



Oxford Cambridge and RSA

**Tuesday 7 June 2022 – Afternoon**

**A Level Mathematics A**

**H240/01 Pure Mathematics**

**Time allowed: 2 hours**



**You must have:**

- the Printed Answer Booklet
- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [ ].
- This document has **8** pages.

**ADVICE**

- Read each question carefully before you start your answer.

## Formulae

### A Level Mathematics A (H240)

#### Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

#### Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

#### Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

#### Differentiation

| $f(x)$                   | $f'(x)$                          |
|--------------------------|----------------------------------|
| $\tan kx$                | $k \sec^2 kx$                    |
| $\sec x$                 | $\sec x \tan x$                  |
| $\cot x$                 | $-\operatorname{cosec}^2 x$      |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

#### Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

#### Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

#### Small angle approximations

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2, \quad \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

**Trigonometric identities**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

**Numerical methods**

Trapezium rule:  $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$ , where  $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving  $f(x) = 0$ :  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

**Probability**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

**Standard deviation**

$$\sqrt{\frac{\sum(x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

**The binomial distribution**

If  $X \sim B(n, p)$  then  $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ , mean of  $X$  is  $np$ , variance of  $X$  is  $np(1-p)$

**Hypothesis test for the mean of a normal distribution**

If  $X \sim N(\mu, \sigma^2)$  then  $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$  and  $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

**Percentage points of the normal distribution**

If  $Z$  has a normal distribution with mean 0 and variance 1 then, for each value of  $p$ , the table gives the value of  $z$  such that  $P(Z \leq z) = p$ .

|     |       |       |       |       |       |       |        |       |        |
|-----|-------|-------|-------|-------|-------|-------|--------|-------|--------|
| $p$ | 0.75  | 0.90  | 0.95  | 0.975 | 0.99  | 0.995 | 0.9975 | 0.999 | 0.9995 |
| $z$ | 0.674 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 2.807  | 3.090 | 3.291  |

**Kinematics**

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

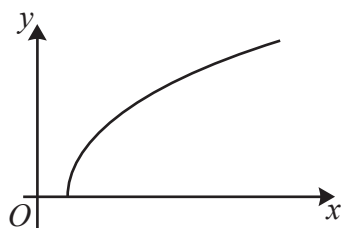
$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Answer **all** the questions.

1



The diagram shows part of the curve  $y = \sqrt{x^2 - 1}$ .

- (a) Use the trapezium rule with 4 intervals to find an estimate for  $\int_1^3 \sqrt{x^2 - 1} \, dx$ .

Give your answer correct to **3** significant figures.

[4]

- (b) State whether the value from part (a) is an under-estimate or an over-estimate, giving a reason for your answer.

[1]

- (c) Explain how the trapezium rule could be used to obtain a more accurate estimate.

[1]

- 2 (a) Given that  $a$  and  $b$  are real numbers, find a counterexample to disprove the statement that, if  $a > b$ , then  $a^2 > b^2$ .

[1]

- (b) A student writes the statement that  $\sin x^\circ = 0.5 \iff x^\circ = 30^\circ$ .

(i) Explain why this statement is incorrect.

[1]

(ii) Write a corrected version of this statement.

[1]

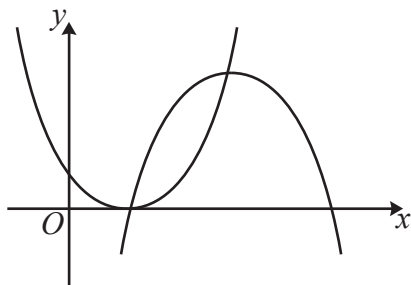
- (c) Prove that the sum of four consecutive multiples of 4 is always a multiple of 8.

[3]

**3 (a) In this question you must show detailed reasoning.**

Find the coordinates of the points of intersection of the curves with equations  $y = x^2 - 2x + 1$  and  $y = -x^2 + 6x - 5$ . [4]

- (b) The diagram shows the curves  $y = x^2 - 2x + 1$  and  $y = -x^2 + 6x - 5$ . This diagram is repeated in the Printed Answer Booklet.



On the diagram in the Printed Answer Booklet, draw the line  $y = 2x - 2$ . [2]

- (c) Show on your diagram in the Printed Answer Booklet the region of the  $x$ - $y$  plane within which all three of the following inequalities are satisfied.

$$y \geq x^2 - 2x + 1 \qquad y \leq -x^2 + 6x - 5 \qquad y \leq 2x - 2$$

You should indicate the region for which all the inequalities hold by labelling the region  $R$ . [1]

- 4 (a)** Write  $2x^2 + 6x + 7$  in the form  $p(x+q)^2 + r$ , where  $p$ ,  $q$  and  $r$  are constants. [3]

- (b) State the coordinates of the minimum point on the graph of  $y = 2x^2 + 6x + 7$ . [2]

- (c) Hence deduce

- the minimum value of  $2 \tan^2 \theta + 6 \tan \theta + 7$ ,
- the smallest positive value of  $\theta$ , in degrees, for which the minimum value occurs. [3]

- 5 (a)** The graph of  $y = 2^x$  can be transformed to the graph of  $y = 2^{x+4}$  **either** by a translation **or** by a stretch.

- (i) Give full details of the translation. [2]

- (ii) Give full details of the stretch. [2]

- (b) In this question you must show detailed reasoning.**

Solve the equation  $\log_2(8x) = 1 - \log_2(1-x)$ . [4]

- 6 (a) Find the first four terms in the expansion of  $(3 + 2x)^5$  in ascending powers of  $x$ . [4]
- (b) Hence determine the coefficient of  $y^3$  in the expansion of  $(3 + 2y + 4y^2)^5$ . [4]

- 7 A curve has equation  $2x^3 + 6xy - 3y^2 = 2$ .

Show that there are no points on this curve where the tangent is parallel to  $y = x$ . [8]

- 8 (a) Substance  $A$  is decaying exponentially such that its mass is  $m$  grams at time  $t$  minutes. Find the missing values of  $m$  and  $t$  in the following table.

|     |      |     |     |    |
|-----|------|-----|-----|----|
| $t$ | 0    | 10  |     | 50 |
| $m$ | 1250 | 750 | 450 |    |

[2]

- (b) Substance  $B$  is also decaying exponentially, according to the model  $m = 160e^{-0.055t}$ , where  $m$  grams is its mass after  $t$  minutes.

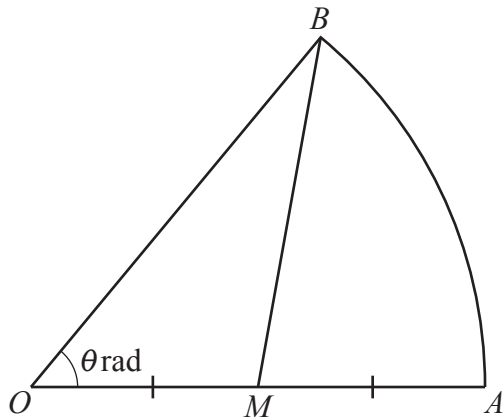
(i) Determine the value of  $t$  for which the mass of substance  $B$  is half of its original mass. [3]

(ii) Determine the rate of decay of substance  $B$  when  $t = 15$ . [3]

- (c) State whether substance  $A$  or substance  $B$  is decaying at a faster rate, giving a reason for your answer. [1]

- 9 Use the substitution  $x = 2 \sin \theta$  to show that  $\int_1^{\sqrt{3}} \sqrt{4 - x^2} \, dx = \frac{1}{3}\pi$ . [7]

10



The diagram shows a sector  $OAB$  of a circle with centre  $O$  and radius  $OA$ . The angle  $AOB$  is  $\theta$  radians.  $M$  is the mid-point of  $OA$ . The ratio of areas  $OMB : MAB$  is  $2:3$ .

- (a) Show that  $\theta = 1.25 \sin \theta$ . [4]

The equation  $\theta = 1.25 \sin \theta$  has only one root for  $\theta > 0$ .

- (b) This root can be found by using the iterative formula  $\theta_{n+1} = 1.25 \sin \theta_n$  with a starting value of  $\theta_1 = 0.5$ .
- Write down the values of  $\theta_2$ ,  $\theta_3$  and  $\theta_4$ .
  - Hence find the value of this root correct to **3** significant figures. [3]
- (c) The diagram in the Printed Answer Booklet shows the graph of  $y = 1.25 \sin \theta$ , for  $0 \leq \theta \leq \pi$ .
- Use this diagram to show how the iterative process used in (b) converges to this root.
  - State the type of convergence. [3]
- (d) Draw a suitable diagram to show why using an iterative process with the formula  $\theta_{n+1} = \sin^{-1}(0.8\theta_n)$  does not converge to the root found in (b). [2]

- 11 The gradient function of a curve is given by  $\frac{dy}{dx} = \frac{3x^2 \ln x}{e^{3y}}$ .

The curve passes through the point  $(e, 1)$ .

- (a) Find the equation of this curve, giving your answer in the form  $e^{3y} = f(x)$ . [6]
- (b) Show that, when  $x = e^2$ , the  $y$ -coordinate of this curve can be written as  $y = a + \frac{1}{3} \ln(b e^3 + c)$ , where  $a$ ,  $b$  and  $c$  are constants to be determined. [3]

12 A curve has parametric equations  $x = \frac{1}{t}$ ,  $y = 2t$ . The point  $P$  is  $\left(\frac{1}{p}, 2p\right)$ .

(a) Show that the equation of the tangent at  $P$  can be written as  $y = -2p^2x + 4p$ . [4]

The tangent to this curve at  $P$  crosses the  $x$ -axis at the point  $A$  and the normal to this curve at  $P$  crosses the  $x$ -axis at the point  $B$ .

(b) Show that the ratio  $PA:PB$  is  $1:2p^2$ . [8]

**END OF QUESTION PAPER**

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