OCR
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Tuesday 7 June 2022 – Afternoon

A Level Mathematics A

H240/01 Pure Mathematics

Time allowed: 2 hours

9085460550

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim\limits_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

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Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

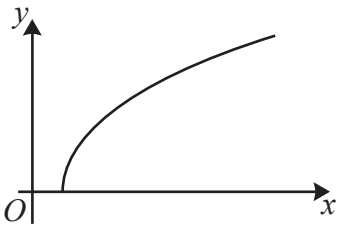
Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u+v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

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Answer **all** the questions.

1



The diagram shows part of the curve $y = \sqrt{x^2 - 1}$.

(a) Use the trapezium rule with 4 intervals to find an estimate for $\int_1^3 \sqrt{x^2 - 1}\,\mathrm{d}x$.

Give your answer correct to **3** significant figures. **[4]**

(b) State whether the value from part **(a)** is an under-estimate or an over-estimate, giving a reason for your answer. **[1]**

(c) Explain how the trapezium rule could be used to obtain a more accurate estimate. **[1]**

2 **(a)** Given that a and b are real numbers, find a counterexample to disprove the statement that, if $a > b$, then $a^2 > b^2$. **[1]**

(b) A student writes the statement that $\sin x° = 0.5 \iff x° = 30°$.

(i) Explain why this statement is incorrect. **[1]**

(ii) Write a corrected version of this statement. **[1]**

(c) Prove that the sum of four consecutive multiples of 4 is always a multiple of 8. **[3]**

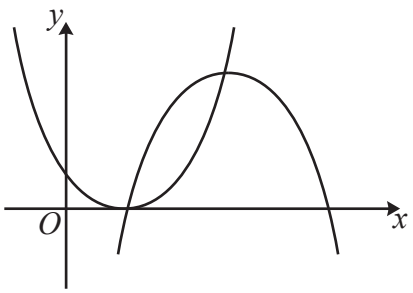
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3 (a) In this question you must show detailed reasoning.

Find the coordinates of the points of intersection of the curves with equations $y = x^2 - 2x + 1$ and $y = -x^2 + 6x - 5$. **[4]**

(b) The diagram shows the curves $y = x^2 - 2x + 1$ and $y = -x^2 + 6x - 5$.
This diagram is repeated in the Printed Answer Booklet.



On the diagram in the Printed Answer Booklet, draw the line $y = 2x - 2$. **[2]**

(c) Show on your diagram in the Printed Answer Booklet the region of the x-y plane within which all three of the following inequalities are satisfied.

$y \geqslant x^2 - 2x + 1$ $\quad$ $y \leqslant -x^2 + 6x - 5$ $\quad$ $y \leqslant 2x - 2$

You should indicate the region for which all the inequalities hold by labelling the region R. **[1]**

4 (a) Write $2x^2 + 6x + 7$ in the form $p(x+q)^2 + r$, where p, q and r are constants. **[3]**

(b) State the coordinates of the minimum point on the graph of $y = 2x^2 + 6x + 7$. **[2]**

(c) Hence deduce

- the minimum value of $2\tan^2\theta + 6\tan\theta + 7$,
- the smallest positive value of θ, in degrees, for which the minimum value occurs. **[3]**

5 (a) The graph of $y = 2^x$ can be transformed to the graph of $y = 2^{x+4}$ **either** by a translation **or** by a stretch.

(i) Give full details of the translation. **[2]**

(ii) Give full details of the stretch. **[2]**

(b) In this question you must show detailed reasoning.

Solve the equation $\log_2(8x) = 1 - \log_2(1-x)$. **[4]**

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6 **(a)** Find the first four terms in the expansion of $(3+2x)^5$ in ascending powers of x. **[4]**

(b) Hence determine the coefficient of y^3 in the expansion of $(3+2y+4y^2)^5$. **[4]**

7 A curve has equation $2x^3+6xy-3y^2=2$.

Show that there are no points on this curve where the tangent is parallel to $y=x$. **[8]**

8 **(a)** Substance A is decaying exponentially such that its mass is m grams at time t minutes. Find the missing values of m and t in the following table.

t	0	10		50
m	1250	750	450	

[2]

(b) Substance B is also decaying exponentially, according to the model $m=160\mathrm{e}^{-0.055t}$, where m grams is its mass after t minutes.

(i) Determine the value of t for which the mass of substance B is half of its original mass. **[3]**

(ii) Determine the rate of decay of substance B when $t=15$. **[3]**

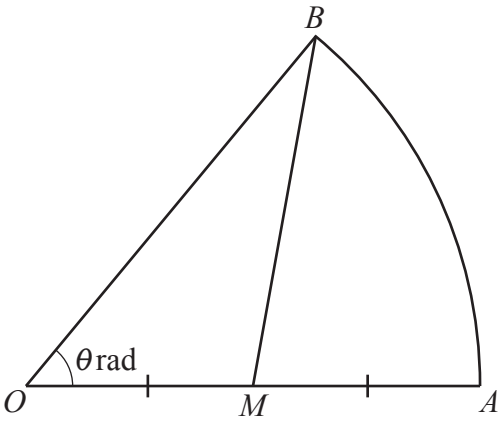
(c) State whether substance A or substance B is decaying at a faster rate, giving a reason for your answer. **[1]**

9 Use the substitution $x=2\sin\theta$ to show that $\int_1^{\sqrt{3}}\sqrt{4-x^2}\,\mathrm{d}x=\frac{1}{3}\pi$. **[7]**

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10



The diagram shows a sector OAB of a circle with centre O and radius OA. The angle AOB is θ radians. M is the mid-point of OA. The ratio of areas OMB : MAB is 2:3.

(a) Show that $\theta = 1.25\sin\theta$. **[4]**

The equation $\theta = 1.25\sin\theta$ has only one root for $\theta > 0$.

(b) This root can be found by using the iterative formula $\theta_{n+1} = 1.25\sin\theta_n$ with a starting value of $\theta_1 = 0.5$.

- Write down the values of θ_2, θ_3 and θ_4.
- Hence find the value of this root correct to **3** significant figures. **[3]**

(c) The diagram in the Printed Answer Booklet shows the graph of $y = 1.25\sin\theta$, for $0 \leqslant \theta \leqslant \pi$.

- Use this diagram to show how the iterative process used in **(b)** converges to this root.
- State the type of convergence. **[3]**

(d) Draw a suitable diagram to show why using an iterative process with the formula $\theta_{n+1} = \sin^{-1}(0.8\theta_n)$ does not converge to the root found in **(b)**. **[2]**

11 The gradient function of a curve is given by $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3x^2\ln x}{\mathrm{e}^{3y}}$.

The curve passes through the point (e, 1).

(a) Find the equation of this curve, giving your answer in the form $\mathrm{e}^{3y} = \mathrm{f}(x)$. **[6]**

(b) Show that, when $x = \mathrm{e}^2$, the y-coordinate of this curve can be written as $y = a + \frac{1}{3}\ln(b\mathrm{e}^3 + c)$, where a, b and c are constants to be determined. **[3]**

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12 A curve has parametric equations $x = \frac{1}{t}$, $y = 2t$. The point P is $\left(\frac{1}{p}, 2p\right)$.

(a) Show that the equation of the tangent at P can be written as $y = -2p^2x + 4p$. **[4]**

The tangent to this curve at P crosses the x-axis at the point A and the normal to this curve at P crosses the x-axis at the point B.

(b) Show that the ratio $PA : PB$ is $1 : 2p^2$. **[8]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a)

$h = (3 - 1) \div 4 = 0.5$

x:	1	1.5	2	2.5	3
y:	0	$\sqrt{1.25}$	$\sqrt{3}$	$\sqrt{5.25}$	$\sqrt{8}$
	$= 0$	$= 1.1180$	$= 1.7321$	$= 2.2913$	$= 2.8284$

$$\int \approx \frac{1}{2} \times 0.5 \times \{0 + 2.8284 + 2(1.1180 + 1.7321 + 2.2913)\}$$

$= 0.25 \times \{2.8284 + 10.2829\} = 0.25 \times 13.1113$

$= 3.2777... = 3.28$ (3 s.f.)

1 (b)

Under-estimate, because the curve is concave (its gradient decreases), so the tops of the trapezia lie below the curve.

1 (c)

Use more trapezia (more intervals), each of a smaller width, between the same limits $x = 1$ and $x = 3$.

2 (a)

Take $a = 1$, $b = -2$: $1 > -2$, but $a^2 = 1$ and $b^2 = (-2)^2 = 4$, and $1 < 4$, so $a^2 > b^2$ is false. Counterexample found.

2 (b)(i)

$\sin 150° = 0.5$ as well, so $\sin x° = 0.5$ does not imply $x° = 30°$ (x could be 150°, 390°, ...), so the $\Leftrightarrow$ is wrong.

2 (b)(ii)

$\sin x° = 0.5 \Leftarrow x° = 30°$

Question number

2 (c) Let the four consecutive multiples of 4 be

$4n$, $4n + 4$, $4n + 8$, $4n + 12$, where n is an integer.

Sum $= 4n + (4n + 4) + (4n + 8) + (4n + 12) = 16n + 24$

$= 8(2n + 3)$

$2n + 3$ is an integer, so $8(2n + 3)$ is a multiple of 8.

$\therefore$ the sum of four consecutive multiples of 4 is always a multiple of 8.

3 (a) At intersection: $x^2 - 2x + 1 = -x^2 + 6x - 5$

$2x^2 - 8x + 6 = 0$

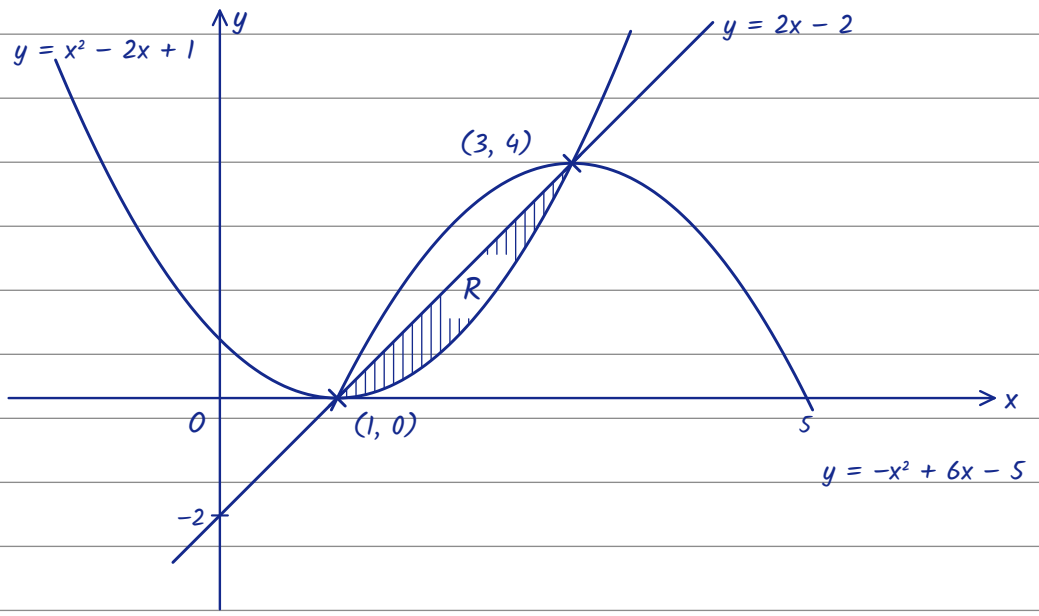
$x^2 - 4x + 3 = 0$

$(x - 1)(x - 3) = 0 \quad \Rightarrow \quad x = 1$ or $x = 3$

$x = 1$: $y = 1 - 2 + 1 = 0$; $\quad x = 3$: $y = 9 - 6 + 1 = 4$

Points of intersection: $(1, 0)$ and $(3, 4)$

3 (b)



3 (c) Region R is shaded and labelled R on the diagram in 3 (b).

Question number	
4 (a)	$2x^2 + 6x + 7 = 2(x^2 + 3x) + 7$
	$= 2[(x + 1.5)^2 - 2.25] + 7 = 2(x + 1.5)^2 - 4.5 + 7$
	$= 2(x + 1.5)^2 + 2.5$ $\quad (p = 2, q = 1.5, r = 2.5)$
4 (b)	Minimum point $(-1.5, 2.5)$
4 (c)	Put $x = \tan\theta$: $2\tan^2\theta + 6\tan\theta + 7 = 2(\tan\theta + 1.5)^2 + 2.5$
	Minimum value of $2\tan^2\theta + 6\tan\theta + 7$ is 2.5
	It occurs when $\tan\theta = -1.5 \Rightarrow \theta = \tan^{-1}(-1.5) = -56.309...°$
	Smallest positive $\theta = -56.309...° + 180° = 123.69...° = 124°$
5 (a)(i)	Translation by vector $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$, i.e. 4 units in the negative x-direction
5 (a)(ii)	$2^{x+4} = 2^4 \times 2^x = 16 \times 2^x$
	Stretch parallel to the y-axis (in the y-direction), scale factor 16
5 (b)	$\log_2(8x) + \log_2(1 - x) = 1$
	$\log_2(8x(1 - x)) = 1$
	$8x(1 - x) = 2^1 = 2$
	$8x - 8x^2 = 2 \Rightarrow 8x^2 - 8x + 2 = 0 \Rightarrow 4x^2 - 4x + 1 = 0$
	$(2x - 1)^2 = 0 \Rightarrow x = 0.5$
	(check: $8x = 4 > 0$ and $1 - x = 0.5 > 0$, so valid) $\quad x = 0.5$
6 (a)	$(3 + 2x)^5 = 3^5 + 5 \times 3^4(2x) + 10 \times 3^3(2x)^2 + 10 \times 3^2(2x)^3 + ...$
	$= 243 + 5 \times 81 \times 2x + 10 \times 27 \times 4x^2 + 10 \times 9 \times 8x^3 + ...$
	$= 243 + 810x + 1080x^2 + 720x^3 + ...$

Question number

6 (b)

$3 + 2y + 4y^2 = 3 + 2(y + 2y^2)$, so put $x = y + 2y^2$ in (a):

$(3 + 2y + 4y^2)^5 = 243 + 810(y + 2y^2) + 1080(y + 2y^2)^2 + 720(y + 2y^2)^3 + \ldots$

y^3 terms: $1080(y + 2y^2)^2 = 1080(y^2 + 4y^3 + 4y^4)$ gives $4320y^3$

$720(y + 2y^2)^3 = 720(y^3 + \ldots)$ gives $720y^3$

(higher terms of the expansion only give powers of $y \geq 4$)

Coefficient of $y^3 = 4320 + 720 = 5040$

7

Differentiate $2x^3 + 6xy - 3y^2 = 2$ implicitly with respect to x:

$$6x^2 + 6y + 6x\frac{dy}{dx} - 6y\frac{dy}{dx} = 0$$

Tangent parallel to $y = x \Rightarrow \frac{dy}{dx} = 1$:

$6x^2 + 6y + 6x - 6y = 0 \Rightarrow 6x^2 + 6x = 0 \Rightarrow x(x + 1) = 0$

$x = 0$ or $x = -1$

$x = 0$: $0 + 0 - 3y^2 = 2 \Rightarrow y^2 = -\frac{2}{3}$, impossible as $y^2 \geq 0$ for real y,

so no points with $x = 0$.

$x = -1$: $-2 - 6y - 3y^2 = 2 \Rightarrow 3y^2 + 6y + 4 = 0$

discriminant $b^2 - 4ac = 6^2 - 4 \times 3 \times 4 = 36 - 48 = -12$

$-12 < 0$, so there are no real values of y.

$\therefore$ there are no points on the curve where the tangent is parallel to $y = x$.

8 (a)

Exponential decay: equal time steps give equal ratios.

$750 \div 1250 = 0.6$ per 10 minutes; $450 \div 750 = 0.6$, so 10 minutes later:

$t = 20$ (minutes)

At $t = 50$: $m = 1250 \times 0.6^5 = 97.2$ (grams)

Question number

8 (b)(i) Original mass = 160 g, so half = 80 g

$160e^{-0.055t} = 80 \Rightarrow e^{-0.055t} = 0.5$

$-0.055t = \ln 0.5$

$t = \dfrac{\ln 0.5}{-0.055} = 12.6026\ldots = 12.6$ minutes

8 (b)(ii) $\dfrac{dm}{dt} = 160 \times (-0.055)e^{-0.055t} = -8.8e^{-0.055t}$

$t = 15$: $\dfrac{dm}{dt} = -8.8e^{-0.055 \times 15} = -3.8564\ldots$

Rate of decay = 3.86 grams per minute

8 (c) A: $m = 1250e^{-kt}$, $e^{-10k} = 0.6 \Rightarrow k = \dfrac{\ln(5/3)}{10} = 0.05108\ldots$

$\dfrac{dm}{dt} = -63.85e^{-0.0511t}$; at $t = 15$ rate of decay = 29.7 g per minute

This is greater than 3.86 g per minute for B, so substance A is decaying at a faster rate.

Question number

9

$x = 2\sin\theta \Rightarrow \frac{dx}{d\theta} = 2\cos\theta, \quad dx = 2\cos\theta\, d\theta$

$x = 1$: $\sin\theta = \frac{1}{2}$, $\theta = \frac{\pi}{6}$; $\quad x = \sqrt{3}$: $\sin\theta = \frac{\sqrt{3}}{2}$, $\theta = \frac{\pi}{3}$

$$\int_1^{\sqrt{3}} \sqrt{(4 - x^2)}\, dx = \int_{\pi/6}^{\pi/3} \sqrt{(4 - 4\sin^2\theta)} \times 2\cos\theta\, d\theta$$

$$= \int_{\pi/6}^{\pi/3} \sqrt{(4\cos^2\theta)} \times 2\cos\theta\, d\theta = \int_{\pi/6}^{\pi/3} 4\cos^2\theta\, d\theta \quad (\cos\theta > 0)$$

$$= \int_{\pi/6}^{\pi/3} (2\cos 2\theta + 2)\, d\theta \qquad \text{using } \cos 2\theta = 2\cos^2\theta - 1$$

$$= [\sin 2\theta + 2\theta]_{\pi/6}^{\pi/3}$$

$$= \left(\sin\frac{2\pi}{3} + \frac{2\pi}{3}\right) - \left(\sin\frac{\pi}{3} + \frac{\pi}{3}\right)$$

$$= \left(\frac{\sqrt{3}}{2} + \frac{2\pi}{3}\right) - \left(\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right) = \frac{1}{3}\pi \quad \text{as required}$$

10 (a) Let the radius be r, so OM = ½r.

Area OMB $= \frac{1}{2} \times \frac{r}{2} \times r \times \sin\theta = \frac{1}{4} r^2 \sin\theta$

Area MAB = sector OAB − OMB $= \frac{1}{2} r^2\theta - \frac{1}{4} r^2 \sin\theta$

OMB : MAB = 2 : 3 $\Rightarrow \quad 3\left(\frac{1}{4} r^2 \sin\theta\right) = 2\left(\frac{1}{2} r^2\theta - \frac{1}{4} r^2 \sin\theta\right)$

$\frac{3}{4}\sin\theta = \theta - \frac{1}{2}\sin\theta \Rightarrow \quad \theta = \frac{5}{4}\sin\theta$

$\theta = 1.25\sin\theta$ as required

Question number

10 (b)

$\theta_2 = 1.25 \sin 0.5 = 0.59928\ldots = 0.599$

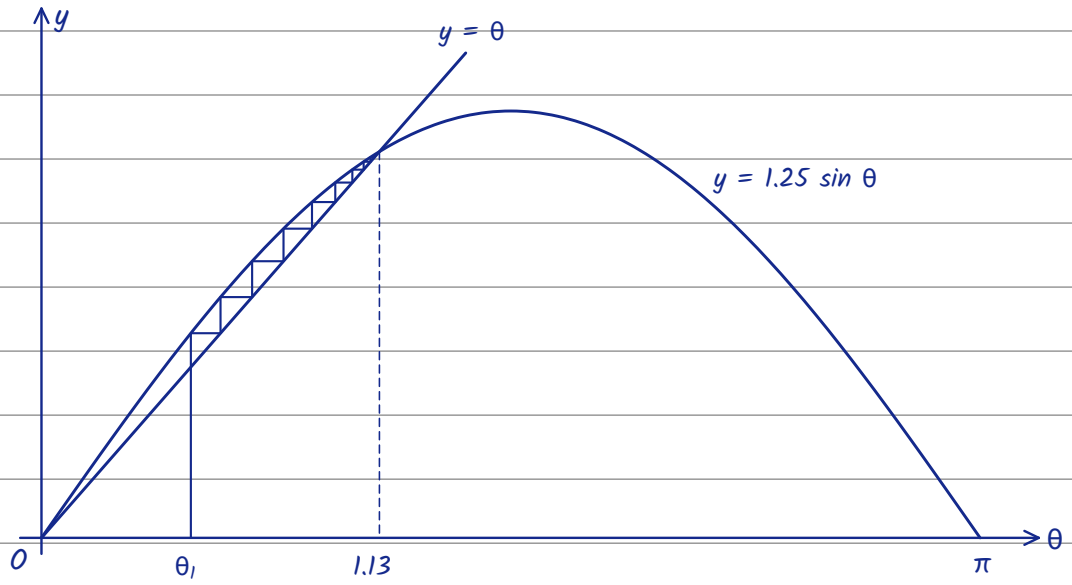
$\theta_3 = 1.25 \sin 0.59928\ldots = 0.70506\ldots = 0.705$

$\theta_4 = 1.25 \sin 0.70506\ldots = 0.81010\ldots = 0.810$

Continuing: 0.905, 0.983, ... → 1.1311...

Root $\theta = 1.13$ (3 s.f.)

10 (c)

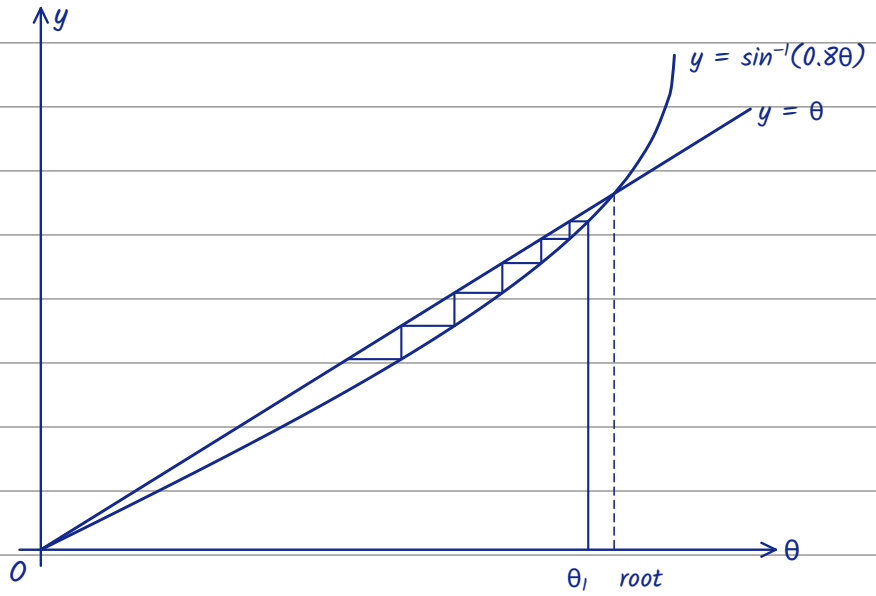


The iterates step up the staircase towards the root $\theta \approx 1.13$:

this is staircase convergence.

Question number

10 (d)



At the root the gradient of $y = \sin^{-1}(0.8\theta)$ is greater than 1, so the staircase moves away from the root: the iteration diverges.

Question number

11 (a) Separate the variables:

$\int e^{3y}\,dy = \int 3x^2 \ln x\,dx$

LHS: $\int e^{3y}\,dy = \frac{1}{3}e^{3y}$

RHS by parts: $u = \ln x$, $\frac{dv}{dx} = 3x^2 \Rightarrow \frac{du}{dx} = \frac{1}{x}$, $v = x^3$

$\int 3x^2 \ln x\,dx = x^3 \ln x - \int x^3 \times \frac{1}{x}\,dx = x^3 \ln x - \int x^2\,dx$

$= x^3 \ln x - \frac{1}{3}x^3 + c$

So $\frac{1}{3}e^{3y} = x^3 \ln x - \frac{1}{3}x^3 + c$

At $(e, 1)$: $\frac{1}{3}e^3 = e^3 \ln e - \frac{1}{3}e^3 + c \quad \Rightarrow \quad c = -\frac{1}{3}e^3$

$\frac{1}{3}e^{3y} = x^3 \ln x - \frac{1}{3}x^3 - \frac{1}{3}e^3$

$e^{3y} = 3x^3 \ln x - x^3 - e^3$

11 (b) $x = e^2$: $e^{3y} = 3e^6 \ln(e^2) - e^6 - e^3 = 6e^6 - e^6 - e^3$

$e^{3y} = 5e^6 - e^3 = e^3(5e^3 - 1)$

$3y = \ln(e^3(5e^3 - 1)) = \ln e^3 + \ln(5e^3 - 1) = 3 + \ln(5e^3 - 1)$

$y = 1 + \frac{1}{3}\ln(5e^3 - 1)$, so $a = 1$, $b = 5$, $c = -1$

Question number

12 (a)

$\frac{dx}{dt} = -\frac{1}{t^2}, \quad \frac{dy}{dt} = 2$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2}{-1/t^2} = -2t^2$

At P, $t = p$, so the gradient of the tangent is $-2p^2$

$y - 2p = -2p^2(x - \frac{1}{p})$

$y - 2p = -2p^2x + 2p$

$y = -2p^2x + 4p$ as required

12 (b)

Gradient of normal at P $= \frac{1}{2p^2}$

Normal: $y - 2p = \frac{1}{2p^2}(x - \frac{1}{p})$

B ($y = 0$): $-2p = \frac{1}{2p^2}(x - \frac{1}{p}) \Rightarrow x - \frac{1}{p} = -4p^3 \Rightarrow x = \frac{1}{p} - 4p^3$

A ($y = 0$ in tangent): $2p^2x = 4p \Rightarrow x = \frac{2}{p}$

$PA^2 = (\frac{2}{p} - \frac{1}{p})^2 + (2p)^2 = \frac{1}{p^2} + 4p^2 = \frac{1 + 4p^4}{p^2}$

$PB^2 = (\frac{1}{p} - 4p^3 - \frac{1}{p})^2 + (2p)^2 = 16p^6 + 4p^2 = 4p^2(4p^4 + 1)$

$PA = \frac{1}{p}\sqrt{(4p^4 + 1)}, \quad PB = 2p\sqrt{(4p^4 + 1)}$

$PA : PB = \frac{1}{p}\sqrt{(4p^4 + 1)} : 2p\sqrt{(4p^4 + 1)} = \frac{1}{p} : 2p$

Multiply by p: $PA : PB = 1 : 2p^2$ as required