



Oxford Cambridge and RSA

Tuesday 21 June 2022 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours



You must have:

- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **12** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae

A Level Mathematics A (H240)

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2, \quad \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Standard deviation

$$\sqrt{\frac{\sum(x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np , variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Section A: Pure Mathematics

Answer **all** the questions.

1 Solve the equation $|2x - 3| = 9$. [3]

2 (a) Give full details of the single transformation that transforms the graph of $y = x^3$ to the graph of $y = x^3 - 8$. [2]

The function f is defined by $f(x) = x^3 - 8$.

(b) Find an expression for $f^{-1}(x)$. [2]

(c) State how the graphs of $y = f(x)$ and $y = f^{-1}(x)$ are related geometrically. [1]

3 The points P and Q have coordinates $(2, -5)$ and $(3, 1)$ respectively.

Determine the equation of the circle that has PQ as a diameter. Give your answer in the form $x^2 + y^2 + ax + by + c = 0$, where a , b and c are integers. [4]

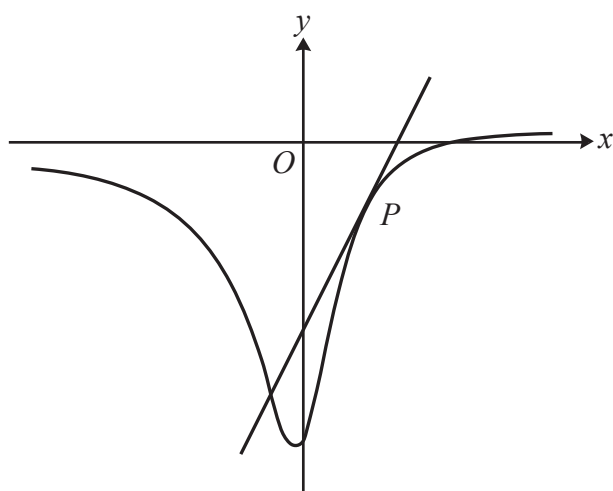
4 The positive integers x , y and z are the first, second and third terms, respectively, of an arithmetic progression with common difference -4 .

Also, x , $\frac{15}{y}$ and z are the first, second and third terms, respectively, of a geometric progression.

(a) Show that y satisfies the equation $y^4 - 16y^2 - 225 = 0$. [4]

(b) Hence determine the sum to infinity of the geometric progression. [4]

5 In this question you must show detailed reasoning.



The diagram shows the curve with equation $y = \frac{2x-3}{4x^2+1}$. The tangent to the curve at the point P has gradient 2.

- (a) Show that the x -coordinate of P satisfies the equation

$$4x^3 + 3x - 3 = 0. \quad [5]$$

- (b) Show by calculation that the x -coordinate of P lies between 0.5 and 1. [2]

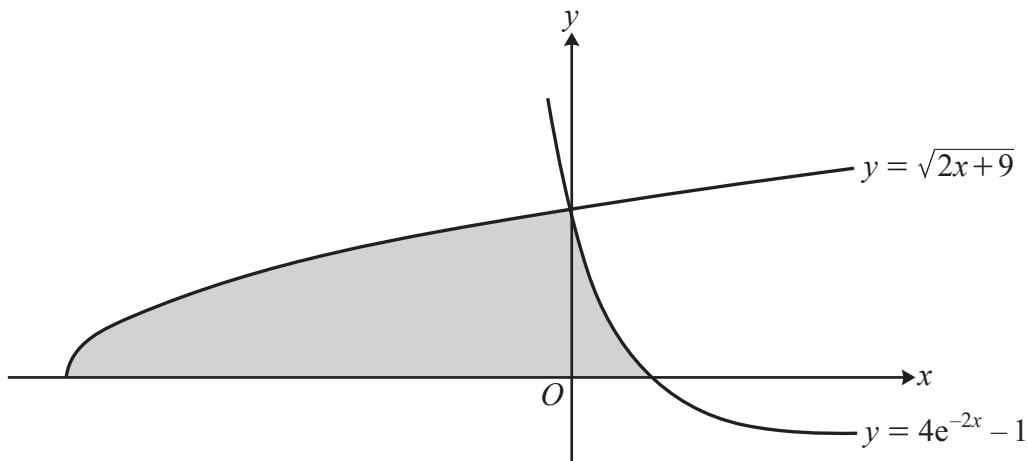
- (c) Show that the iteration

$$x_{n+1} = \frac{3-4x_n^3}{3}$$

cannot converge to the x -coordinate of P whatever starting value is used. [2]

- (d) Use the Newton-Raphson method, with initial value 0.5, to determine the coordinates of P correct to 5 decimal places. [5]

6 In this question you must show detailed reasoning.



The diagram shows the curves $y = \sqrt{2x+9}$ and $y = 4e^{-2x} - 1$ which intersect on the y -axis. The shaded region is bounded by the curves and the x -axis.

Determine the area of the shaded region, giving your answer in the form $p + q \ln 2$ where p and q are constants to be determined. [8]

7 In this question you must show detailed reasoning.

(a) Show that the equation $m \sec \theta + 3 \cos \theta = 4 \sin \theta$ can be expressed in the form

$$m \tan^2 \theta - 4 \tan \theta + (m + 3) = 0. \quad [3]$$

(b) It is given that there is only one value of θ , for $0 < \theta < \pi$, satisfying the equation $m \sec \theta + 3 \cos \theta = 4 \sin \theta$.

Given also that m is a negative integer, find this value of θ , correct to 3 significant figures. [5]

Section B: Mechanics

Answer **all** the questions.

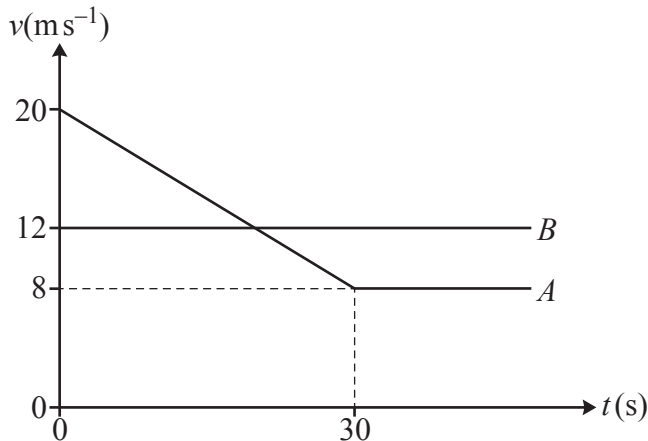
8



A child attempts to drag a sledge along horizontal ground by means of a rope attached to the sledge. The rope makes an angle of 15° with the horizontal (see diagram).

Given that the sledge remains at rest and that the frictional force acting on the sledge is 60 N , find the tension in the rope. [2]

9

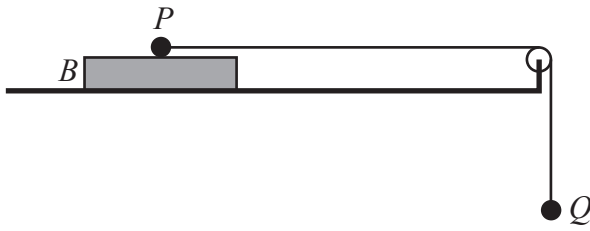


The diagram shows a velocity-time graph representing the motion of two cars A and B which are both travelling along a horizontal straight road. At time $t = 0$, car B , which is travelling with constant speed 12 m s^{-1} , is overtaken by car A which has initial speed 20 m s^{-1} .

From $t = 0$ car A travels with constant deceleration for 30 seconds. When $t = 30$ the speed of car A is 8 m s^{-1} and the car maintains this speed in its subsequent motion.

(a) Calculate the deceleration of car A . [2]

(b) Determine the value of t when B overtakes A . [4]

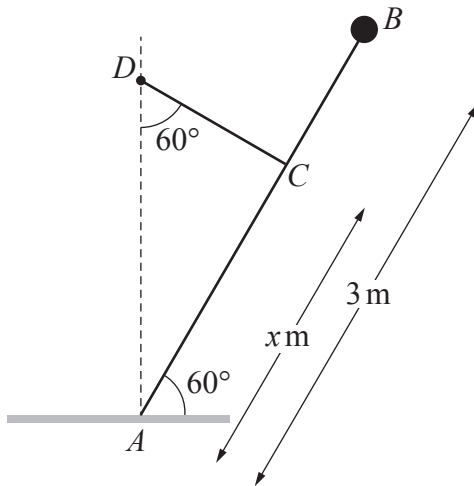


A rectangular block B is at rest on a horizontal surface. A particle P of mass 2.5 kg is placed on the upper surface of B . The particle P is attached to one end of a light inextensible string which passes over a smooth fixed pulley. A particle Q of mass 3 kg is attached to the other end of the string and hangs freely below the pulley. The part of the string between P and the pulley is horizontal (see diagram).

The particles are released from rest with the string taut. It is given that B remains in equilibrium while P moves on the upper surface of B . The tension in the string while P moves on B is 16.8 N .

- (a) Find the acceleration of Q while P and B are in contact. [2]
- (b) Determine the coefficient of friction between P and B . [3]
- (c) Given that the coefficient of friction between B and the horizontal surface is $\frac{5}{49}$, determine the least possible value for the mass of B . [3]

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A uniform rod AB of mass 4 kg and length 3 m rests in a vertical plane with A on rough horizontal ground.

A particle of mass 1 kg is attached to the rod at B . The rod makes an angle of 60° with the horizontal and is held in limiting equilibrium by a light inextensible string CD . D is a fixed point vertically above A and CD makes an angle of 60° with the vertical. The distance AC is $x\text{ m}$ (see diagram).

(a) Find, in terms of g and x , the tension in the string. [3]

The coefficient of friction between the rod and the ground is $\frac{9\sqrt{3}}{35}$.

(b) Determine the value of x . [4]

- 12** In this question the unit vectors **i** and **j** are in the directions east and north respectively.

A particle P is moving on a smooth horizontal surface under the action of a single force $\mathbf{F}$ N. At time t seconds, where $t \geq 0$, the velocity $\mathbf{v}$ ms⁻¹ of P , relative to a fixed origin O , is given by

$$\mathbf{v} = (1 - 2t)\mathbf{i} + (2t^2 + t - 13)\mathbf{j}.$$

- (a) Show that P is never stationary. [2]

- (b) Find, in terms of **i** and **j**, the acceleration of P at time t . [1]

The mass of P is 0.5 kg.

- (c) Determine the magnitude of $\mathbf{F}$ when P is moving in the direction of the vector $-2\mathbf{i} + \mathbf{j}$. Give your answer correct to 3 significant figures. [5]

When $t = 1$, P is at the point with position vector $\frac{1}{6}\mathbf{j}$.

- (d) Determine the bearing of P from O at time $t = 1.5$. [5]

- 13** A small ball B moves in the plane of a fixed horizontal axis Ox , which lies on horizontal ground, and a fixed vertically upwards axis Oy . B is projected from O with a velocity whose components along Ox and Oy are U ms⁻¹ and V ms⁻¹, respectively. The units of x and y are metres.

B is modelled as a particle moving freely under gravity.

- (a) Show that the path of B has equation $2U^2y = 2UVx - gx^2$. [3]

During its motion, B just clears a vertical wall of height $\frac{1}{2}a$ m at a horizontal distance a m from O . B strikes the ground at a horizontal distance $3a$ m beyond the wall.

- (b) Determine the angle of projection of B . Give your answer in degrees correct to 3 significant figures. [5]

- (c) Given that the speed of projection of B is 54.6 ms⁻¹, determine the value of a . [2]

- (d) Hence find the maximum height of B above the ground during its motion. [3]

- (e) State **one** refinement of the model, other than including air resistance, that would make it more realistic. [1]

END OF QUESTION PAPER

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