OCR
Oxford Cambridge and RSA

Wednesday 12 June 2019 – Morning

A Level Mathematics A

H240/02 Pure Mathematics and Statistics

Time allowed: 2 hours

7663502571

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

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Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient rule $y = \frac{u}{v}, \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

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Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\overline{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \overline{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\overline{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \overline{x}^2}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0,1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u+v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

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Section A: Pure Mathematics

Answer **all** the questions.

1 **(a)** Differentiate the following.

(i) $\dfrac{x^2}{2x+1}$ **[3]**

(ii) $\tan(x^2-3x)$ **[2]**

(b) Use the substitution $u=\sqrt{x}-1$ to integrate $\dfrac{1}{\sqrt{x}-1}$. **[4]**

(c) Integrate $\dfrac{x-2}{2x^2-8x-1}$. **[2]**

2 **(a)** Find the coefficient of x^5 in the expansion of $(3-2x)^8$. **[1]**

(b) **(i)** Expand $(1+3x)^{0.5}$ as far as the term in x^3. **[3]**

(ii) State the range of values of x for which your expansion is valid. **[1]**

A student suggests the following check to determine whether the expansion obtained in part **(b)(i)** may be correct.

"Use the expansion to find an estimate for $\sqrt{103}$, correct to five decimal places, and compare this with the value of $\sqrt{103}$ given by your calculator."

(iii) Showing your working, carry out this check on your expansion from part **(b)(i)**. **[3]**

3 **(a)** A circle is defined by the parametric equations $x=3+2\cos\theta$, $y=-4+2\sin\theta$.

(i) Find a cartesian equation of the circle. **[2]**

(ii) Write down the centre and radius of the circle. **[1]**

(b) **In this question you must show detailed reasoning.**

The curve S is defined by the parametric equations $x=4\cos t$, $y=2\sin t$. The line L is a tangent to S at the point given by $t=\frac{1}{6}\pi$.

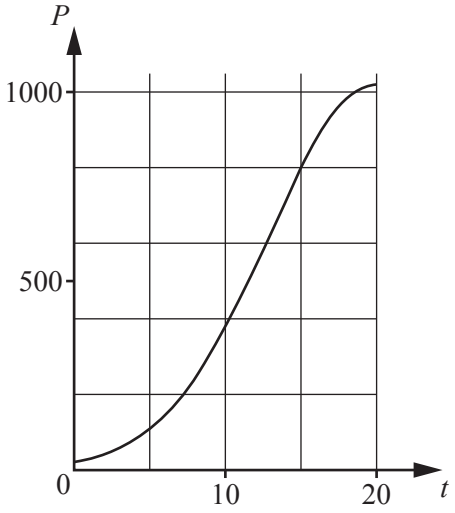
Find where the line L cuts the x-axis. **[6]**

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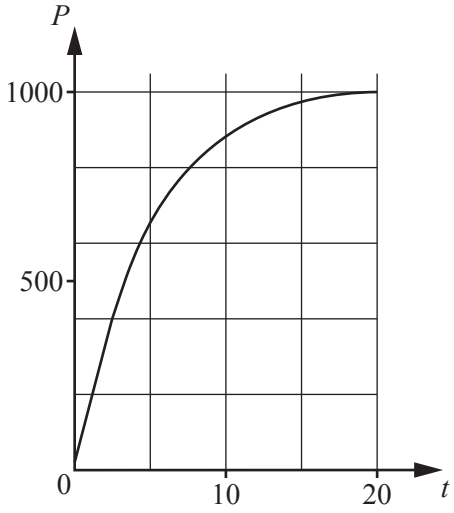
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4 A species of animal is to be introduced onto a remote island. Their food will consist only of various plants that grow on the island. A zoologist proposes two possible models for estimating the population P after t years. The diagrams show these models as they apply to the first 20 years.

Model A



Model B



(a) Without calculation, describe briefly how the rate of growth of P will vary for the first 20 years, according to each of these two models. **[1]**

The equation of the curve for model A is $P = 20 + 1000\mathrm{e}^{-\frac{(t-20)^2}{100}}$.

The equation of the curve for model B is $P = 20 + 1000\left(1 - \mathrm{e}^{-\frac{t}{5}}\right)$.

(b) Describe the behaviour of P that is predicted for $t > 20$

(i) using model A, **[1]**

(ii) using model B. **[1]**

There is only a limited amount of food available on the island, and the zoologist assumes that the size of the population depends on the amount of food available and on no other external factors.

(c) State what is suggested about the long-term food supply by

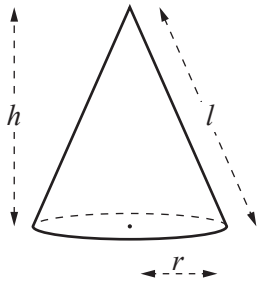
(i) model A, **[1]**

(ii) model B. **[1]**

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5



For a cone with base radius r, height h and slant height l, the following formulae are given.

Curved surface area, $S = \pi rl$

Volume, $V = \frac{1}{3}\pi r^2 h$

A container is to be designed in the shape of an inverted cone with no lid. The base radius is r m and the volume is V m^3.

The area of the material to be used for the cone is 4π m^2.

(a) Show that $V = \frac{1}{3}\pi\sqrt{16r^2 - r^6}$. **[4]**

(b) In this question you must show detailed reasoning.

It is given that V has a maximum value for a certain value of r.

Find the maximum value of V, giving your answer correct to 3 significant figures. **[5]**

6 Shona makes the following claim.

"n is an **even** positive integer greater than $2 \Rightarrow 2^n - 1$ is **not** prime"

Prove that Shona's claim is true. **[4]**

7 **In this question you must show detailed reasoning.**

Use the substitution $u = 6x^2 + x$ to solve the equation $36x^4 + 12x^3 + 7x^2 + x - 2 = 0$. **[5]**

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Section B: Statistics

Answer **all** the questions.

8 The stem-and-leaf diagram shows the heights, in centimetres, of 17 plants, measured correct to the nearest centimetre.

5	5 7 9 9
6	3 4 5 5 5 9 9
7	4 5 7 9 9
8	
9	9

Key: 5 | 6 means 56

(a) Find the median and inter-quartile range of these heights. **[3]**

(b) Calculate the mean and standard deviation of these heights. **[2]**

(c) State one advantage of using the median rather than the mean as a measure of average for these heights. **[1]**

9 **(a)** The masses, in grams, of plums of a certain kind have the distribution N(55, 18).

(i) Find the probability that a plum chosen at random has a mass between 50.0 and 60.0 grams. **[1]**

(ii) The heaviest 5% of plums are classified as extra large.

Find the minimum mass of extra large plums. **[1]**

(iii) The plums are packed in bags, each containing 10 randomly selected plums.

Find the probability that a bag chosen at random has a total mass of less than 530 g. **[4]**

(b) The masses, in grams, of apples of a certain kind have the distribution N(67, σ^2). It is given that half of the apples have masses between 62 g and 72 g.

Determine σ. **[5]**

10 The level, in grams per millilitre, of a pollutant at different locations in a certain river is denoted by the random variable X, where X has the distribution N(μ, 0.000 040 9).

In the past the value of μ has been 0.0340.

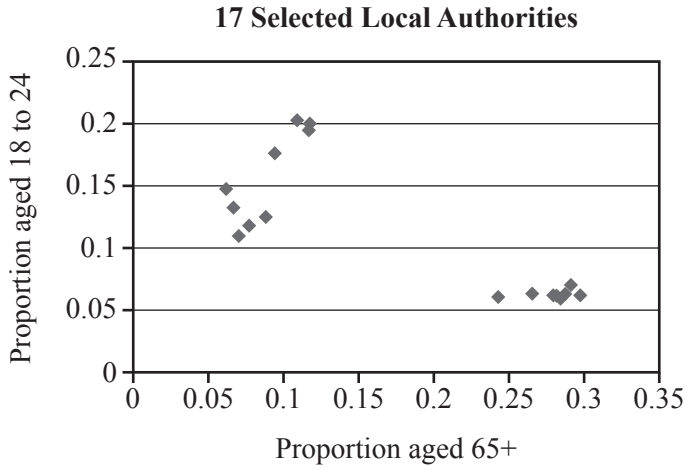
This year the mean level of the pollutant at 50 randomly chosen locations was found to be 0.0325 grams per millilitre.

Test, at the 5% significance level, whether the mean level of pollutant has changed. **[7]**

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11 A trainer was asked to give a lecture on population profiles in different Local Authorities (LAs) in the UK. Using data from the 2011 census, he created the following scatter diagram for 17 selected LAs.



He selected the 17 LAs using the following method. The proportions of people aged 18 to 24 and aged 65+ in any Local Authority are denoted by P_{young} and P_{senior} respectively. The trainer used a spreadsheet to calculate the value of $k = \dfrac{P_{\text{young}}}{P_{\text{senior}}}$ for each of the 348 LAs in the UK. He then used specific ranges of values of k to select the 17 LAs.

(a) Estimate the ranges of values of k that he used to select these 17 LAs. **[2]**

(b) Using the 17 LAs the trainer carried out a hypothesis test with the following hypotheses.

H_0: There is no linear correlation in the population between P_{young} and P_{senior}.
H_1: There is negative linear correlation in the population between P_{young} and P_{senior}.

He found that the value of Pearson's product-moment correlation coefficient for the 17 LAs is -0.797, correct to 3 significant figures.

(i) Use the table on page 9 to show that this value is significant at the 1% level. **[2]**

The trainer concluded that there is evidence of negative linear correlation between P_{young} and P_{senior} in the population.

(ii) Use the diagram to comment on the reliability of this conclusion. **[2]**

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(c) Describe one outstanding feature of the population in the areas represented by the points in the bottom right hand corner of the diagram. **[1]**

(d) The trainer's audience included representatives from several universities.

Suggest a reason why the diagram might be of particular interest to these people. **[1]**

Critical values of Pearson's product-moment correlation coefficient

1-tail test	5%	2.5%	1%	0.5%
2-tail test	10%	5%	2%	1%
n				
1	-	-	-	-
2	-	-	-	-
3	0.9877	0.9969	0.9995	0.9999
4	0.9000	0.9500	0.9800	0.9900
5	0.8054	0.8783	0.9343	0.9587
6	0.7293	0.8114	0.8822	0.9172
7	0.6694	0.7545	0.8329	0.8745
8	0.6215	0.7067	0.7887	0.8343
9	0.5822	0.6664	0.7498	0.7977
10	0.5494	0.6319	0.7155	0.7646
11	0.5214	0.6021	0.6851	0.7348
12	0.4973	0.5760	0.6581	0.7079
13	0.4762	0.5529	0.6339	0.6835
14	0.4575	0.5324	0.6120	0.6614
15	0.4409	0.5140	0.5923	0.6411
16	0.4259	0.4973	0.5742	0.6226
17	0.4124	0.4821	0.5577	0.6055
18	0.4000	0.4683	0.5425	0.5897
19	0.3887	0.4555	0.5285	0.5751
20	0.3783	0.4438	0.5155	0.5614
21	0.3687	0.4329	0.5034	0.5487
22	0.3598	0.4227	0.4921	0.5368
23	0.3515	0.4132	0.4815	0.5256
24	0.3438	0.4044	0.4716	0.5151
25	0.3365	0.3961	0.4622	0.5052
26	0.3297	0.3882	0.4534	0.4958
27	0.3233	0.3809	0.4451	0.4869
28	0.3172	0.3739	0.4372	0.4785
29	0.3115	0.3673	0.4297	0.4705
30	0.3061	0.3610	0.4226	0.4629

Turn over for questions 12 and 13

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12 A random variable X has probability distribution defined as follows.

$$\mathrm{P}(X = x) = \begin{cases} kx & x = 1, 2, 3, 4, 5, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Show that $\mathrm{P}(X = 3) = 0.2$. **[3]**

(b) Show in a table the values of X and their probabilities. **[2]**

(c) Two independent values of X are chosen, and their total T is found.

(i) Find $\mathrm{P}(T = 7)$. **[3]**

(ii) Given that $T = 7$, determine the probability that one of the values of X is 2. **[4]**

13 It is known that 26% of adults in the UK use a certain app. A researcher selects a random sample of 5000 adults in the UK. The random variable X is defined as the number of adults in the sample who use the app.

Given that $\mathrm{P}(X < n) < 0.025$, calculate the largest possible value of n. **[5]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a)(i)

$y = \frac{x^2}{2x+1}$ $\quad u = x^2,\ u' = 2x;\quad v = 2x+1,\ v' = 2$

$\frac{dy}{dx} = \frac{(2x+1)\times 2x - x^2 \times 2}{(2x+1)^2} = \frac{2x^2+2x}{(2x+1)^2} = \frac{2x(x+1)}{(2x+1)^2}$

1 (a)(ii)

$y = \tan(x^2 - 3x) \quad \Rightarrow \quad \frac{dy}{dx} = (2x-3)\sec^2(x^2-3x)$

1 (b)

$u = \sqrt{x} - 1 \quad \Rightarrow \quad x = (u+1)^2, \quad \frac{dx}{du} = 2(u+1) \quad$ so $\quad dx = 2(u+1)\,du$

$\int \frac{1}{\sqrt{x}-1}\,dx = \int \frac{1}{u} \times 2(u+1)\,du = 2\int \left(1 + \frac{1}{u}\right) du$

$= 2(u + \ln|u|) + c$

$= 2(\sqrt{x} - 1 + \ln|\sqrt{x} - 1|) + c$

1 (c)

$d/dx(2x^2 - 8x - 1) = 4x - 8 = 4(x-2)$

$\int \frac{x-2}{2x^2-8x-1}\,dx = \frac{1}{4}\int \frac{4x-8}{2x^2-8x-1}\,dx = \frac{1}{4}\ln|2x^2 - 8x - 1| + c$

2 (a)

Term in x^5: ${}^8C_5 \times 3^3 \times (-2x)^5 = 56 \times 27 \times (-32)x^5$

Coefficient of $x^5 = -48384$

2 (b)(i)

$(1+3x)^{0.5} = 1 + 0.5(3x) + \frac{0.5 \times (-0.5)}{2!}(3x)^2 + \frac{0.5 \times (-0.5) \times (-1.5)}{3!}(3x)^3 + \ldots$

$= 1 + \frac{3}{2}x - \frac{9}{8}x^2 + \frac{27}{16}x^3 + \ldots$

2 (b)(ii)

Valid for $|3x| < 1$, i.e. $-\frac{1}{3} < x < \frac{1}{3}$

Question number	
2 (b)(iii)	$\sqrt{103} = \sqrt{(100 \times 1.03)} = 10\sqrt{1.03} = 10(1 + 3x)^{0.5}$ with $x = 0.01$
	$(1.03)^{0.5} \approx 1 + 0.015 - 0.0001125 + 0.0000016875 = 1.0148891875$
	From series: $\sqrt{103} \approx 10 \times 1.0148891875 = 10.148891875 = 10.14889$ (5 d.p.)
	From calculator: $\sqrt{103} = 10.14889157\ldots = 10.14889$ (5 d.p.)
	The values agree to 5 d.p., so the expansion may be correct.
3 (a)(i)	$x - 3 = 2\cos\theta, \quad y + 4 = 2\sin\theta$
	$(x - 3)^2 + (y + 4)^2 = 4\cos^2\theta + 4\sin^2\theta = 4(\cos^2\theta + \sin^2\theta)$
	$(x - 3)^2 + (y + 4)^2 = 4$
3 (a)(ii)	Centre $(3, -4)$, radius 2
3 (b)	$\frac{dx}{dt} = -4\sin t, \quad \frac{dy}{dt} = 2\cos t$
	$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{2\cos t}{-4\sin t} = -\frac{1}{2}\cot t$
	At $t = \frac{\pi}{6}$: $\frac{dy}{dx} = -\frac{1}{2}\cot\frac{\pi}{6} = -\frac{1}{2} \times \sqrt{3} = -\frac{\sqrt{3}}{2}$
	$x = 4\cos\frac{\pi}{6} = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}, \quad y = 2\sin\frac{\pi}{6} = 2 \times \frac{1}{2} = 1$
	Equation of L: $y - 1 = -\frac{\sqrt{3}}{2}(x - 2\sqrt{3}) \quad \Rightarrow \quad y = -\frac{\sqrt{3}}{2}x + 4$
	Cuts x-axis when $y = 0$: $0 = -\frac{\sqrt{3}}{2}x + 4 \quad \Rightarrow \quad x = \frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{3}$
	L cuts the x-axis at $\left(\frac{8\sqrt{3}}{3}, 0\right)$ $(= (4.62, 0)$ to 3 s.f.)
4 (a)	Model A: the rate of growth increases at first, then decreases
	(P grows slowly, then quickly, then slowly again).
	Model B: the rate of growth decreases throughout
	(P grows quickly at first, then more and more slowly).
4 (b)(i)	Model A: P decreases and tends to 20 (P never goes below 20).
4 (b)(ii)	Model B: P keeps increasing slowly and tends to 1020 (never exceeds 1020).

Question number	
4 (c)(i)	Model A suggests the food will (almost) run out, so in the long term it will only support a population of about 20.
4 (c)(ii)	Model B suggests the food supply will stay constant: enough to sustain a population of about 1020 but not enough for the population to grow beyond it.
5 (a)	Area: $\pi r l = 4\pi \Rightarrow l = \frac{4}{r}$ $h^2 = l^2 - r^2 = \frac{16}{r^2} - r^2 \Rightarrow h = \sqrt{\frac{16}{r^2} - r^2}$ $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 \sqrt{\frac{16}{r^2} - r^2} = \frac{1}{3}\pi\sqrt{r^4\left(\frac{16}{r^2} - r^2\right)}$ $V = \frac{1}{3}\pi\sqrt{16r^2 - r^6}$ as required
5 (b)	$V = \frac{\pi}{3}(16r^2 - r^6)^{1/2}$ $\frac{dV}{dr} = \frac{\pi}{3} \times \frac{1}{2}(16r^2 - r^6)^{-1/2} \times (32r - 6r^5) = \frac{\pi(32r - 6r^5)}{6\sqrt{(16r^2 - r^6)}}$ $\frac{dV}{dr} = 0 \Rightarrow 32r - 6r^5 = 0 \Rightarrow 2r(16 - 3r^4) = 0$ $r = 0$ or $r^4 = \frac{16}{3} \Rightarrow r = \frac{2}{\sqrt[4]{3}} = 1.51967\ldots$ (or $r = -1.51967\ldots$) $r = 0$ gives no cone and r must be positive, so $r = 0$ and $r = -1.52$ are invalid. $V_{max} = \frac{\pi}{3}\sqrt{16 \times 1.51967\ldots^2 - 1.51967\ldots^6} = 5.19747\ldots$ Maximum value of $V = 5.20\ m^3$ (3 s.f.)
6	n is even and $n > 2$, so $n = 2k$ where k is an integer with $k \geq 2$. $2^n - 1 = 2^{2k} - 1 = (2^k)^2 - 1 = (2^k - 1)(2^k + 1)$ Since $k \geq 2$: $2^k - 1 \geq 2^2 - 1 = 3 > 1$ and $2^k + 1 \geq 5 > 1$. So $2^n - 1$ is the product of two integers, each greater than 1, hence $2^n - 1$ is not prime. ∴ Shona's claim is true.

Question number	
7	$u = 6x^2 + x \Rightarrow u^2 = (6x^2 + x)^2 = 36x^4 + 12x^3 + x^2$
	So $36x^4 + 12x^3 + 7x^2 + x - 2 = (36x^4 + 12x^3 + x^2) + (6x^2 + x) - 2$
	$= u^2 + u - 2$
	$u^2 + u - 2 = 0 \Rightarrow (u + 2)(u - 1) = 0 \Rightarrow u = -2$ or $u = 1$
	$u = -2$: $6x^2 + x + 2 = 0$, discriminant $= 1^2 - 4 \times 6 \times 2 = 1 - 48 = -47 < 0$,
	so there are no real roots.
	$u = 1$: $6x^2 + x - 1 = 0 \Rightarrow (3x - 1)(2x + 1) = 0$
	$x = \frac{1}{3}$ or $x = -\frac{1}{2}$
8 (a)	$n = 17$, so the median is the 9th value: median = 65 cm
	Lower quartile = mean of 4th and 5th values $= (59 + 63) \div 2 = 61$
	Upper quartile = mean of 13th and 14th values $= (75 + 77) \div 2 = 76$
	IQR $= 76 - 61 = 15$ cm
8 (b)	$\Sigma x = 1173$, $\Sigma x^2 = 82811$
	Mean $= \frac{1173}{17} = 69$ cm
	sd $= \sqrt{\frac{82811}{17} - 69^2} = \sqrt{110.235\ldots} = 10.499\ldots = 10.5$ cm (3 s.f.)
8 (c)	The median is not affected by the outlier 99, whereas the mean is
	pulled up by the outlier 99.
9 (a)(i)	$X \sim N(55, 18)$: $P(50.0 < X < 60.0) = 0.76140\ldots = 0.761$ (3 s.f.)
9 (a)(ii)	$P(X > m) = 0.05 \Rightarrow P(X < m) = 0.95$
	$m = 55 + 1.6449 \times \sqrt{18} = 61.9785\ldots$ Minimum mass = 62.0 g (3 s.f.)

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9 (a)(iii)	Total mass $T = X_1 + X_2 + \ldots + X_{10}$
	$T \sim N(10 \times 55, 10 \times 18) = N(550, 180)$
	$P(T < 530) = P(Z < (530 - 550) \div \sqrt{180}) = P(Z < -1.4907\ldots)$
	$= 0.068018\ldots = 0.0680$ (3 s.f.)
9 (b)	$Y \sim N(67, \sigma^2)$. 62 and 72 are symmetric about the mean 67, so
	$P(67 < Y < 72) = 0.25 \Rightarrow P(Y < 72) = 0.75$
	$\Phi^{-1}(0.75) = 0.67449\ldots$
	$\frac{72 - 67}{\sigma} = 0.67449\ldots \Rightarrow \sigma = \frac{5}{0.67449\ldots} = 7.4130\ldots$
	$\sigma = 7.41$ (3 s.f.)
10	H_0: $\mu = 0.034$, H_1: $\mu \neq 0.034$,
	where μ is the population mean level of pollutant (g per ml).
	Two-tailed test at 5%: 2.5% in each tail.
	If H_0 is true, $\bar{X} \sim N\left(0.034, \frac{0.0000409}{50}\right) = N(0.034, 0.000000818)$
	$P(\bar{X} < 0.0325) = 0.048608\ldots = 0.0486$
	0.0486 > 0.025, so the result is not significant.
	Do not reject H_0.
	There is insufficient evidence at the 5% level that the mean level of
	pollutant has changed.
11 (a)	Top left group: $k = P_{young} \div P_{senior}$ is at least about $0.11 \div 0.075$,
	so $k > 1.4$
	Bottom right group: k is at most about $0.07 \div 0.28$, so $k < 0.25$
	Ranges used: $k < 0.25$ and $k > 1.4$
11 (b)(i)	$n = 17$, 1-tail test at 1%: critical value = 0.5577
	$-0.797 < -0.5577$ ($\lvert -0.797 \rvert = 0.797 > 0.5577$),
	so the value is significant at the 1% level (reject H_0).

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11 (b)(ii) The points form two separate clusters, and within each cluster there is no clear negative correlation. The large value of r is caused by the gap between the clusters, so r is misleading and the conclusion is unreliable.

11 (c) These areas have a high proportion of people aged 65+ (about 0.25 to 0.3), about 4 or 5 times the proportion of people aged 18 to 24.

11 (d) The top left points show LAs with a high proportion of 18 to 24 year olds, so these are likely to be areas with universities (or areas where the universities could recruit students).

12 (a) $\Sigma P(X = x) = 1$: $k(1 + 2 + 3 + 4 + 5) = 1 \Rightarrow 15k = 1$

$k = \frac{1}{15}$ $\quad P(X = 3) = 3k = 3 \times \frac{1}{15} = \frac{3}{15} = 0.2$ as required

12 (b)

x	1	2	3	4	5
$P(X = x)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

12 (c)(i) $T = 7$ from (2, 5), (5, 2), (3, 4), (4, 3)

$P(T = 7) = 2 \times \left(\frac{2}{15} \times \frac{5}{15} + \frac{3}{15} \times \frac{4}{15} \right) = 2 \times \frac{22}{225} = \frac{44}{225} = 0.196$ (3 s.f.)

12 (c)(ii) $P(\text{one value is 2 and } T = 7) = P(2, 5) + P(5, 2) = 2 \times \frac{2}{15} \times \frac{5}{15} = \frac{20}{225} = \frac{4}{45}$

$P(\text{one value is 2} \mid T = 7) = \frac{P(\text{one value is 2 and } T = 7)}{P(T = 7)} = \frac{4/45}{44/225} = \frac{20}{44}$

$= \frac{5}{11}$ (= 0.455 to 3 s.f.)

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13

$X \sim B(5000, 0.26)$: mean $= 5000 \times 0.26 = 1300$,

variance $= 5000 \times 0.26 \times 0.74 = 962$; approximately $X \sim N(1300, 962)$

Estimate: $1300 - 1.96 \times \sqrt{962} = 1239.2\ldots$, so try values of n near 1239

Using $B(5000, 0.26)$:

$P(X < 1239) = P(X \leqslant 1238) = 0.023261\ldots = 0.0233 < 0.025$

$P(X < 1240) = P(X \leqslant 1239) = 0.025112\ldots = 0.0251 > 0.025$

So $P(X < n) < 0.025$ for $n \leqslant 1239$, and the largest possible value is $n = 1239$