OCR
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Tuesday 21 June 2022 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours

9086358730

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient rule $y = \dfrac{u}{v},\ \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta,\ \cos\theta \approx 1 - \frac{1}{2}\theta^2,\ \tan\theta \approx \theta$ where θ is measured in radians

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Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

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Section A: Pure Mathematics

Answer **all** the questions.

1 Solve the equation $|2x-3|=9$. **[3]**

2 **(a)** Give full details of the single transformation that transforms the graph of $y=x^3$ to the graph of $y=x^3-8$. **[2]**

The function f is defined by $f(x)=x^3-8$.

(b) Find an expression for $f^{-1}(x)$. **[2]**

(c) State how the graphs of $y=f(x)$ and $y=f^{-1}(x)$ are related geometrically. **[1]**

3 The points P and Q have coordinates $(2, -5)$ and $(3, 1)$ respectively.

Determine the equation of the circle that has PQ as a diameter. Give your answer in the form $x^2+y^2+ax+by+c=0$, where a, b and c are integers. **[4]**

4 The positive integers x, y and z are the first, second and third terms, respectively, of an arithmetic progression with common difference -4.

Also, x, $\frac{15}{y}$ and z are the first, second and third terms, respectively, of a geometric progression.

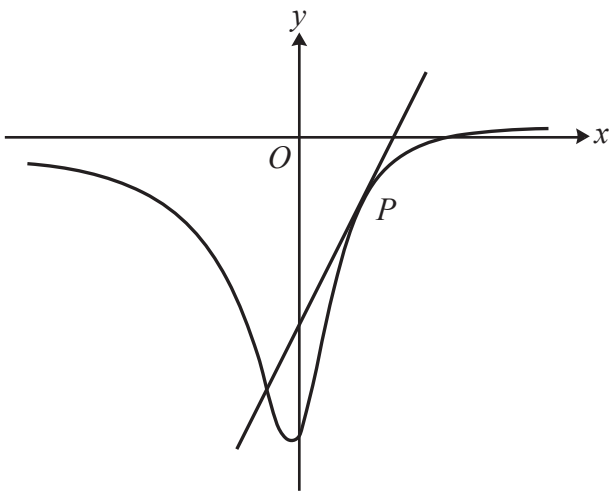
(a) Show that y satisfies the equation $y^4-16y^2-225=0$. **[4]**

(b) Hence determine the sum to infinity of the geometric progression. **[4]**

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5 **In this question you must show detailed reasoning.**



The diagram shows the curve with equation $y = \dfrac{2x-3}{4x^2+1}$. The tangent to the curve at the point P has gradient 2.

(a) Show that the x-coordinate of P satisfies the equation

$$4x^3 + 3x - 3 = 0.$$ **[5]**

(b) Show by calculation that the x-coordinate of P lies between 0.5 and 1. **[2]**

(c) Show that the iteration

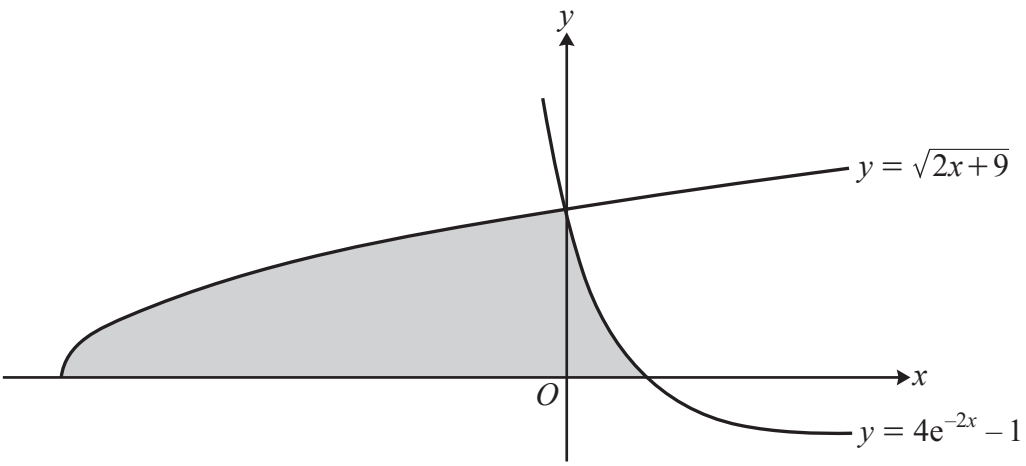
$$x_{n+1} = \frac{3-4x_n^3}{3}$$

cannot converge to the x-coordinate of P whatever starting value is used. **[2]**

(d) Use the Newton-Raphson method, with initial value 0.5, to determine the coordinates of P correct to **5** decimal places. **[5]**

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6 In this question you must show detailed reasoning.



The diagram shows the curves $y = \sqrt{2x+9}$ and $y = 4e^{-2x} - 1$ which intersect on the y-axis. The shaded region is bounded by the curves and the x-axis.

Determine the area of the shaded region, giving your answer in the form $p + q\ln 2$ where p and q are constants to be determined. **[8]**

7 In this question you must show detailed reasoning.

(a) Show that the equation $m\sec\theta + 3\cos\theta = 4\sin\theta$ can be expressed in the form

$m\tan^2\theta - 4\tan\theta + (m+3) = 0.$ **[3]**

(b) It is given that there is only one value of θ, for $0 < \theta < \pi$, satisfying the equation $m\sec\theta + 3\cos\theta = 4\sin\theta$.

Given also that m is a negative integer, find this value of θ, correct to **3** significant figures. **[5]**

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Section B: Mechanics

Answer **all** the questions.

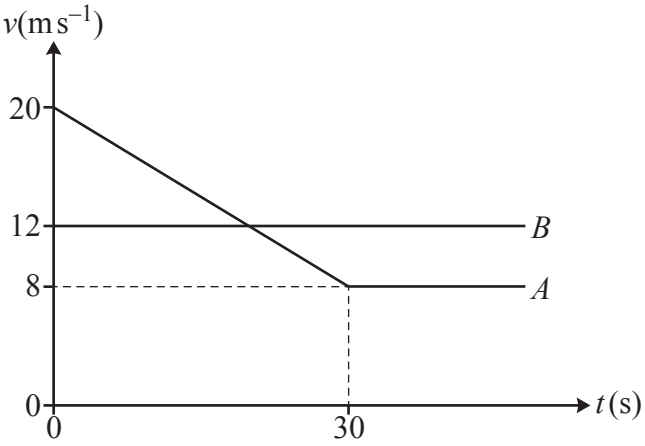
8



A child attempts to drag a sledge along horizontal ground by means of a rope attached to the sledge. The rope makes an angle of 15° with the horizontal (see diagram).

Given that the sledge remains at rest and that the frictional force acting on the sledge is 60 N, find the tension in the rope. **[2]**

9



The diagram shows a velocity-time graph representing the motion of two cars A and B which are both travelling along a horizontal straight road. At time $t = 0$, car B, which is travelling with constant speed $12\,\text{m s}^{-1}$, is overtaken by car A which has initial speed $20\,\text{m s}^{-1}$.

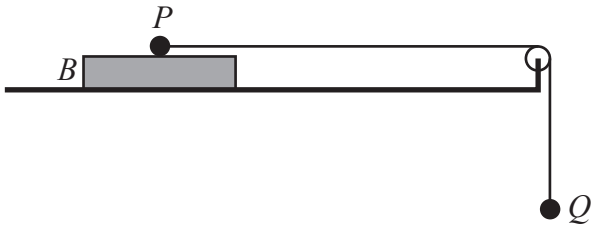
From $t = 0$ car A travels with constant deceleration for 30 seconds. When $t = 30$ the speed of car A is $8\,\text{m s}^{-1}$ and the car maintains this speed in its subsequent motion.

(a) Calculate the deceleration of car A. **[2]**

(b) Determine the value of t when B overtakes A. **[4]**

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10



A rectangular block B is at rest on a horizontal surface. A particle P of mass 2.5 kg is placed on the upper surface of B. The particle P is attached to one end of a light inextensible string which passes over a smooth fixed pulley. A particle Q of mass 3 kg is attached to the other end of the string and hangs freely below the pulley. The part of the string between P and the pulley is horizontal (see diagram).

The particles are released from rest with the string taut. It is given that B remains in equilibrium while P moves on the upper surface of B. The tension in the string while P moves on B is 16.8 N.

(a) Find the acceleration of Q while P and B are in contact. **[2]**

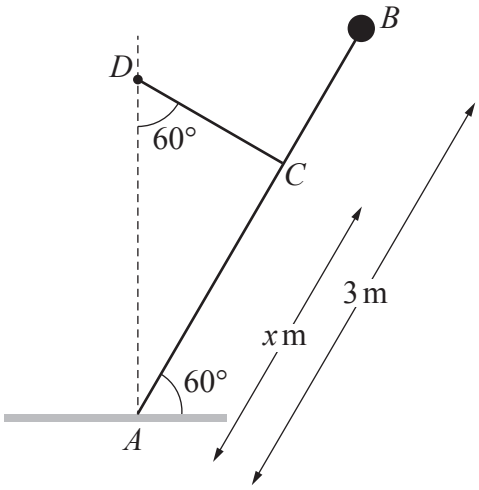
(b) Determine the coefficient of friction between P and B. **[3]**

(c) Given that the coefficient of friction between B and the horizontal surface is $\frac{5}{49}$, determine the least possible value for the mass of B. **[3]**

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A uniform rod AB of mass 4 kg and length 3 m rests in a vertical plane with A on rough horizontal ground.

A particle of mass 1 kg is attached to the rod at B. The rod makes an angle of 60° with the horizontal and is held in limiting equilibrium by a light inextensible string CD. D is a fixed point vertically above A and CD makes an angle of 60° with the vertical. The distance AC is x m (see diagram).

(a) Find, in terms of g and x, the tension in the string. **[3]**

The coefficient of friction between the rod and the ground is $\frac{9\sqrt{3}}{35}$.

(b) Determine the value of x. **[4]**

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12 In this question the unit vectors **i** and **j** are in the directions east and north respectively.

A particle P is moving on a smooth horizontal surface under the action of a single force $\mathbf{F}$ N. At time t seconds, where $t \geqslant 0$, the velocity $\mathbf{v}\,\text{m s}^{-1}$ of P, relative to a fixed origin O, is given by

$\mathbf{v} = (1-2t)\mathbf{i} + (2t^2 + t - 13)\mathbf{j}$.

(a) Show that P is never stationary. **[2]**

(b) Find, in terms of **i** and **j**, the acceleration of P at time t. **[1]**

The mass of P is 0.5 kg.

(c) Determine the magnitude of $\mathbf{F}$ when P is moving in the direction of the vector $-2\mathbf{i} + \mathbf{j}$. Give your answer correct to **3** significant figures. **[5]**

When $t = 1$, P is at the point with position vector $\frac{1}{6}\mathbf{j}$.

(d) Determine the bearing of P from O at time $t = 1.5$. **[5]**

13 A small ball B moves in the plane of a fixed horizontal axis Ox, which lies on horizontal ground, and a fixed vertically upwards axis Oy. B is projected from O with a velocity whose components along Ox and Oy are $U\,\text{m s}^{-1}$ and $V\,\text{m s}^{-1}$, respectively. The units of x and y are metres.

B is modelled as a particle moving freely under gravity.

(a) Show that the path of B has equation $2U^2y = 2UVx - gx^2$. **[3]**

During its motion, B just clears a vertical wall of height $\frac{1}{2}a$ m at a horizontal distance a m from O. B strikes the ground at a horizontal distance $3a$ m beyond the wall.

(b) Determine the angle of projection of B. Give your answer in degrees correct to **3** significant figures. **[5]**

(c) Given that the speed of projection of B is $54.6\,\text{m s}^{-1}$, determine the value of a. **[2]**

(d) Hence find the maximum height of B above the ground during its motion. **[3]**

(e) State **one** refinement of the model, other than including air resistance, that would make it more realistic. **[1]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1

$|2x - 3| = 9 \Rightarrow 2x - 3 = 9$ or $2x - 3 = -9$

$2x = 12$ or $2x = -6$

$x = 6$ or $x = -3$

2 (a)

Translation of -8 units parallel to the y-axis

(i.e. 8 units in the negative y-direction)

2 (b)

$y = x^3 - 8 \Rightarrow x^3 = y + 8 \Rightarrow x = (y + 8)^{1/3}$

$f^{-1}(x) = \sqrt[3]{(x + 8)}$ i.e. $f^{-1}(x) = (x + 8)^{1/3}$

2 (c)

The graph of $y = f^{-1}(x)$ is the reflection of the graph of $y = f(x)$

in the line $y = x$.

3

Centre = midpoint of PQ $= \left(\frac{2 + 3}{2}, \frac{-5 + 1}{2}\right) = \left(\frac{5}{2}, -2\right)$

$d^2 = PQ^2 = (3 - 2)^2 + (1 - (-5))^2 = 1 + 36 = 37 \Rightarrow r^2 = \frac{37}{4}$

Circle: $\left(x - \frac{5}{2}\right)^2 + (y + 2)^2 = \frac{37}{4}$

$x^2 - 5x + \frac{25}{4} + y^2 + 4y + 4 = \frac{37}{4}$

$x^2 + y^2 - 5x + 4y + \frac{25 + 16 - 37}{4} = 0$

$x^2 + y^2 - 5x + 4y + 1 = 0$

4 (a)

AP with $d = -4$: $y - x = -4$, $z - y = -4$

$\Rightarrow x = y + 4$, $z = y - 4$

GP: $x, \frac{15}{y}, z \Rightarrow$ common ratio $r = \frac{15/y}{x} = \frac{z}{15/y}$

$\Rightarrow \left(\frac{15}{y}\right)^2 = xz \Rightarrow \frac{225}{y^2} = (y + 4)(y - 4)$

$225 = y^2(y + 4)(y - 4) = y^2(y^2 - 4y + 4y - 16) = y^4 - 16y^2$

Question number

$\therefore \quad y^4 - 16y^2 - 225 = 0 \quad$ as required

4 (b)

$(y^2 - 25)(y^2 + 9) = 0 \quad \Rightarrow \quad y^2 = 25 \quad (y^2 = -9$ has no real solutions)

$y = \pm 5$, but y is a positive integer so $y = 5$

$x = 5 + 4 = 9, \quad z = 5 - 4 = 1, \quad \frac{15}{y} = 3$: GP is 9, 3, 1, ...

$r = \frac{3}{9} = \frac{1}{3} \quad$ (check: $r^2 = \frac{z}{x} = \frac{1}{9}$) and $|r| < 1$ so S_∞ exists

$S_\infty = \frac{a}{1 - r} = \frac{9}{1 - 1/3} = \frac{9}{2/3} = \frac{27}{2}$

$S_\infty = 13.5$

5 (a)

$y = \frac{2x - 3}{4x^2 + 1} \quad$ Quotient rule:

$\frac{dy}{dx} = \frac{(4x^2 + 1)(2) - (2x - 3)(8x)}{(4x^2 + 1)^2} = \frac{8x^2 + 2 - 16x^2 + 24x}{(4x^2 + 1)^2} = \frac{2(1 + 12x - 4x^2)}{(4x^2 + 1)^2}$

Gradient 2 at P: $\frac{2(1 + 12x - 4x^2)}{(4x^2 + 1)^2} = 2$

$1 + 12x - 4x^2 = (4x^2 + 1)^2 = 16x^4 + 8x^2 + 1$

$16x^4 + 12x^2 - 12x = 0$

$4x(4x^3 + 3x - 3) = 0$

From the diagram P has positive x-coordinate, so $x \neq 0$

$\therefore \quad 4x^3 + 3x - 3 = 0 \quad$ as required

5 (b)

Let $f(x) = 4x^3 + 3x - 3$

$f(0.5) = 4(0.125) + 1.5 - 3 = -1 < 0$

$f(1) = 4 + 3 - 3 = 4 > 0$

f is continuous and there is a change of sign between $x = 0.5$ and $x = 1$,

so the root, the x-coordinate of P, lies between 0.5 and 1.

5 (c)

Let $h(x) = \frac{3 - 4x^3}{3} \quad \Rightarrow \quad h'(x) = \frac{-12x^2}{3} = -4x^2$

The root α lies in $(0.5, 1)$, so $h'(\alpha) = -4\alpha^2 < -4(0.5)^2 = -1$

$|h'(\alpha)| > 1$ for every value in this interval, so the iteration cannot converge

to the x-coordinate of P, whatever starting value is used.

Question number

5 (d)

$f(x) = 4x^3 + 3x - 3 \quad \Rightarrow \quad f'(x) = 12x^2 + 3$

$x_{n+1} = x_n - \dfrac{4x_n^3 + 3x_n - 3}{12x_n^2 + 3}$

$x_0 = 0.5$

$x_1 = 0.5 - \dfrac{-1}{6} = \dfrac{2}{3} = 0.6666666...$

$x_2 = \dfrac{29}{45} = 0.6444444...$

$x_3 = 0.6439551...$

$x_4 = 0.6439548... \qquad$ so $x = 0.64395$ (5 d.p.)

$y = \dfrac{2(0.6439548...) - 3}{4(0.6439548...)^2 + 1} = -0.6439548...$

P is $(0.64395, -0.64395)$

6

Curves meet on the y-axis at $(0, 3)$.

$y = \sqrt{(2x + 9)}$ meets the x-axis when $2x + 9 = 0$, $x = -\dfrac{9}{2}$

$\displaystyle\int_{-9/2}^{0} (2x + 9)^{1/2}\, dx = \left[\frac{1}{3}(2x + 9)^{3/2} \right]_{-9/2}^{0}$

$= \dfrac{1}{3}(9)^{3/2} - \dfrac{1}{3}(0) = \dfrac{27}{3} = 9$

$y = 4e^{-2x} - 1 = 0 \quad \Rightarrow \quad e^{-2x} = \dfrac{1}{4} \quad \Rightarrow \quad -2x = \ln \dfrac{1}{4}$

$x = -\dfrac{1}{2} \ln \dfrac{1}{4} = \dfrac{1}{2} \ln 4 = \ln 2$

$\displaystyle\int_{0}^{\ln 2} (4e^{-2x} - 1)\, dx = \left[-2e^{-2x} - x \right]_{0}^{\ln 2}$

$= (-2e^{-2\ln 2} - \ln 2) - (-2e^{0} - 0)$

$= (-2 \times \dfrac{1}{4} - \ln 2) + 2 = \dfrac{3}{2} - \ln 2$

Area $= 9 + \dfrac{3}{2} - \ln 2 = \dfrac{21}{2} - \ln 2$

$p = \dfrac{21}{2}, \quad q = -1$

7 (a)

$m \sec\theta + 3\cos\theta = 4\sin\theta$

Divide by $\cos\theta$: $\quad m \sec^2\theta + 3 = 4\tan\theta$

Use $\sec^2\theta = 1 + \tan^2\theta$: $\quad m(1 + \tan^2\theta) + 3 = 4\tan\theta$

$m + m\tan^2\theta + 3 = 4\tan\theta$

$\therefore \quad m\tan^2\theta - 4\tan\theta + (m + 3) = 0 \quad$ as required

Question number	
7 (b)	For $0 < \theta < \pi$ ($\theta \neq \frac{\pi}{2}$), $\tan\theta$ takes each non-zero real value exactly once
	(and $\tan\theta = 0$ does not satisfy the original equation for $0 < \theta < \pi$),
	so only one value of θ means the quadratic in $\tan\theta$ has exactly one root:
	discriminant $\Delta = (-4)^2 - 4m(m+3) = 0$
	$16 - 4m^2 - 12m = 0 \Rightarrow m^2 + 3m - 4 = 0 \Rightarrow (m+4)(m-1) = 0$
	$m = -4$ or $m = 1$; m is a negative integer so $m = -4$
	$-4\tan^2\theta - 4\tan\theta - 1 = 0 \Rightarrow 4\tan^2\theta + 4\tan\theta + 1 = 0$
	$(2\tan\theta + 1)^2 = 0$
	$\tan\theta = -0.5 \Rightarrow \theta = \pi - \tan^{-1}(0.5) = 2.677945\ldots$
	$\theta = 2.68$ (3 s.f.)
8	Sledge at rest: resolve horizontally
	$T\cos 15° = 60$
	$T = \frac{60}{\cos 15°} = 62.11657\ldots$
	$T = 62.1$ N
9 (a)	$v = u + at$: $8 = 20 + 30a \Rightarrow a = -0.4$
	Deceleration of A is 0.4 m s^{-2}
9 (b)	Distance travelled by B in t s: $12t$
	Distance by A in first 30 s: $\frac{1}{2}(20+8)(30) = 420$ m
	(B travels $12 \times 30 = 360$ m in this time, so A is still ahead at $t = 30$)
	For $t > 30$, distance by A: $420 + 8(t-30)$
	B overtakes A when $12t = 420 + 8(t-30)$
	$12t = 8t + 180 \Rightarrow 4t = 180$
	$t = 45$
10 (a)	N2L for Q (downwards): $3g - 16.8 = 3a$
	$a = \frac{3(9.8) - 16.8}{3} = \frac{12.6}{3} = 4.2$ m s^{-2}

Question number

10 (b)

N2L for P (horizontally): $16.8 - F_P = 2.5 \times 4.2$

$F_P = 16.8 - 10.5 = 6.3$ N

$R_P = 2.5g = 24.5$ N, P is sliding so $F_P = \mu R_P$

$6.3 = \mu(2.5 \times 9.8) \Rightarrow \mu = \frac{6.3}{24.5} = \frac{9}{35} = 0.257142...$

$\mu = 0.257$ (3 s.f.)

10 (c)

Let the mass of B be M kg. Resolve vertically for B: $R_B = 2.5g + Mg$

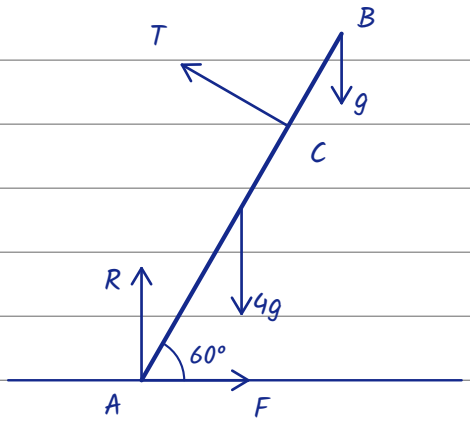
Friction from P on B is 6.3 N; B stays in equilibrium so $6.3 \leq \mu R_B$

$6.3 \leq \frac{5}{49}(2.5g + Mg) = \frac{5}{49} \times 9.8(2.5 + M) = 2.5 + M$

$M \geq 3.8$

Least possible mass of B is 3.8 kg

11 (a)



$TC \perp AB$ (angle $DCA = 180° - 30° - 60° = 90°$)

$AM = 1.5$ m, $AB = 3$ m, $AC = x$ m

Angle DAC = 30°, angle ADC = 60°, so the string is perpendicular to AB.

Moments about A: $T \times x = 4g \times 1.5\cos 60° + g \times 3\cos 60°$

$Tx = 3g + 1.5g = 4.5g$

$T = \frac{9g}{2x}$ N

11 (b)

Resolve vertically: $T\cos 60° + R = 4g + g$

$R = 5g - \frac{1}{2} \times \frac{9g}{2x} = 5g - \frac{9g}{4x}$

Resolve horizontally: $F = T\sin 60° = \frac{\sqrt{3}}{2} \times \frac{9g}{2x} = \frac{9\sqrt{3}\,g}{4x}$

Limiting equilibrium: $F = \mu R$

Question number

$\frac{9\sqrt{3}\,g}{4x} = \frac{9\sqrt{3}}{35}\left(5g - \frac{9g}{4x}\right)$

$\frac{35}{4x} = 5 - \frac{9}{4x} \Rightarrow \frac{44}{4x} = 5 \Rightarrow \frac{11}{x} = 5$

$x = 2.2$

12 (a) P stationary requires both components of v to be zero at the same time.

i: $1 - 2t = 0 \Rightarrow t = 0.5$

j: $2t^2 + t - 13 = 0 \Rightarrow t = \frac{-1 \pm \sqrt{105}}{4} = 2.3117...$ or $-2.8117...$

(at $t = 0.5$: $2(0.25) + 0.5 - 13 = -12 \neq 0$)

No value of t makes both components zero, so P is never stationary.

12 (b) $a = \frac{dv}{dt} = -2i + (4t + 1)j \quad m\,s^{-2}$

12 (c) v parallel to $-2i + j$: $\frac{2t^2 + t - 13}{1 - 2t} = \frac{1}{-2}$

$-2(2t^2 + t - 13) = 1 - 2t \Rightarrow -4t^2 - 2t + 26 = 1 - 2t$

$4t^2 - 25 = 0$

$t = 2.5 \quad (t \geq 0)$ check: $v = -4i + 2j = 2(-2i + j)$ ✓

$F = ma = 0.5(-2i + (4(2.5) + 1)j) = 0.5(-2i + 11j) = -i + 5.5j$

$|F| = \sqrt{((-1)^2 + 5.5^2)} = \sqrt{31.25} = 5.590169...$

$|F| = 5.59$ N

12 (d) $r = \int v\,dt = (t - t^2)i + (\frac{2}{3}t^3 + \frac{1}{2}t^2 - 13t)j + c$

$t = 1$: $r = 0i + (\frac{2}{3} + \frac{1}{2} - 13)j + c = -\frac{71}{6}j + c = \frac{1}{6}j \Rightarrow c = 12j$

$t = 1.5$: $r = (1.5 - 2.25)i + (\frac{2}{3}(3.375) + \frac{1}{2}(2.25) - 19.5 + 12)j$

$r = -\frac{3}{4}i - \frac{33}{8}j$ (P is south-west of O)

Angle west of south $= \tan^{-1}(\frac{3/4}{33/8}) = 10.30...°$

Bearing $= 180° + 10.30...° = 190.30...°$

Bearing of P from O is 190°

Question number

13 (a)

Horizontally: $x = Ut \quad \Rightarrow \quad t = \frac{x}{U}$

Vertically ($s = ut + \frac{1}{2}at^2$): $y = Vt - \frac{1}{2}gt^2$

$y = V(\frac{x}{U}) - \frac{1}{2}g(\frac{x}{U})^2 = \frac{Vx}{U} - \frac{gx^2}{2U^2}$

Multiply by $2U^2$: $2U^2y = 2UVx - gx^2$ as required

13 (b)

B passes through $(a, \frac{1}{2}a)$: $2U^2(\frac{1}{2}a) = 2UVa - ga^2$

$\Rightarrow \quad U^2 = 2UV - ga \quad (1)$

B lands at $x = a + 3a = 4a$, $y = 0$: $0 = 2UV(4a) - g(4a)^2$

$\Rightarrow \quad 8UVa = 16ga^2 \quad \Rightarrow \quad UV = 2ga \quad (2)$

Sub (2) into (1): $U^2 = 4ga - ga = 3ga \quad \Rightarrow \quad U = \sqrt{(3ga)}$

$V = \frac{2ga}{U} = \frac{2ga}{\sqrt{(3ga)}} = \frac{2\sqrt{(ga)}}{\sqrt{3}} \qquad (V^2 = \frac{4}{3}ga)$

$\tan\theta = \frac{V}{U} = \frac{2ga}{U^2} = \frac{2ga}{3ga} = \frac{2}{3}$

$\theta = \tan^{-1}(\frac{2}{3}) = 33.690...°$

Angle of projection = 33.7° (3 s.f.)

13 (c)

$U^2 + V^2 = 54.6^2$: $3ga + \frac{4}{3}ga = \frac{13}{3}ga = 2981.16$

$a = \frac{3 \times 2981.16}{13 \times 9.8} = 70.2$

13 (d)

At maximum height vertical velocity is 0: $0 = V^2 - 2gH$

$H = \frac{V^2}{2g} = \frac{(4/3)ga}{2g} = \frac{2}{3}a$

$H = \frac{2}{3}(70.2) = 46.8 \text{ m}$

13 (e)

Take into account the size of B (model B as a ball, not a particle),

e.g. so that the ball's radius is allowed for when it clears the wall.