



Oxford Cambridge and RSA

**Wednesday 5 June 2019 – Morning**

**A Level Mathematics A**

**H240/01 Pure Mathematics**

**Time allowed: 2 hours**



**You must have:**

- Printed Answer Booklet

**You may use:**

- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

**INFORMATION**

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [ ].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

## Formulae

### A Level Mathematics A (H240)

#### Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

#### Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

#### Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

#### Differentiation

| $f(x)$                   | $f'(x)$                          |
|--------------------------|----------------------------------|
| $\tan kx$                | $k \sec^2 kx$                    |
| $\sec x$                 | $\sec x \tan x$                  |
| $\cot x$                 | $-\operatorname{cosec}^2 x$      |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

#### Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

#### Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

#### Small angle approximations

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2, \quad \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

**Trigonometric identities**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

**Numerical methods**

Trapezium rule:  $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$ , where  $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving  $f(x) = 0$ :  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

**Probability**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

**Standard deviation**

$$\sqrt{\frac{\sum(x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

**The binomial distribution**

If  $X \sim B(n, p)$  then  $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ , mean of  $X$  is  $np$ , variance of  $X$  is  $np(1-p)$

**Hypothesis test for the mean of a normal distribution**

If  $X \sim N(\mu, \sigma^2)$  then  $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$  and  $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

**Percentage points of the normal distribution**

If  $Z$  has a normal distribution with mean 0 and variance 1 then, for each value of  $p$ , the table gives the value of  $z$  such that  $P(Z \leq z) = p$ .

|     |       |       |       |       |       |       |        |       |        |
|-----|-------|-------|-------|-------|-------|-------|--------|-------|--------|
| $p$ | 0.75  | 0.90  | 0.95  | 0.975 | 0.99  | 0.995 | 0.9975 | 0.999 | 0.9995 |
| $z$ | 0.674 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 2.807  | 3.090 | 3.291  |

**Kinematics**

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Answer **all** the questions.

**1 In this question you must show detailed reasoning.**

Solve the inequality  $10x^2 + x - 2 > 0$ . [4]

**2** The point  $A$  is such that the magnitude of  $\overrightarrow{OA}$  is 8 and the direction of  $\overrightarrow{OA}$  is  $240^\circ$ .

(a) (i) Show the point  $A$  on the axes provided in the Printed Answer Booklet. [1]

(ii) Find the position vector of point  $A$ .  
Give your answer in terms of  $\mathbf{i}$  and  $\mathbf{j}$ . [3]

The point  $B$  has position vector  $6\mathbf{i}$ .

(b) Find the exact area of triangle  $AOB$ . [2]

The point  $C$  is such that  $OABC$  is a parallelogram.

(c) Find the position vector of  $C$ .  
Give your answer in terms of  $\mathbf{i}$  and  $\mathbf{j}$ . [2]

**3** The function  $f$  is defined by  $f(x) = (x-3)^2 - 17$  for  $x \geq k$ , where  $k$  is a constant.

(a) Given that  $f^{-1}(x)$  exists, state the least possible value of  $k$ . [1]

(b) Evaluate  $ff(5)$ . [2]

(c) Solve the equation  $f(x) = x$ . [3]

(d) Explain why your solution to part (c) is also the solution to the equation  $f(x) = f^{-1}(x)$ . [1]

**4** Sam starts a job with an annual salary of £16 000. It is promised that the salary will go up by the same amount every year. In the second year Sam is paid £17 200.

(a) Find Sam's salary in the tenth year. [2]

(b) Find the number of complete years needed for Sam's **total** salary to first exceed £500 000. [4]

(c) Comment on how realistic this model may be in the long term. [1]

5 A curve has equation  $x^3 - 3x^2y + y^2 + 1 = 0$ .

(a) Show that  $\frac{dy}{dx} = \frac{6xy - 3x^2}{2y - 3x^2}$ . [4]

(b) Find the equation of the normal to the curve at the point (1, 2). [4]

6 Let  $f(x) = 2x^3 + 3x$ . Use differentiation from first principles to show that  $f'(x) = 6x^2 + 3$ . [6]

7 In this question you must show detailed reasoning.

A sequence  $u_1, u_2, u_3 \dots$  is defined by  $u_n = 25 \times 0.6^n$ .

Use an algebraic method to find the smallest value of  $N$  such that  $\sum_{n=1}^{\infty} u_n - \sum_{n=1}^N u_n < 10^{-4}$ . [8]

8 A cylindrical tank is initially full of water. There is a small hole at the base of the tank out of which the water leaks.

The height of water in the tank is  $x$  m at time  $t$  seconds. The rate of change of the height of water may be modelled by the assumption that it is proportional to the square root of the height of water.

When  $t = 100$ ,  $x = 0.64$  and, at this instant, the height is decreasing at a rate of  $0.0032 \text{ ms}^{-1}$ .

(a) Show that  $\frac{dx}{dt} = -0.004\sqrt{x}$ . [2]

(b) Find an expression for  $x$  in terms of  $t$ . [4]

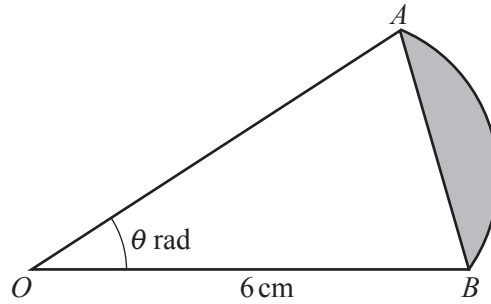
(c) Hence determine at what time, according to this model, the tank will be empty. [2]

9 (a) Express  $3 \cos 3x + 7 \sin 3x$  in the form  $R \cos(3x - \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{1}{2}\pi$ . [3]

(b) Give full details of a sequence of three transformations needed to transform the curve  $y = \cos x$  to the curve  $y = 3 \cos 3x + 7 \sin 3x$ . [4]

(c) Determine the **greatest** value of  $3 \cos 3x + 7 \sin 3x$  as  $x$  varies and give the smallest positive value of  $x$  for which it occurs. [2]

(d) Determine the **least** value of  $3 \cos 3x + 7 \sin 3x$  as  $x$  varies and give the smallest positive value of  $x$  for which it occurs. [2]



The diagram shows a sector  $AOB$  of a circle with centre  $O$  and radius 6 cm.

The angle  $AOB$  is  $\theta$  radians.

The area of the segment bounded by the chord  $AB$  and the arc  $AB$  is  $7.2 \text{ cm}^2$ .

(a) Show that  $\theta = 0.4 + \sin \theta$ . [3]

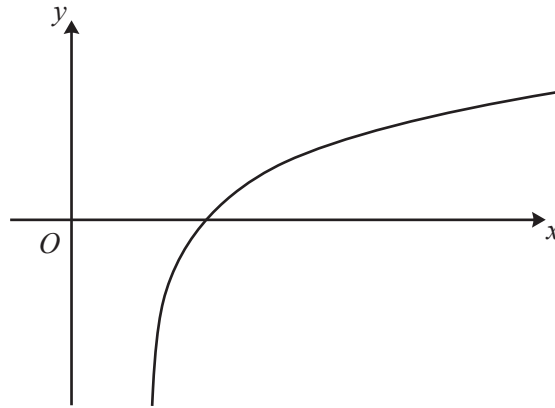
(b) Let  $F(\theta) = 0.4 + \sin \theta$ .

By considering the value of  $F'(\theta)$  where  $\theta = 1.2$ , explain why using an iterative method based on the equation in part (a) will converge to the root, assuming that 1.2 is sufficiently close to the root. [2]

(c) Use the iterative formula  $\theta_{n+1} = 0.4 + \sin \theta_n$  with a starting value of 1.2 to find the value of  $\theta$  correct to 4 significant figures. You should show the result of each iteration. [3]

(d) Use a change of sign method to show that the value of  $\theta$  found in part (c) is correct to 4 significant figures. [3]

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The diagram shows part of the curve  $y = \ln(x-4)$ .

- (a) Use integration by parts to show that  $\int \ln(x-4) dx = (x-4) \ln|x-4| - x + c$ . [5]
- (b) State the equation of the vertical asymptote to the curve  $y = \ln(x-4)$ . [1]
- (c) Find the total area enclosed by the curve  $y = \ln(x-4)$ , the  $x$ -axis and the lines  $x = 4.5$  and  $x = 7$ . Give your answer in the form  $a \ln 3 + b \ln 2 + c$  where  $a$ ,  $b$  and  $c$  are constants to be found. [4]

12 A curve has equation  $y = a^{3x^2}$ , where  $a$  is a constant greater than 1.

- (a) Show that  $\frac{dy}{dx} = 6xa^{3x^2} \ln a$ . [3]
- (b) The tangent at the point  $(1, a^3)$  passes through the point  $(\frac{1}{2}, 0)$ .

Find the value of  $a$ , giving your answer in an exact form. [4]

- (c) By considering  $\frac{d^2y}{dx^2}$  show that the curve is convex for all values of  $x$ . [5]

**END OF QUESTION PAPER**

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