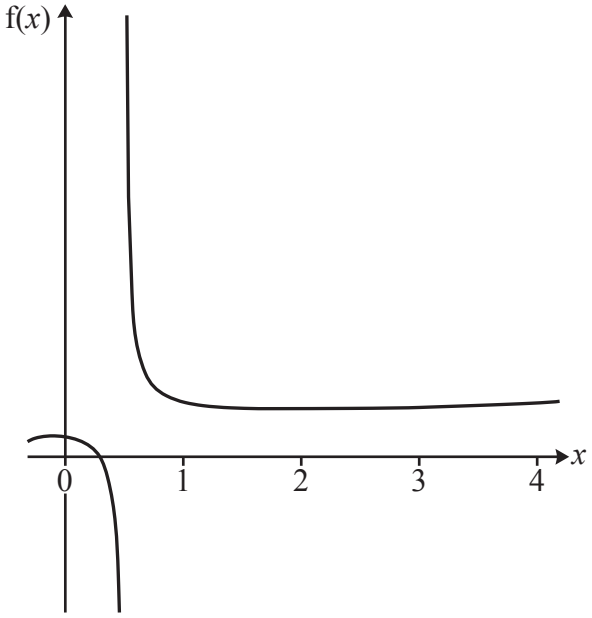
(b) Determine $\frac{dB}{dF}$ when $t = 4$. **[4]**

(c) Suggest a reason why this model may not be valid for values of t greater than 12. **[1]**

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The diagram shows part of the curve $f(x) = \dfrac{e^x}{4x^2 - 1} + 2$. The equation $f(x) = 0$ has a positive root α close to $x = 0.3$.

(a) Explain why using the sign change method with $x = 0$ and $x = 1$ will fail to locate α. **[1]**

(b) Show that the equation $f(x) = 0$ can be written as $x = \dfrac{1}{4}\sqrt{(4 - 2e^x)}$. **[2]**

(c) Use the iterative formula $x_{n+1} = \dfrac{1}{4}\sqrt{(4 - 2e^{x_n})}$ with a starting value of $x_1 = 0.3$ to find the value of α correct to **4** significant figures, showing the result of each iteration. **[3]**

(d) An alternative iterative formula is $x_{n+1} = F(x_n)$, where $F(x_n) = \ln(2 - 8x_n^2)$.

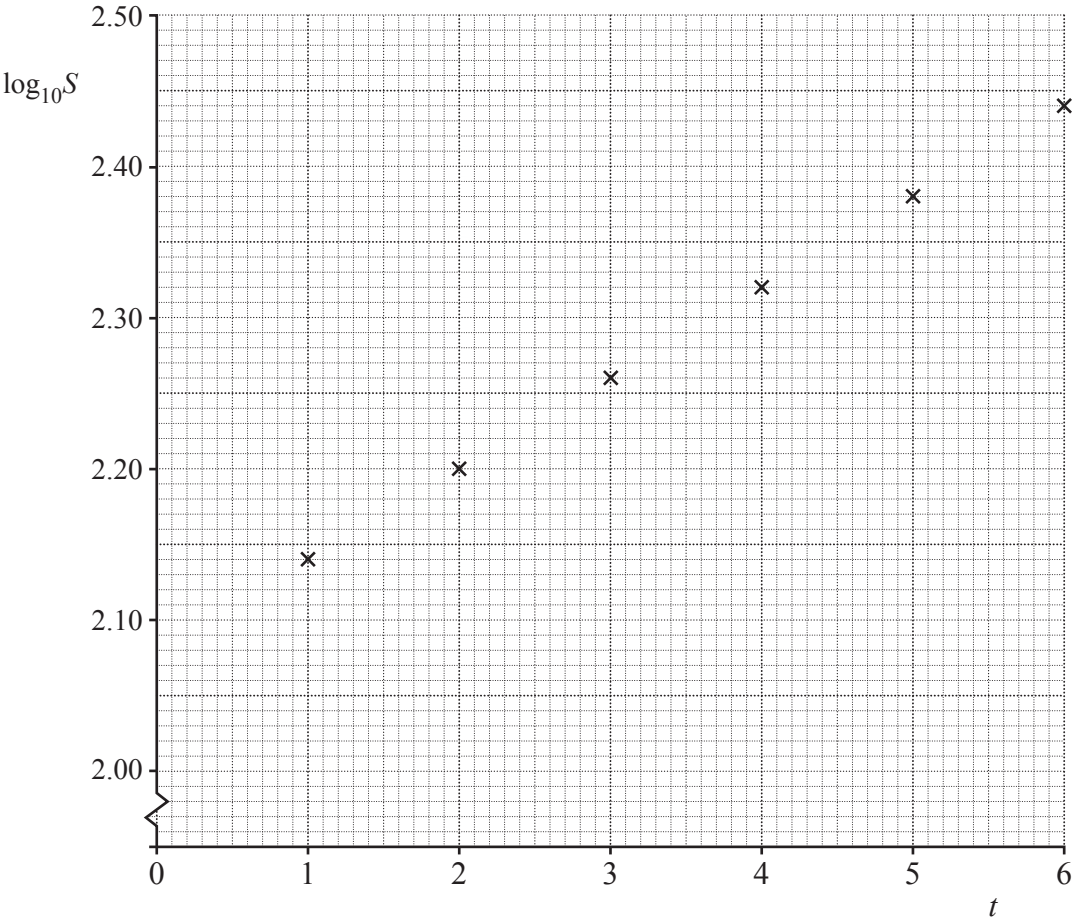
By considering $F'(0.3)$ explain why this iterative formula will not find α. **[3]**

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11 The owners of an online shop believe that their sales can be modelled by $S = ab^t$, where a and b are both positive constants, S is the number of items sold in a month and t is the number of complete months since starting their online shop.

The sales for the first six months are recorded, and the values of $\log_{10} S$ are plotted against t in the graph below. The graph is repeated in the Printed Answer Booklet.



(a) Explain why the graph suggests that the given model is appropriate. **[3]**

The owners believe that $a = 120$ and $b = 1.15$ are good estimates for the parameters in the model.

(b) Show that the graph supports these estimates for the parameters. **[2]**

(c) Use the model $S = 120 \times 1.15^t$ to predict the number of items sold in the **seventh** month after opening. **[2]**

(d) (i) Use the model $S = 120 \times 1.15^t$ to predict the number of months after opening when the **total** number of items sold after opening will first exceed 70 000. **[4]**

(ii) Comment on how reliable this prediction may be. **[1]**

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12 **(a)** Use the substitution $u = \mathrm{e}^x - 2$ to show that

$$\int \frac{7\mathrm{e}^x - 8}{(\mathrm{e}^x - 2)^2}\mathrm{d}x = \int \frac{7u + 6}{u^2(u + 2)}\mathrm{d}u.$$

[3]

(b) Hence show that

$$\int_{\ln 4}^{\ln 6} \frac{7\mathrm{e}^x - 8}{(\mathrm{e}^x - 2)^2}\mathrm{d}x = a + \ln b$$

where a and b are rational numbers to be determined. **[7]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a)

$BC^2 = 6^2 + 15^2 - 2 \times 6 \times 15 \times \cos 30°$

$BC^2 = 261 - 90\sqrt{3} = 105.115...$

$BC = 10.2525... = 10.3 \text{ cm}$

1 (b)

$\frac{\sin ADB}{6} = \frac{\sin 30°}{4}$

$\sin ADB = \frac{6 \sin 30°}{4} = 0.75$

$ADB = \sin^{-1}(0.75) = 48.5903...° = 48.6°$

or $ADB = 180° - 48.5903...° = 131.4096...° = 131°$

Angle $ADB = 48.6°$ or $131°$

2 (a)(i)

$\frac{1}{3 - 2\sqrt{x}} + \frac{1}{3 + 2\sqrt{x}} = \frac{(3 + 2\sqrt{x}) + (3 - 2\sqrt{x})}{(3 - 2\sqrt{x})(3 + 2\sqrt{x})}$

$= \frac{6}{9 - 4x}$ $\quad (a = 6, b = 9, c = -4)$

2 (a)(ii)

$\frac{6}{9 - 4x} = 2 \Rightarrow 6 = 18 - 8x$

$8x = 12 \Rightarrow x = \frac{3}{2}$

2 (b)

Let $u = 2^y$: $(2^y)^2 - 7 \times 2^y - 8 = 0$

$u^2 - 7u - 8 = 0 \Rightarrow (u - 8)(u + 1) = 0$

$2^y = 8$ or $2^y = -1$

Question number	
	$2^y = 8 \Rightarrow y = \log_2 8 = 3$
	$2^y = -1$ has no solutions since $2^y > 0$ for all y
	So $y = 3$ only
3 (a)	$f(x + h) - f(x) = ((x + h)^2 + 2(x + h)) - (x^2 + 2x)$
	$= x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x$
	$= 2xh + h^2 + 2h$
	$\frac{f(x + h) - f(x)}{h} = \frac{2xh + h^2 + 2h}{h} = 2x + h + 2$
	$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} (2x + h + 2)$
	$f'(x) = 2x + 2$ as required
3 (b)	$y = \int (2x + 2)\, dx = x^2 + 2x + c$
	Through $(-1, 5)$: $5 = (-1)^2 + 2(-1) + c = 1 - 2 + c$
	$c = 6$
	$y = x^2 + 2x + 6$
4 (a)	$AB = (3i + 5j) - (i + j) = 2i + 4j$
	$\lvert AB \rvert = \sqrt{(2^2 + 4^2)} = \sqrt{20} = 2\sqrt{5}$
4 (b)	$BC = (pi + pj) - (3i + 5j) = (p - 3)i + (p - 5)j$
	$\lvert BC \rvert = \lvert AB \rvert$: $(p - 3)^2 + (p - 5)^2 = 20$
	$p^2 - 6p + 9 + p^2 - 10p + 25 = 20$
	$2p^2 - 16p + 14 = 0 \Rightarrow p^2 - 8p + 7 = 0$
	$(p - 7)(p - 1) = 0 \Rightarrow p = 7$ ($p = 1$ rejected as $p > 1$)
	Position vector of C is $7i + 7j$
4 (c)	$OM = \frac{1}{2}((i + j) + (7i + 7j)) = 4i + 4j$

Question number	
	$\overrightarrow{BM} = (4\mathbf{i} + 4\mathbf{j}) - (3\mathbf{i} + 5\mathbf{j}) = \mathbf{i} - \mathbf{j}$
	$\overrightarrow{MD} = 2\overrightarrow{BM} = 2\mathbf{i} - 2\mathbf{j}$
	$\overrightarrow{OD} = \overrightarrow{OM} + \overrightarrow{MD} = (4\mathbf{i} + 4\mathbf{j}) + (2\mathbf{i} - 2\mathbf{j}) = 6\mathbf{i} + 2\mathbf{j}$
	Position vector of D is $6\mathbf{i} + 2\mathbf{j}$
4 (d)	ABCD is a kite.
	$\overrightarrow{AD} = (6\mathbf{i} + 2\mathbf{j}) - (\mathbf{i} + \mathbf{j}) = 5\mathbf{i} + \mathbf{j}$, $\lvert AD\rvert = \sqrt{26}$
	$\overrightarrow{CD} = (6\mathbf{i} + 2\mathbf{j}) - (7\mathbf{i} + 7\mathbf{j}) = -\mathbf{i} - 5\mathbf{j}$, $\lvert CD\rvert = \sqrt{26}$
	So $AB = BC = 2\sqrt{5}$ and $AD = CD = \sqrt{26}$: two pairs of
	adjacent sides equal (and $AB \neq AD$), so ABCD is a kite.
5 (a)(i)	$y = \lvert ax - b\rvert + c$ has its vertex where $ax - b = 0$,
	so the vertex is at $(\frac{b}{a}, c) = (3, 1) \Rightarrow c = 1,\ b = 3a$
	At $(0, 7)$: $7 = \lvert -b\rvert + 1 \Rightarrow b = 6\ \ (b > 0)$
	$b = 3a \Rightarrow a = 2$
	$a = 2,\ b = 6,\ c = 1$
5 (a)(ii)	f is a many-to-one function, eg $f(0) = f(6) = 6$,
	so f is not one-to-one and has no inverse function.
5 (b)(i)	For $x \geq \frac{q}{p}$, $px - q \geq 0$ so $g(x) = px - q$
	$y = px - q \Rightarrow px = y + q \Rightarrow x = \frac{y + q}{p}$
	$g^{-1}(x) = \frac{x + q}{p} = \frac{1}{p}x + \frac{q}{p}$
	Range of g is $g(x) \geq 0$, so the domain of g^{-1} is $x \geq 0$

Question number	
5 (b)(ii)	g and g^{-1} meet on $y = x$; for no solutions the gradient
	of g must be at most 1 (else g crosses $y = x$ for $x \geqslant \frac{q}{p}$).
	$0 < p \leqslant 1$
6 (a)	$y = e^{x^2 + 3x}$
	$\frac{dy}{dx} = (2x + 3)e^{x^2 + 3x}$
	Stationary points: $(2x + 3)e^{x^2 + 3x} = 0$
	$e^{x^2 + 3x} > 0$ for all x, so it is never zero
	$2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$ (only stationary point)
6 (b)	Product rule on $(2x + 3)e^{x^2 + 3x}$:
	$\frac{d^2y}{dx^2} = 2e^{x^2 + 3x} + (2x + 3)(2x + 3)e^{x^2 + 3x}$
	$\frac{d^2y}{dx^2} = (2 + (2x + 3)^2)e^{x^2 + 3x}$
	The curve is convex where $\frac{d^2y}{dx^2} > 0$
	$(2x + 3)^2 \geqslant 0$ for all $x \Rightarrow 2 + (2x + 3)^2 \geqslant 2 > 0$
	and $e^{x^2 + 3x} > 0$ for all x
	So $\frac{d^2y}{dx^2} > 0$ for all x, hence the curve is convex for all x.
7 (a)	Replace B with $-B$:
	$\cos(A - B) = \cos A \cos(-B) - \sin A \sin(-B)$
	$\cos(-B) = \cos B$ and $\sin(-B) = -\sin B$
	$\cos(A - B) = \cos A \cos B - \sin A(-\sin B)$

Question number	
	$\cos(A - B) = \cos A \cos B + \sin A \sin B$ as required
7 (b)	$f(\theta) = (\cos\theta \cos 30° - \sin\theta \sin 30°)(\cos\theta \cos 30° + \sin\theta \sin 30°)$
	$= (\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta)(\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta)$
	$= \frac{3}{4}\cos^2\theta - \frac{1}{4}\sin^2\theta$
	$= \frac{3}{4}\cos^2\theta - \frac{1}{4}(1 - \cos^2\theta)$
	$= \cos^2\theta - \frac{1}{4}$ as required
7 (c)(i)	$0 \leq \cos^2\theta \leq 1$. Max when $\cos^2\theta = 1$:
	Maximum value of $f(\theta) = 1 - \frac{1}{4} = \frac{3}{4}$
	$\cos\theta = \pm 1$, smallest positive θ: $\theta = 180°$
7 (c)(ii)	Min when $\cos^2\theta = 0$:
	Minimum value of $f(\theta) = 0 - \frac{1}{4} = -\frac{1}{4}$
	$\cos\theta = 0$, smallest positive θ: $\theta = 90°$
8 (a)	$(4 + 3x)^{3/2} = 4^{3/2}(1 + \frac{3}{4}x)^{3/2} = 8(1 + \frac{3}{4}x)^{3/2}$
	$(1 + \frac{3}{4}x)^{3/2} = 1 + (\frac{3}{2})(\frac{3}{4}x) + \frac{(3/2)(1/2)}{2!}(\frac{3}{4}x)^2 + \ldots$
	$= 1 + \frac{9}{8}x + \frac{3}{8} \times \frac{9}{16}x^2 + \ldots = 1 + \frac{9}{8}x + \frac{27}{128}x^2 + \ldots$

Question number

$(4 + 3x)^{3/2} = 8 + 9x + \frac{27}{16}x^2 + \ldots$

8 (b) Valid for $|\frac{3}{4}x| < 1$, ie $|x| < \frac{4}{3}$ $\quad (-\frac{4}{3} < x < \frac{4}{3})$

8 (c) $(1 + ax)^2 = 1 + 2ax + a^2x^2$

$(8 + 9x + \frac{27}{16}x^2 + \ldots)(1 + 2ax + a^2x^2)$

Coefficient of x^2: $8a^2 + 18a + \frac{27}{16}$

$8a^2 + 18a + \frac{27}{16} = \frac{107}{16} \Rightarrow 8a^2 + 18a - 5 = 0$

$(2a + 5)(4a - 1) = 0$

$a = -\frac{5}{2}$ or $a = \frac{1}{4}$

9 (a) $\frac{dB}{dF}$ is the rate of change of the number of bees with respect to the number of wildflower plants.

9 (b) $\frac{dF}{dt} = 5e^{0.1t}$ $\quad \frac{dB}{dt} = 2 - 3\sin 3t$

$\frac{dB}{dF} = \frac{dB}{dt} \times \frac{dt}{dF} = \frac{2 - 3\sin 3t}{5e^{0.1t}}$

$t = 4$: $\frac{dB}{dF} = \frac{2 - 3\sin 12}{5e^{0.4}} = \frac{3.6097\ldots}{7.4591\ldots} = 0.48393\ldots$

$\frac{dB}{dF} = 0.484$

Question number	
9 (c)	The study is over a summer; after 12 weeks the summer is ending, so in autumn there will be fewer flowers and bees and the pattern of the model is unlikely to continue.
10 (a)	$f(0) = 1 > 0$ and $f(1) = 2.906\ldots > 0$, so there is no sign change (the curve has an asymptote at $x = \frac{1}{2}$ between them).
10 (b)	$\frac{e^x}{4x^2 - 1} + 2 = 0 \Rightarrow \frac{e^x}{4x^2 - 1} = -2$ $e^x = -8x^2 + 2 \Rightarrow 8x^2 = 2 - e^x$ $16x^2 = 4 - 2e^x \Rightarrow 4x = \sqrt{(4 - 2e^x)}$ (positive root) $x = \frac{1}{4}\sqrt{(4 - 2e^x)}$ as required
10 (c)	$x_1 = 0.3$ $x_2 = 0.285074813\ldots$ $x_3 = 0.289425829\ldots$ $x_4 = 0.288170876\ldots$ $x_5 = 0.288533960\ldots$ $x_6 = 0.288429006\ldots$ $x_7 = 0.288459352\ldots$ $x_8 = 0.288450579\ldots$ x_7 and x_8 both round to 0.2885, so $\alpha = 0.2885$ (4 s.f.)
10 (d)	$F(x) = \ln(2 - 8x^2) \Rightarrow F'(x) = \frac{-16x}{2 - 8x^2}$ $F'(0.3) = \frac{-16 \times 0.3}{2 - 8 \times 0.09} = \frac{-4.8}{1.28} = -3.75$ For convergence we need $\lvert F'(\alpha)\rvert < 1$ near the root. $\alpha \approx 0.3$ and $\lvert F'(0.3)\rvert = 3.75 > 1$ ($F' < -1$), so the iteration diverges and will not find α.

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11 (a) $\log_{10}S = \log_{10}(ab^t) = \log_{10}a + \log_{10}b^t$

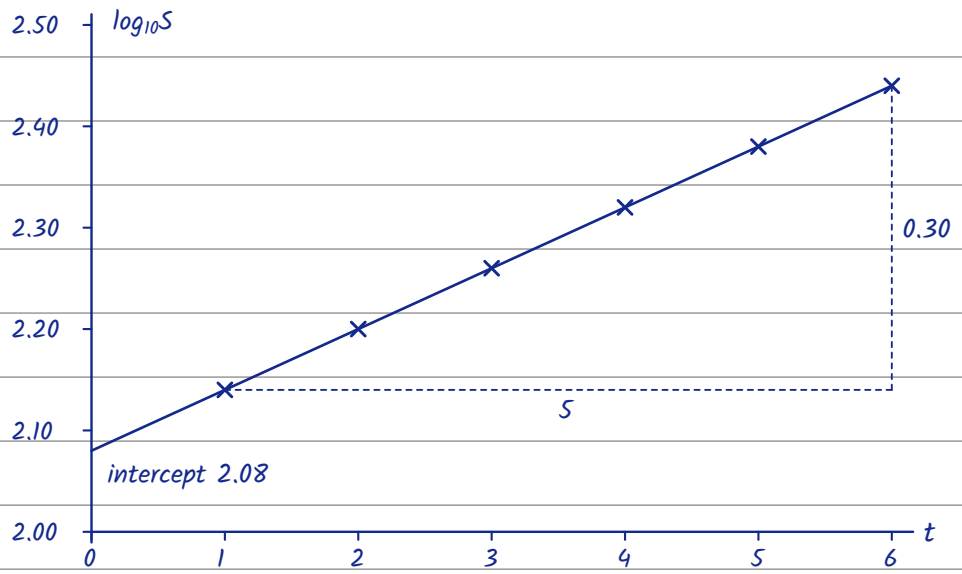
$\log_{10}S = (\log_{10}b)t + \log_{10}a$

This is of the form $Y = mX + c$ with $Y = \log_{10}S$, $X = t$,

so the model predicts a straight line. The plotted points

lie on a straight line, so the model is appropriate.

11 (b)



Gradient of line $= \frac{2.44 - 2.14}{6 - 1} = 0.06 = \log_{10}b$

$b = 10^{0.06} = 1.148\ldots \approx 1.15$ $(\log_{10}1.15 = 0.0607)$

Intercept $= 2.08 = \log_{10}a \Rightarrow a = 10^{2.08} = 120.2\ldots \approx 120$

$(\log_{10}120 = 2.079)$, so the graph supports $a = 120$, $b = 1.15$

11 (c) Seventh month: $t = 7$

$S = 120 \times 1.15^7 = 319.20\ldots$

Predicted sales: 319 items

11 (d)(i) Months $t = 1, 2, 3, \ldots$: GP with first term $120 \times 1.15 = 138$,

common ratio $r = 1.15$

$S_n = \frac{138(1.15^n - 1)}{1.15 - 1} > 70\,000$

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$1.15^n - 1 > \frac{70\,000 \times 0.15}{138} = 76.087... \Rightarrow 1.15^n > 77.087...$

$n > \frac{\ln 77.087...}{\ln 1.15} = 31.088...$

So the total first exceeds 70 000 after 32 months

11 (d)(ii) Not very reliable: it extrapolates far beyond the 6 months of data, and sales are unlikely to keep growing at 15% a month (the market could become saturated).

12 (a) $u = e^x - 2 \Rightarrow \frac{du}{dx} = e^x \Rightarrow dx = \frac{du}{e^x} = \frac{du}{u+2}$

$e^x = u + 2$, so $7e^x - 8 = 7(u+2) - 8 = 7u + 6$

$\int \frac{7e^x - 8}{(e^x - 2)^2}\,dx = \int \frac{7(u+2) - 8}{u^2} \times \frac{1}{u+2}\,du$

$= \int \frac{7u + 14 - 8}{u^2(u+2)}\,du = \int \frac{7u+6}{u^2(u+2)}\,du$ as required

12 (b) $\frac{7u+6}{u^2(u+2)} \equiv \frac{A}{u} + \frac{B}{u^2} + \frac{C}{u+2}$

$7u + 6 \equiv Au(u+2) + B(u+2) + Cu^2$

$u = 0$: $6 = 2B \Rightarrow B = 3$ $\quad u = -2$: $-8 = 4C \Rightarrow C = -2$

u^2: $0 = A + C \Rightarrow A = 2$

$\frac{7u+6}{u^2(u+2)} = \frac{2}{u} + \frac{3}{u^2} - \frac{2}{u+2}$

Limits: $x = \ln 4 \Rightarrow u = 4 - 2 = 2$; $x = \ln 6 \Rightarrow u = 6 - 2 = 4$

$\int_2^4 \left(\frac{2}{u} + \frac{3}{u^2} - \frac{2}{u+2}\right)du = [2\ln u - \frac{3}{u} - 2\ln(u+2)]$

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$= (2 \ln 4 - \frac{3}{4} - 2 \ln 6) - (2 \ln 2 - \frac{3}{2} - 2 \ln 4)$

$= (\frac{3}{2} - \frac{3}{4}) + 2 \ln 4 - 2 \ln 6 - 2 \ln 2 + 2 \ln 4$

$= \frac{3}{4} + \ln \frac{4^2 \times 4^2}{6^2 \times 2^2} = \frac{3}{4} + \ln \frac{256}{144}$

$\int_{\ln 4}^{\ln 6} \frac{7e^x - 8}{(e^x - 2)^2} dx = \frac{3}{4} + \ln \frac{16}{9}$ $\quad a = \frac{3}{4}, \quad b = \frac{16}{9}$